AGRICULTURAL MATHEMATICS
(ANAKĀPALLE)

BY

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INTRODUCTION.

THIS book is meant for use in the Agricultural Middle School, Anakāpalle. It is written to aid a teacher of the secondary grade training to handle mathematics in a practical manner.

2. In choosing the topics for presentation the syllabus once prepared for the use of the Agricultural Middle School, Taliparamba, has been adapted with due deference to the local needs and conditions at Anakāpalle and in personal consultation with the officers of the Agricultural Department as advised by Mr. H. C. Sampson, B.Sc., ex-Director of Agriculture, Madras.

3. Even though some detailed instructions are given in the treatment of some of the topics for revision, the teacher will do well to test first if the topics have been well grasped by the pupils already and he need deal with those aspects only in detail which have not been well understood by them. Also, items like the estimation, at sight, of lengths, capacity, etc., of weight by direct handling, rough estimate of results, etc., must not be restricted to a lesson or two in connection with the sub-section of the topics concerned, but must be repeated every now and then during the course until the pupils are well up at estimating. In the treatment of fresh topics, one or more illustrations may happen

to be given and the teacher need not use all of them if he finds that one of them serves his purpose. The teacher may also freely choose variants so as to suit the occasion. A large variety of problems is given in most of the topics so that the teacher may make his own selections as would appeal to him and to his pupils in a future date. In regard to questions involving prices, the figures may have to be modified in accordance with the local rates prevalent at the time of instruction.

4. The book may find a fitting place in the libraries of the Higher Elementary, Secondary and Training schools, where education with an agricultural bias may be thought desirable to be imparted.

SAIDAPET, V. RAGHUNATHA AYYAR.
4th November 1922.

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AGRICULTURAL MATHEMATICS (ANAKĀPALLE).

TOPIC 1.—REVISION OF NOTATION AND NUMERATION.

(**Scope of the revision.**—*The pupils are expected to be already familiar with the method of expressing integers in words and in figures. During the revision some ideas may be given about the comparative bigness and smallness of the numbers used. Numbers up to a lac would suffice and those higher than ten lacs need not be presented to pupils, for it is hard to expect of them to have any conception of such numbers. It must also be seen if they are able to write down integers in figures without mistakes and also to read correctly the integers so written. Mistakes like reading or writing* 11,000 *in place of* 10,100 *may be of common occurrence among pupils and the teacher should set them right by pointing out the appropriate place-values of the digits used.*)

Big and small numbers in the decimal scale.

Ask pupils to express in words the answers to the following :—

Suppose guavas* (Jama Pandlu) from the gardens near Anakāpalle are packed in baskets containing a hundred each. How many Jama Pandlu will there be in two, four, six, nine and ten baskets respectively ?

If these baskets are carried in single bullock carts to the railway station at ten for each cart, how many Jama Pandlu will there be in three, five, eight, nine and ten carts respectively ?

If these cart-loads are emptied in railway compartments at ten for each compartment, how many Jama Pandlu will

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* Instead of guavas, sugarcane from Anakāpalle, plantain suckers from Samalkota, coconuts from Amalāpuram, limes from Pārvatipuram, etc., may also be proposed and the exercises adapted accordingly.

there be in two, five, seven, nine and ten compartments respectively?

If a wagon will hold the contents of ten compartments, how many Jama Pandlu could be sent in two, three, four, five, six, eight, nine and ten wagons respectively? How many cart-loads will a wagon hold?

Let pupils derive the following :—

A basket contains one hundred,
A cart contains ten hundred or one thousand,
A compartment holds ten thousand,
A wagon holds ten ten-thousand or a hundred thousand, i.e., a lac.

If in the course of the working the pupils are not found to be already familiar with the word "lac," point out to them that just as "ten tens" form a "hundred" and "a ten hundred" is a "thousand", so also "a hundred thousand" is called "a lac."

Next propose the following exercises :—

While selling the guavas obtained from the gardens in our taluk some are sold singly, some in tens, some in baskets containing a hundred each and sometimes these basket-loads are sent in carts and wagons as mentioned above to important markets. During a year the number of guavas sent from some gardens has been found to be as follows :—

—	Number sold or sent.					
	Wagon-loads or lacs.	Compartment loads or ten thousands.	Cart-loads or thousands.	Basketful or hundreds.	Tens.	Singles.
1. From the gardens in Thumapala.	Two.	One.	Seven.	Five.	...	...
2. From the gardens in Kasimkota.	One.	One.	Seven.	Four.	Three.	Two.
3. From S. Patrudu's gardens ...	...	...	Five.	Six.	Four.	Nine.

—	Number sold or sent.					
	Wagon-loads or lacs.	Compartment loads or ten thousands.	Cart-loads or thousands.	Basketsful or hundreds.	Tens.	Singles.
4. From Sanyasi Raju's gardens .	...	One.	Four.	Four.	Three.	Six.
5. From B. Hannumanta Rao's gardens ...	...	Two.	Five.	Six.	Seven.	Five.
6. From S. Narayanamurti's gardens ...	One.	One.	Two.	Three.	Six.	Three.

Express in words the number sent from each.

Place-value of digits.

Ask pupils to write down in words the number of guavas obtained from S. Patrudu's gardens in (3) above and point out that to write the numbers in less space the symbols from 1 to 9 (also known as *Figures* or *Digits*) are used to denote the number-words from one to nine and, by using them, the above number may be written briefly as 5,649. Let them now state what number each digit actually denotes. Hence lead them to infer that in writing in symbols it is assumed that the symbol in the extreme right denotes units, the next one to the left represents tens, the next one still to the left represents hundreds and the next one still further to the left represents thousands.

Similarly, let pupils write down in figures the number of guavas sent from each of the gardens (5) and (6) and lead them to note that the digit written to the left of the thousands' place indicates so many ten thousands and that written still to the left so many lacs and so on.

The use of cipher.

Propose :—

The guavas picked one day from Gopala Raju's gardens came to 5 basketsful and 4 single ones. Write in figures the total number picked

Point out that the number should *not be written as* 54 but should be written as 504 in order that "5" may indicate 5 hundreds instead of 5 tens. Hence show them the use of the symbol "0" (read as "*Cipher,*" "*Nought,*" "*Zero*") to indicate the absence of any number in a place and at the same time to fix the position of the figures to the left of it so that they may indicate the suitable place-values.

Ask pupils to write down in figures the number of guavas in gardens (1) to (6) above.

Marking off into periods.

Propose :—

"The number of guavas sent one year from the gardens near our farm was 2 lacs 25 thousand and 176. Express solely in figures the number sent."

If the pupils write the number as 2 2 5 1 7 6 point out that it may be written as 2,25,176. Let them read one and the same number, first when written as 448765 and then *when written as* 4,48,765. *Thereby* lead them to feel that it would facilitate accurate and rapid reading if a comma be put in immediately to the right of the lacs' place and also to the right of the thousands' place.

Propose the following exercises :—

Write in figures the numbers contained in the following statements :—

1. There are a *lac and five thousand* guava trees in our taluk.

2. Every year our district sends away ***twelve lacs and forty thousand*** bags of groundnut, ***a lac and two thousand bags*** of gingelly and ***eight lacs and two thousand candies*** of jaggery.

3. In a garden where our farm-manager would advise only "***forty thousand four hundred***" sets of sugarcane to be planted, the owner, adhering to the old custom, plants "***eighty four thousand***" sets.

4. The guava gardens near our school are valued at "***ten-thousand and one hundred***" rupees.

5. Some years ago, sugarcane cultivation was about to become extinct in the Gōdāvari district owing to disease. But the Agricultural Department opened the Samālkōta Farm and advised in time how the disaster may be overcome. Consequently the ryots in the Gōdāvari district are realizing "*two lacs and seven thousand rupees*" extra every year.

6. This year owing to timely rains, the Yellamanchili taluk may expect to get "*forty-four thousand five hundred*" bustas more of ragi than last year.

Express in words the numbers contained in the following statements :—

7. If we want to erect an oil-mill we must have a capital of Rs. 1,10,000.

8. The annual income that may be obtained from the sale of oil and cake from the mill would be Rs. 4,35,600.

9. Out of the sale-proceeds may be paid the cost of gingelly and groundnut bought to the value of Rs. 3,98,428.

10. Thus a profit of Rs. 37,172 may be expected for a year.

11. If we want to run a moderate-sized sugar factory the machinery and buildings would cost Rs. 80,000. Besides, for paying in advance to planters we must have a capital of Rs. 1,80,000.

12. The Bobbili estate pays a *peshkush* of Rs. 83,652 and land cess amounting to Rs. 32,090.

13. In which chest is there more money :—

A chest containing Rs. 1,01,000,

or

another which contains Rs. 99,999 ?

14. B. Sitarama Patrudu has Rs. 9,900 cash with him. K. Satyam has Rs. 10,100 cash. Who has more cash with him ?

TOPIC 2.—REVISION OF ADDITION.

(**Scope of the revision.**—*The pupils are expected to know addition by the method of writing figures of like place-value one below another and then adding them up. In the course of the revision a few examples of such addition may be given. But the main object should be to develop quick and accurate oral work, particularly in finding the sum of two numbers which contain two or three effective digits.*)

1. Addition possible only with quantities denoting things of the same kind.

Propose :—

1. A land-owner got 10 puttis of ganti by letting out one field and 20 puttis of ganti by letting out another. How much did he get on the whole?

Point out that the answer can be expressed as 30 puttis of ganti. Inform that no similar addition is possible in the case of the following question :—" A ryot got 10 puttis of ganti from one field and 30 puttis of jaggery from sugarcane cultivation in another. How much did he get on the whole ? " Impress that there is no meaning in adding so many puttis of ganti and so many puttis of jaggery, and that only quantities denoting things of the same kind could be added together. Point out also that the total value of the produce mentioned in the question can be expressed, only if the money value of each kind of produce is given.

2. The signs " + " and " = ".

Next introduce the sign of addition " + " and the sign of equality " = " as contained in the statement Rs. 2 + Rs. 8 = Rs. 10 to represent that Rs. 2 added to Rs. 8 gives Rs. 10.

3. Drill in addition.

Give pupils the following drill in addition :—

(a) Starting from given numbers, ask them to count forwards by 5's, 6's, 7's, 9's, 10's as in

25, 30, 35, 40	...	...	...
25, 31, 37, 43	...	...	...
35, 42, 49, 56	...	...	...
49, 58, 67, 76	...	...	...
28, 38, 48, 58	...	...	...

Do not allow pupils to say 25 plus 6 equals 31, 31 plus 6 gives 37, but direct them to give out quickly 25, 31, 37, etc.

(b) Give pairs of numbers like 65, 20; 45, 30; 75, 60; 55, 50; 65, 70; 85, 90; etc., and ask pupils to add them up quickly. Point out that in finding the sum of 65 and 20 we find the number of tens in it by adding the digits in the tens' place.

(c) Give pairs of numbers whose sum is 100 and ask pupils to find out quickly and orally their sum. Thus make them familiar with such pairs as 80, 20; 70, 30; 93, 7; 85, 15; 83, 17; 65, 35; 45, 55; 25, 75; 63, 37; 27, 73; etc., which give a total of 100.

(d) Give drill in adding a number to itself, e.g., 15 to 15, 22 to 22, 12 to 12, 24 to 24, 48 to 48, etc.

4. Adding orally, beginning with the figures of the highest place-value.

Propose :—

A cultivator bought a monsoon plough for Rs. 23 and a cart for Rs. 79. How much did he spend on the whole?

The pupils would be inclined to write the figures down on paper and add. Direct them to use the following oral method :—

79 added to 20 gives 99, 99 added to 3 gives 102, therefore 79 added to 23 gives 102.

Therefore the cultivator spent Rs. 102.

Point out that the symbol " ∴ " will hereafter be used to denote the word "therefore" which will be of frequent occurrence.

In working the above, do not allow the pupils to repeat the steps in full as appearing above, but allow them to say this much only; 79, 20, 99, 3, 102, i.e., *see that the numbers are taken in mentally.*

If the boys have a temptation to whisper, or to move their lips and work or to make signs with their fingers as

if they are writing something on the desk or on any other object, *that* should be sternly checked.

Propose :—

1. A cultivator requires seed for Rs. 65 and manure for Rs. 95. How much should he spend on the whole?

(*Ans.* Rs. 160.)

2. I got 54 puttis of ragi from my lands at Gouripala and 59 puttis of ragi from my lands at Duvvada. How many puttis of ragi did I get on the whole?

(*Ans.* 113 puttis of ragi.)

3. For watering the sugar-cane plantations in my gardens I got 2 wells (without any masonry work) dug, one costing Rs. 85 and another costing Rs. 95. How much did I spend on the whole? (*Ans.* Rs. 180.)

4. I bought a cow for Rs. 25 and a buffalo for Rs. 85. How much did they cost me on the whole? (*Ans.* Rs. 110.)

5. Gourisam realized Rs. 75 by selling the paddy and Rs. 35 by selling the straw which he got from a field. How much did he realize on the whole? (*Ans.* Rs. 110.)

6. In the course of a week a cultivator at Mulakapakka sent to a sowcar at Anakapalle 15, 18, 25, 35, 45, 25 and 15 maunds of jaggery. How many maunds of jaggery did he send on the whole?

(*Ans.* 178 maunds of jaggery.)

7. At the Thummapala market a ryot bought bullocks, a carting pair for Rs. 300 and a ploughing pair for Rs. 100. How much did he spend on the whole? (*Ans.* Rs. 400.)

In this exercise substitute in place of 300 and 100 numbers like 250, 80 ; 350, 120 ; 250, 150 ; 275, 150 ; 275, 120 ; etc., and ask pupils to perform the addition quickly and orally. Point out that in adding two numbers, e.g., 275, 150 the following method may be adopted :—

275 + 100 gives 375. 375 + 50 gives 425. Let pupils apply the above method and work additions like those involved in the following :—

8. A ryot bought a piece of land for Rs. 1,500 and another for Rs. 1,600. What amount did he give in all?

(*Ans.* Rs. 3,100.)

9. A gentleman got Rs. 216 in connection with the marriage of his first son and Rs. 816 in connection with

the marriage of his second son. How much did he thus get on the whole ? (*Ans.* Rs. 1,032.)

10. A landlord has houses worth Rs. 1,500, lands worth Rs. 7,500, live-stock worth Rs. 500, and cash to the extent of Rs. 2,500. How much is he worth on the whole ? (*Ans.* Rs. 12,000.)

11. Sanyasi Raju has 256 puttis of jonna in one granary and 164 puttis of jonna in another granary. How many puttis of jonna has he altogether in the two granaries ? (*Ans.* 420.)

In working this exercise direct pupils to adopt the following method :—

164 + (200) = 364, + (50) = 414, + (6) = 420.

Insist on the numbers enclosed within brackets being taken in mentally by the pupils.

Propose :—

12. A ryot requires 25,600 sets of sugarcane for being planted in one garden, 26,780 sets for a second garden and 24,480 sets for a third. How many sets does he require on the whole ?

Impress that the usual method of writing one number below another facilitates the addition of digits of like place-values. Ask one of the pupils to work the addition on the board. If he falters, show the following working :—

Working :	Ten thousands.	*Thousands.*	*Hundreds.*	*Tens.*	*Units.*
25,600 =	2	5	6	0	0
26,780 =	2	6	7	8	0
24,480 =	2	4	4	8	0
76,860 =	7	6	8	6	0

Point out that the total in the units' column is "0", in the tens' column it is "16" and as ten tens make one hundred, the figure 6 may be inserted in the tens' column, and the figure 1 carried to be added to the hundreds' column; the total in the hundreds' column is 18 and as ten hundreds make one thousand, the figure 8 is inserted in the hundreds' column and the figure 1 is carried to the thousands' column ;

the total in the thousands' column is 16 and as ten thousands make one ten thousand, the figure 6 is retained in the thousands' column and the figure 1 carried to the ten thousands' column; the total in the ten thousands' column is 7, and therefore, the total is 76,860.

Therefore the number of sets required by the cultivator is 76,860.

Checking the work.—Point out that if the addition in each place above is begun from the bottom and worked up to the top, the total obtained may be checked by beginning the addition from the top and working down to the bottom.

Propose:—

13. A cultivator got Rs. 556 by selling paddy obtained from his fields, Rs. 826 by selling the jaggery from his sugarcane plantations and Rs. 726 by selling turmeric. How much did he get by the sale? (*Ans*. 2,108.)

14. A ryot bought a cart for Rs. 85, a carting pair for Rs. 475, two ploughing pairs for Rs. 325, and ploughing implements for Rs. 48. How much did he spend in all? (*Ans*. Rs. 933.)

15. One year a *Bhukamanthu* spent Rs. 550 for marriages, Rs. 375 in buying cattle, Rs. 255 in digging wells, Rs. 150 for erecting sheds, Rs. 300 in cultivation and Rs. 400 for food and clothing. How much did he spend during the year? (*Ans*. Rs. 2,030.)

16. For propping sugarcane Gopal Razu requires 25,500 bamboo poles for his gardens at Mulakapakka, 18,680 poles for his gardens at Kasimkota, 25,670 poles for his gardens at Gouripala and 28,690 poles for his gardens at Thummapala. How many bamboo poles should be procured in all for purposes of propping? (*Ans*. 98,640 bamboos.)

17. A gentleman owes Rs. 2,560 to D. Suryanarayana, Rs. 2,800 to B. Hanumantha Rao and Rs. 5,700 to Narasimham Pantulu. How much does he owe on the whole? (*Ans*. Rs. 11,060.)

18. The following give the items of income to a landlord during one year:—

			Rs.
From the sale of	paddy	... ...	2,856
Do.	ragi	... ...	1,544
Do.	jonna	... ...	1,648

		RS.
From the sale of jaggery ...	...	11,560
Do. groundnut ...	...	2,568
Do. turmeric ...	...	1,264
Do. chillies ...	...	5,684
Do. onions ...	...	5,244
Do. guavas ...	...	288

Find the total income for the year. (*Ans.* Rs. 32,656.)

Substitute other numbers in place of those in the rupees' column in the above exercise and give two more exercises, if pupils betray slowness and inaccuracy in work.

TOPIC 3.—REVISION OF SUBTRACTION.

(**Scope of the revision.**—*A few examples may be given on the decomposition method with which the pupils are already familiar. Also, oral drill may be given in the subtraction of one number of two or three effective digits from another.*)

Interpretation of subtraction as finding remainder and as finding complement.

Propose :—

1. A ryot had 100 puttis of paddy in his "puri." He sells away 80 puttis. How many puttis of paddy will remain?

Here point out that we simply find how many puttis of paddy will remain, if out of 100 puttis we take away 80 puttis and we express the result as 20 puttis of paddy.

Propose :—

2. A ryot promises to supply a sowcar Rs. 500 worth of jaggery in return for some money borrowed during the "*sistu*" time, but his sugarcane cultivation yielded only Rs. 486 worth of jaggery. How many rupees worth of jaggery should he secure, in addition, to pay up his debts?

Here point out that we find the sum to be added to Rs. 486 to get Rs. 500.

The sign of subtraction.

Next introduce the sign "—" to denote subtraction as contained in the statement "120 puttis of paddy—100 puttis of paddy = 20 puttis of paddy," which simply represents that if 100 puttis of paddy be taken away from 120 puttis, the remainder would be 20 puttis of paddy.

Subtraction when possible.

By means of an example like, "Find out the balance remaining when 100 puttis of paddy are taken away from 120 puttis of paddy," point out that the subtraction of one quantity from another is possible only when they are both

of the same kind and impress the fact that there is no meaning in stating that 100 puttis of jaggery should be taken away from 120 puttis of paddy.

If in the course of working any question on subtraction the pupils should be found to attempt wrongly the subtraction of a larger number from a smaller number, point out that such subtraction cannot be done and that *only from a larger number a smaller number can be subtracted.*

Oral drill in subtraction.

(*a*) Starting from given numbers ask pupils to give out rapidly the results of counting backwards by 5's, 10's, 6's, 7's, 8's, 9's, 11's, 12's, 20's, 30's, etc., as in the following :—

158,	153,	148,	143,	138,	...	...	...
158,	148,	138,	128,	118,	...	...	...
158,	152,	146,	140,	134,	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
158,	147,	136,	125,	114,	...	...	...
158,	146,	134,	122,	110,	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...

They must not be allowed to say 158 less 5 equals 153, 153 less 5 equals 148, 148 less 5 equals 143, and so on, but they must simply say 158, 153, 148 . . . *taking in the numbers as well as the operations mentally.* Practices like those of moving the lips, muttering, folding or opening fingers, making signs with the fingers on the desk or on any part of the body should be detected and checked.

(*b*) Give exercises in orally finding the difference between pairs of numbers of the types shown below :—

Type 1. $45 - 39 = ?$ $78 - 69 = ?$

Point out that to find the value of $45 - 39$ that number should be found which when added to 39 will give 45 and that the value is 6.

Type 2. $68 - 20 = ?$; $67 - 20 = ?$;
$119 - 10 = ?$; $120 - 50 = ?$;
$704 - 200 = ?$; and $640 - 100 = ?$
(*Ans* **48, 47, 100, 70, 504, 540.**)

Point out that in subtracting 20 from 68 and finding the remainder to be 48, we actually take away the number of tens in 20 from the entire number 68 and in subtracting 200 from 704 we take away the number of hundreds in 200 from the whole number 704.

Type 3. 66 − 26 = ? ; 38 − 25 = ? ;
97 − 44 = ? ; 65 − 24 = ? ;
74 − 41 = ? ; 84 − 21 = ? ;
(*Ans.* 40, 13, 53, 41, 33, 63.)

Type 4. 100 − 28 = ? ; 100 − 52 = ? ;
100 − 36 = ? ; 100 − 37 = ? ;
100 − 27 = ? ; 100 − 63 = ? ;
100 − 54 = ?.
(*Ans.* 72, 48, 64, 63, 73, 37, 46.)

In subtracting a number like 63 from 100, point out that we perform the operation as follows :—

100 − 60 = 40. 40 − 3 = 37.

Point out that in the actual working the whole step should not be given out but only the following numbers should be repeated :—100, 60, 40, 3, 37.

Verifying the correctness of results obtained in subtraction.

Point out to pupils that in each case above, the result obtained by subtraction may be checked by addition ; for instance, if 100 — 63 be obtained to be 37, it may be seen if 63 + 37 gives 100 and if it be so the result obtained may be certified to be correct.

Type 5. *Oral method of subtracting one number of two or three effective digits from a bigger number of the same type.*

Propose :—

Boddedu Deviah offers to give Rs. 256 if I lease out my guava gardens to him while P. Apannah offers to give only Rs. 198 for the same garden. How much more would I get by leasing out the garden to Deviah ?

To find out the result, question which amount should be subtracted from which. Point out that in subtracting

198 from 256 the following oral method may be used :—

$$256 - 100 = 156,\ 156 - 90 = 66,$$
$$66 - 8 = 58.$$

Therefore I would get Rs. 58 more by leasing the garden to Boddedu Deviah.

In performing the subtraction orally, see that the pupils do not repeat the step in full as indicated above, but that they simply give out 256, 156, 66, 58 and that *the operations are taken in mentally.* In this connection point out that to find the value of 145 — 139 it is better to follow the method of finding the complement which when added to 139 gives 145, than the method specified above.

Propose the following exercises :—

1. A tenant and his landlord divided equally between themselves some quantity of jaggery obtained by sugar-cane cultivation in a field. The tenant owing to emergent need for money sold his share for Rs. 595 while the landlord waited for some time and sold his share for Rs. 640. How much more did the landlord get for the jaggery apportioned to him than the tenant for his portion ?

(*Ans.* Rs. 45.)

2. Achayya took his bullocks to a market with a view to sell them and to buy a fresh pair. A broker valued his bullocks for Rs. 286 and promised to get the pair he chose for Rs. 420. After handing over the bullocks to the broker, how much more money should Achayya give to get the pair he wanted ? (*Ans.* Rs. 184.)

3. If the old traditional method of extracting juice from sugarcane had been followed, a ryot would have got only 698 maunds of jaggery out of the cane he got from his fields. But by adopting the method demonstrated in the Government Farm he gets 726 maunds of jaggery. How many maunds more of jaggery does he get by adopting the farm method ? (*Ans.* 28 maunds.)

4. In our farm here it has been found by experiment that a field produces 419 kunchams of paddy when no manure is used and *that* the same field produces 455 kunchams of the same kind of paddy when bone-meal plus green manure is used. How many more kunchams of

paddy are obtained by using the manure instead of using no manure? (*Ans.* 36 kunchams of paddy.)

5. In cultivating sugarcane the country system is to tread down indiscriminately 25,000 sets to the acre whereas the number advised by the Agricultural Department is 10,100 sets. What will be the savings in sets per acre in carrying out the advice of the Agricultural Department? (*Ans.* 14,900 sets.)

6. I bought a pair of bullocks for Rs. 375 and sold them for Rs. 425. How much did I gain? (*Ans.* Rs. 50.)

7. Between the seller and the purchaser there is a middle man. For getting a certain quantity of jaggery he demands Rs. 1,565 from the purchaser and for having the same quantity of jaggery disposed of he offers only Rs. 1,418 to the seller. How much does the middleman get as his profit? (*Ans.* Rs. 147.)

The decomposition method in subtraction.

Propose :—

The total income obtained from the crops raised in some fields is Rs. 5,236 while the expenditure incurred in the cultivation, etc., is Rs. 2,484. What is the net profit in the cultivation?

The pupils generally know the method of decomposition. Hence let them adopt the method. Get them to write the smaller number below the larger so that the figures of like place-value appear in one and the same vertical line and work as follows :—

5,236	*Units* 6, 4, 2, put down 2 in the units' column,
2,484	*Tens* 13, 8, 5, put down 5 in the tens' column,
	Hundreds 11, 4, 7, put down 7 in the hundreds' column,
2,752	*Thousands* 4, 2, 2, put down 2 in the thousands' column.

If the pupils do not check the result, insist on their checking the work. That is, let them add, 2,752 and 2,484 and verify if 5,236 is obtained, repeating only the following :—

Units	**... 2, 4, 6.**
Tens	**... 5, 8, 13, 3 carry 1.**
Hundreds	**... 8, 12, 2, carry 1.**
Thousands	**... 3, 5.**

If the pupils are not familiar with this method of subtraction too, explain the method as follows:—

		Thousands.	Hundreds.	Tens.	Units.
5, 236	=	5	2	3	6
2, 484		2	4	8	4
	=	5	2	3	2
		2	4	8	...
	=	5	1	13	2
		2	4	8	...
	=	5	1	5	2
		2	4	...	...
	=	4	11	5	2
		2	4	...	...
	=	2	7	5	2

As the pupils generally know the decomposition method, that method alone need be followed. If they should happen to know the method of equal additions or that of complementary addition let them work according to the method they know. If the pupils are not already familiar with these methods they need not be taught these anew.

Propose :—

8. In the account book maintained by an agent for his master's lands the "*tzama*" column shows a total of Rs. 12,588 and the "*kartsu*" column shows a total of Rs. 2,864. Find the net income from the lands.

(*Ans.* Rs. 9,724.)

9. Vara Serla Chinnan Nayudu's lands were worth only Rs. 2,660 before the Nagavali channel was opened. After it was opened the same lands are valued at Rs. 30,800. Find how much Vara Serla Chinnan Nayudu Garu gains in the value of his lands by the Nagavali project.

(*Ans.* Rs. 28,140.)

10. Owing to the mango-hopper pest in Alamonda one year, the owner of a garden there found that the trees which yielded 12,600 mangoes in the preceding year yielded only 2,481 mangoes that year. Find the loss in the number of mangoes which he had.

(*Ans.* 10,119 mangoes.)

11. The maistry gave me an estimate for Rs. 6,500 for erecting a house, a granary, a cattle shed and three irrigation wells. But after finishing the work, I found that the cost actually incurred was Rs. 9,128. How much more had I to spend than what was estimated by the maistry ?
(*Ans.* Rs. 2,628.)

12. A tenant got Rs. 888 by the sale of jaggery obtained from sugarcane cultivation. Out of this he has to give away Rs. 298 to the landowner. How much money can he then keep for himself ? (*Ans.* Rs. 590.)

TOPIC 4.—REVISION OF MULTIPLICATION.

(**Scope of the revision**:—*The pupils may already be familiar with the method of multiplying one integer by another. In the course of the revision they may be directed to build up the multiplication table for 11, 12, 15, 16, 20, with which some of them may not be already familiar. A few exercises may be set on picking out a few common multiples of integers for which the multiplication table is known. Also special drill may be given in quick oral work in finding out the products of integers of one or two effective digits and a good grasp of the rationale of the process of multiplication aimed at If in written work the pupils do not already know the method of multiplication beginning with the digit of the highest place-value in the multiplier, the method need not be insisted on to be followed and the method already known to them need alone be revised.*)

Multiplication as a means of finding rapidly the sum of a given number of equal addends.

Refer pupils to an adjoining field which is divided into small beds to serve as nurseries for onions, ragi, etc. Sketch out on the blackboard a plot containing, say, seven rows with six beds in each row. Question pupils how many beds there will be in the plot. Even though the pupils answer that there will be 42 beds, question how the number 42 is arrived at. As the pupils are likely to say 7 times 6 is 42 and nothing more, point out that the number may be arrived at (1) by counting the number of beds in each plot one after another from 1 to 42, or better still, (2) by adding the number of plots, row by row, viz., by finding the sum $6+6+6+6+6+6+6$, that by such addition one gets "7 times 6 is 42" and that the easy remembrance of the equality "*6 taken 7 times is 42*" would help one to find out the same.

The use of the sign "×".

Next introduce the use of the sign "×", e.g., in 6×7 *to denote that 6 should be taken 7 times.* Let pupils compare and infer that it takes decidedly less time and space to indicate 6×7 than to write "seven times six."

The multiplication table.

Point out that to perform multiplication quickly the multiplication tables for 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16 and 20 have been built by a process of addition as indicated above and that the pupils ought to get them memorised. If the pupils are not already familiar with the multiplication table for the numbers, 11, 12, 16 and 20, direct them to construct one by addition and memorize.

The law allowing interchange of multiplicand and multiplier.

Question pupils how much twice seven comes to as well as seven times 2.

Hence point out that $7 \times 2 = 2 \times 7$

Similarly, let pupils deduce that

$$2 \times 5 = 5 \times 2$$
$$3 \times 4 = 4 \times 3$$
$$5 \times 9 = 9 \times 5$$

and generalize the law that the product obtained by multiplying one integer by another is equal to that obtained by multiplying the latter by the former. In this connexion, point out that if pupils know already $7 \times 9 = 63$ they can derive at once that $9 \times 7 = 63$. Give exercises involving the application of the above, e.g.,

from the multiplication table for 7,

$$7 \times 2 = 14$$
$$7 \times 3 = 21$$
$$7 \times 4 = 28$$
$$7 \times 5 = 35,$$

etc.,

let pupils derive

$$2 \times 7 = 14$$
$$3 \times 7 = 21$$
$$4 \times 7 = 28$$
$$5 \times 7 = 35$$

Propose exercises like the following to test the *familiarity of the pupils with the multiplication table* :—

1. In a big bundle of paddy seedlings there are 16 smaller bundles. How many smaller bundles will there be in 8 such big bundles? (*Ans.* 128 smaller bundles.)

2. A cart could carry 8 bustas of paddy at a time. How many bustas could be carried in 12 such carts at a time? (*Ans.* 96 bustas.)

3. For getting paddy seedlings enough for transplanting an acre 7 kunchams of seed are required. How much of seed will be required for transplanting 12 acres? (*Ans.* 84 kunchams.)

4. From a single sugarcane 7 sets could be cut. How many such sets could be got from 16 such canes? (*Ans.* 112 sets.)

5. At Kasimkota oranges sell at 12 for a rupee. At this rate, how many oranges could be got for Rs. 7? (*Ans.* 84 oranges.)

6. In a ridge there are 16 canes growing. How many canes may be expected at this rate from 11 such ridges? (*Ans.* 176 canes.)

7. Jaggery cakes are brought to the market in small carts at 6 cakes in a cart. How many cakes would 16 carts bring at a time? (*Ans.* 96 cakes.)

8. Find the values of each of the following:—

11 bustas of paddy at Rs. 8 a busta. (*Ans.* Rs. 88.)

12 bustas of fish-guano at Rs. 6 a bag. (*Ans.* Rs. 72.)

16 bags of castor cake at Rs. 8 a bag. (*Ans.* Rs. 128.)

11 bags of paddy seed at Rs. 10 a bag. (*Ans.* Rs. 110.)

16 bags of cotton seed at Rs. 6 a bag. (*Ans.* Rs. 96.)

9. A cow gives 4 seers of milk a day. At this rate how many seers of milk will be got from it in 16 days? (*Ans.* 64 seers.)

10. There are three "gadhis" in a landlord's house, each containing 12 puttis of paddy. How many puttis of paddy are there in them on the whole? (*Ans.* 36 puttis.)

(If in using the multiplication table to find out the results required in the above exercises, e.g., in finding the value of 7 times 9 to be 63 the pupils show any

temptation to repeat the multiplication table for 9 from the beginning before giving out the result 63, direct them to get familiarized with the multiplication table better).

Notion of multiples and common multiples.

Get pupils to repeat any line from the multiplication table for any integer, e.g., 7 times 4 is 28. Tell them that 4 multiplied by 7 gives 28 and hence 28 is called a multiple of 4. Get them to note that for a similar reason 28 may be called a multiple of 7 and thus 28 is a common multiple of 4 and 7. Referring to the multiplication tables for 4 and 7, ask them to find out whether there are other common multiples for 4 and 7, lead them to find out 56, 84, etc., as such common multiples.

Repeat the above exercise with any pair of numbers for which the multiplication tables are known, e.g., 10, 15; 8, 6; 4, 3; 6, 10; etc. Next let them find by extending the above method the common multiples of numbers like 2, 3, 4; 5, 6, 8; etc., taking three numbers at a time.

Multiplication at sight by 10, 100, 1,000, etc., and their applications.

Propose :—

1. A plot requires 8 kunchams of paddy seed to be sown on it. How much of seed will be required for 10 such plots? (*Ans.* 80 kunchams).

2. A cultivator at Gollaprole finds that if he uses 1 putti of onion seed bulbs in planting he will realize an yield of 10 puttis. What yield may be expected by using 6 puttis of seed bulbs? (*Ans.* 60 puttis.)

3. In a bunch (gala) of '*Karpura Chakkara Keli*' there are 100 plantains. How many plaintains will there be in 8 bunches; in 12 bunches; in 17 bunches?

Ans. 800 plantains; 1,200 plantains; 1,700 plantains.)

4. In a date palm a bunch contains 1,000 *Ethakaya*. There are 12 bunches in a tree. How many Ethakaya will the tree give? How many Ethakaya will there be in 7, 8, 9, etc., bunches?

(*Ans.* 12,000; 7,000; 8,000; 9,000 Ethakaya.)

Let pupils write down on the board the results of multiplication obtained in the above as follows :—

8 × 10 = 80
8 × 100 = 800
17 × 100 = 1,700
7 × 1,000 = 7,000
12 × 1,000 = 12,000
13 × 1,000 = 13,000,
etc., etc.

Hence lead them to generalize that the *results of multiplying an integer by 10, 100, 1,000, etc., would be obtained by affixing 1, 2, 3, etc., ciphers to the right of the integer.* Next give exercises of the following type :—

1. 10 × 10 = ?
10 × 1,000 = ?
10 × 100 = ?
100 × 100 = ?
100 × 1,000 = ?

2. A landlord has 15 acres of dry lands worth Rs. 500 an acre. Find how much the lands are worth on the whole. (*Ans.* Rs. 7,500.)

In working the question point out how, if 5 × 15 is known to be 75, 500 × 15 may at once be written down as 75 hundreds or 7,500.

3. A ryot digs 8 wells in his garden at Rs. 200 for each well. How much do they cost him on the whole? (*Ans.* Rs. 1,600.)

4. A cultivator has 20 acres of lands to be cultivated. At 30 maunds of castor-cake for an acre, how much of castor-cake would he require to manure the lands? (*Ans.* 600 maunds.)

5. A ryot plants 2,000 sets of sugarcane in a garden. If each set has 5 nodes (*ganupu*) and if from each *ganupu* a cane grows, how many canes may be expected from the garden? (*Ans.* 10,000 canes.)

6. From a brick-kiln 200 cartloads of bricks have been removed with each cartload containing 400 bricks. How many bricks have been removed? (*Ans.* 80,000 bricks.)

7. At 500 palmyra leaves for a cart, how many leaves can be carried in 7 carts? (*Ans.* 3,500 leaves.)

8. A wagon contains 200 bustas of rice. If 5 such wagons be cleared, how many bustas of rice would be got? (*Ans.* 1,000 bustas.)

9. A cart holds 200 bamboo poles. One day 70 such cart-loads of bamboo poles were sold in the Thummapala market. How many bamboo poles were sold on the whole? (*Ans.* 14,000 bamboos.)

10. "*Doosi Sannellu*" sells at Rs. 20 a busta. By selling 40 bustas, how much would be realized? (*Ans.* Rs. 800.)

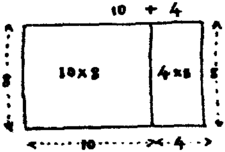
Processes leading to the general method of multiplication.

(1) Refer pupils to any casuarina or guava plantations in the neighbourhood and propose :—

In a plot there are 8 rows with 14 trees in each row. How many trees will there be in the plot?

The pupils would attempt the multiplication of 14 by 8. Point out to them the rationale of the work as follows :—

Suppose a small drainage canal passes through the plot separating the 10th and the 11th plant in each row. Ask pupils to find the number of parts into which the entire plot would be divided and the number of trees in each part.



Draw a diagram as shown in the margin. Hence lead them to infer that $14 \times 8 = (10 \times 8) + (4 \times 8)$. Thus let them find out the value of 14×8 to be $80 + 32$ or 112.

In the above exercises substitute for 14 the numbers 16, 12, 15, 17, 28, 34, 62, etc., and for 8 the numbers 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 16, 20 and ask pupils to find out the number of trees in each case. If they are not quick at the work even at the start, write on the blackboard equivalents like the following for a few questions.

$16 \times 2 = (10 \times 2) + (6 \times 2)$
$17 \times 7 = (10 \times 7) + (7 \times 7)$
$28 \times 5 = (20 \times 5) + (8 \times 5)$
$38 \times 9 = (30 \times 9) + (8 \times 9)$

and point out how in finding, for example, the value of 38×9 the value of 30×9 may be first found to be 270, that of 8×9 may be next found to be 72 and hence the total lastly found to be 342. Insist on the pupils gradually taking in the operations and the numbers *mentally*.

Propose :—

1. A candy of turmeric rhizomes costs Rs. 19. Find the cost of 8 candies. (*Ans.* 152.)

2. A putti of tobacco sells at Rs. 82. Find the cost of 9 puttis. (*Ans.* Rs. 738.)

3. A busta of maize sells at Rs. 8. At this rate find the cost of 17 bustas. (*Ans.* Rs. 136.)

4. A busta of *Akullu* sells at Rs 18. Find the cost of 8 bustas. (*Ans.* Rs. 144.)

5 Ghee sells at Rs. 14 a maund in the Vizianagram market. Find the cost of 9 maunds of ghee. (*Ans.* Rs. 126.)

6. Sunnhemp seed sells at Rs. 13 a garisa. I want 9 garisalu. How much should I pay? (*Ans.* Rs. 117.)

7. A putti of black paddy sells at Rs. 68. A landlord gets 9 puttis of such paddy from his lands. Find how much he may expect to get by selling the paddy. (*Ans.* Rs. 612.)

8. The following gives the prices of articles as sold in the Vizianagram market one Sunday :—

		RS.
1 kantlam of tamarind	...	8
,, onions...	...	9
,, jaggery	...	19
,, turmeric	...	12
a tin of groundnut oil	...	8
a maund of tobacco ...	...	8

Find the cost of each of the following calculated at the above rates :—

(*a*) 19 kantlams of tamarind (*Ans.* Rs. 152.)

(*b*) 17 kantlams of onions (*Ans.* Rs. 153.)

(c) 9 kantlams of jaggery (*Ans.* Rs. 171.)

(d) 17 kantlams of turmeric (*Ans.* Rs. 204.)

(e) 17 tins of groundnut oil (*Ans.* Rs. 186.)

(f) 27 maunds of tobacco (*Ans.* Rs. 216.)

(2) Refer pupils to a guava or casuarina plantation in the neighbourhood and propose:—

A plot contains 19 rows with 17 trees in each row. How many trees would there be in the plot?

If the pupils are inclined to write the numbers and multiply, point out that the work could be done orally by the following method:—

Ask them to assume the existence of a drainage canal separating the 10th and the 11th tree in each row. Make a rough sketch of such a plot in the blackboard and question pupils into how many parts the entire plot is divided by the drainage canal. Let them calculate that

$$17 \times 19 = 10 \times 19 \text{ plus } 7 \times 19$$

Get them next to work out that

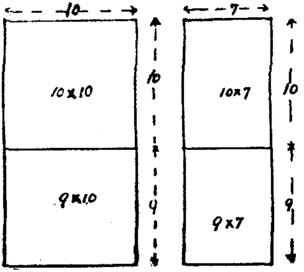
$$10 \times 19 = 10 \times 10 + 9 \times 10$$

and that

$$7 \times 19 = 10 \times 7 + 9 \times 7$$

and hence find that

$$17 \times 19 = 10 \times 10 + 9 \times 10 + 10 \times 7 + 9 \times 7$$
$$= 100 + 90 + 70 + 63 = 323.$$



The pupils may already be familiar with the above method of multiplication. Still, give exercises of the

following type in which the multiplication of integers of two digits is involved and let them work out the products orally and quickly :—

1. A busta of paddy contains 24 kunchams. How many kunchams of paddy will there be in 15 bustas? (*Ans.* 360 kunchams.)

2. A *mudikattu* contains 39 kunchams of paddy. How many kunchams of paddy will there be in 14 such mudikattus? (*Ans.* 546 kunchams.)

3. A busta of *Doosi Sannellu* sells at Rs. 19. How much will be got by selling 15 bustas? (*Ans.* Rs. 285.)

4. A cultivator got 35 barvas of jaggery from sugar-cane cultivation. If a barva sells at Rs. 65 find how much of money he would get by selling the jaggery then. (*Ans.* Rs. 2,275.)

5. Find the cost of 18 *puttis* of jaggery at Rs. 55 a putti. (*Ans.* Rs. 990.)

6. A kantlam of jaggery sells at Rs. 28. Find the cost of 24 kantlams. (*Ans.* Rs. 672.)

In the above exercises replace the numbers given by slightly larger or smaller integers, e.g., those got by adding or subtracting 1 or 2, and give pupils sufficient oral drill in multiplication.

The general method of multiplication.

(3) Propose :—

A landlord has 286 guava trees in a garden. He leases them out at a flat rate of Rs. 4 for a year. How much of money may be got by the lease?

Even though the pupils work out the result by multiplication, remind them of the rationale as follows :—

Point out that from 6 trees Rs. 6 × 4 or Rs. 24 would be got,

from 80 trees Rs. 80 × 4 or Rs. 320 would be got,

from 200 trees Rs. 200 × 4 or Rs. 800 would be got and therefore from 286 trees.

+	Rs. 24	or Rs. 1,144 would be got and that the working is generally simplified as follows :—
+	Rs. 320	
+	Rs. 800	

286	6 × 4 = 24 : put down 4, carry 2,
4	8 × 4 = 32 : + 2 ; 34 : put down 4, carry 3
1,144	2 × 4 = 8 : + 3, 11 ; put down 11.

Propose :—

From an acre of onion cultivation a ryot expects to get an income of Rs. 119. How much of income may be expected from 27 acres at the same rate ?

Question how the above may be worked and *elicit* that in working this question the product of 119 and 27 must be found and to obtain this product 119 may be multiplied by 7 and 20 respectively and the products obtained added as shown below :—

```
  119
   27
 -----
2,380 = 20 times 119
  833 =  7 times 119
 ---------------------
3,213 = 27 times 119.
```

Point out that the work used to be written briefly as

```
  119                 119
   27                  27
  ---                 ---
  238      or as      833
  833                 238
-----               -----
3,213               3,213
```

Revise the general method of multiplying two large numbers. Ask them how they would find the product 1,248 × 692 and elicit that the products 1,248 × 600, 1,248 × 90 and 1,248 × 2 may be found and added as shown below :—

1,248		
692		
748,800	=	1,248 × 600
112,320	=	1,248 × 90
2,496	=	1,248 × 2
863,616	=	1,248 × 692

or as

1,248		
692		
2496	=	1,248 × 2
11232	=	1,248 × 90
7488	=	1,248 × 600
863,616	=	1,248 × 692

If the pupils are not already accustomed to perform the multiplication beginning with the digit of the highest place-value in the multiplier, let them follow the method they are accustomed to use.

Propose the following exercises :—

1. A busta of paddy contains 23 kunchams. How many kunchams of paddy will there be in 212 bustas sent in a wagon ? (*Ans* 4,876 kunchams.)

2. A *mudikattu* contains 40 kunchams of paddy. How many kunchams of paddy can be packed in 27 such mudikattus ? (*Ans.* 1,080 kunchams.)

3. Our farm office ordered for 27 tons of fish-guano. If a ton costs Rs. 119, how much of money would the whole quantity of manure cost ? (*Ans.* Rs. 3,218.)

4. *There are 218 guava trees in Sanyasi Razu's gardens.* Each tree is capable of producing about 850 guavas. Find how many guavas may be expected from the gardens. (*Ans.* 185,300.)

5. In a *mocca jonna pothu* there are 12 varasalu of 25 grains each. There are 2 pothulu in a plant. How many grains will there be in a plant? (*Ans.* 600.)

6. In a plantain bunch (*gela*) there are 5 hands (*petalu*) and each hand contains 10 plantains. How many plantains would there be in 20 bunches? (*Ans.* 1,000.)

7. A man gets 4 seers of milk from his buffalo a day. How many seers of milk could be got at this rate in 180 days? (*Ans.* 720 seers.)

8. A man wants to buy 34 acres of land for which a flat rate of Rs. 875 an acre is demanded. Find how much of money he will have to pay. (*Ans.* Rs. 29,750.)

9. The local method of plantain cultivation in the Gōdāvari district gives an annual net income of Rs. 200 an acre while the South Indian system demonstrated by the Agricultural Department gives an annual net income of Rs. 500 per acre. The ryots in a village adopt the improved method of plantain cultivation in 350 acres of land. What may they expect to gain on the whole thereby?

(*Ans.* Rs. 1,05,000.)

10. Owing to the serious outbreak of red rot disease in sugarcane crop some years ago, sugarcane plantation was about to become extinct in the Ramachandrapuram taluk which was having 4,000 acres under sugarcane.

If by the opening of the Samalkota agricultural station the ryots of Ramachandrapuram taluk were saved from this danger by being shown new varieties immune to red-rot disease, calculate the value of profits thus secured by the above ryots at Rs. 200 per acre of sugarcane.

(*Ans.* Rs. 8,00,000.)

TOPIC 5.—REVISION OF DIVISION.

(**Scope of the revision** :—*The pupils are expected to be familiar with division. Still it may be found that some of them get bewildered and commit mistakes in working out results involving division. Hence in the course of the revision a good deal of drill should be given in the conversion and use of the multiplication table as a division table and sufficient attention should be bestowed on the pupils' guessing the figures in the quotient by means of the trial divisor. Abundant drill should be given to the pupils to secure rapidity and accuracy of work. Cases of exact division may precede those of inexact division and the pupils must be reminded of the fact that the remainder should be less than the divisor at each step in division and that the divisor × the quotient + the remainder would give the dividend. Elementary notions about a half, a fourth, a third, an eighth, should be given as tantamount to division by 2, 4, 3, 8, etc. Also, in the course of the revision, sufficient exercises may be given so that by frequent working the pupils may become familiar with the multiples of numbers, like 19, 21, 23, 24, 31, etc. In connection with exact divisibility, notions of measures, factors and common measures may be imparted*).

Division as grouping and the conversion and use of the multiplication table as a division table.

Propose an exercise like the following :—

Paddy seed has to be stored in "*Mudikattus*" at 40 kunchams a "*Mudikattu.*" How many such "*Mudikattus*" would be required if 360 kunchams have to be stored ?

Though the pupils are able to do the division point out to them that the result "9 Mudikattus" is got by using the multiplication table 40 × 9 = 360 as a division table also, namely, 360 divided by 40 gives 9. Impress that the same result would be got by taking away 40 Kunchams for each "Mudikattu" and by finding the number of "Mudikattus" required by repeated subtraction as shown below :—

360 − 40 = 320,	**320 − 40 = 280,**	**280 − 40 = 240**
240 − 40 = 200,	**200 − 40 = 160,**	**160 − 40 = 120**
120 − 40 = 80,	**80 − 40 = 40,**	**40 − 40 = 0**

Get them to note that such a process would be long, tedious and tiresome and it would be far more easy to get at the result by the process of division.

The division sign and some terminology.

Next introduce the sign " ÷ " (divided by) to denote division, e.g., in taking " 360÷40=9 " to mean 360 divided by 40 gives 9. Tell them that 360 is called the dividend as it is the number divided, 40 is called the divisor as it is the number which divides and that 9 is called the quotient or the result got by dividing.

Propose the following exercises :—

1. At Rs. 3 a bag of rice-bran, how many bags can be got for Rs. 24? (*Ans.* 8 bags.)

2. A ryot has Rs. 280 to spare to dig pitwells (*Uralu Nuyi*) in his gardens. At Rs. 40 a well, how many wells can be sunk in his gardens? (*Ans.* 7 wells.)

3. A tenant has 120 kunchams of paddy seed. By sowing 6 kunchams he expects to get seedlings enough for transplanting an acre. By sowing all the seed he has, how many acres may he expect to transplant with seedlings? (*Ans.* 20 acres.)

4. Fish-guano sells at Rs. 6 a bag. A tenant has Rs. 72 with him to spare for the manure. How many bags of fish-guano can be bought with the money he has? (*Ans.* 12 bags.)

5. A tenant has 144 bags of castor-cake. At 16 bags for an acre of sugarcane, how many acres can be treated with the manure? (*Ans.* 9 acres.)

6. A cultivator wants to buy carting pairs at Rs. 200 a pair. How many carting pairs can be bought for Rs. 600? (*Ans.* 3 pairs.)

7. A landlord pays an assessment of Rs. 96 per year for all his lands. If the assessment is at Rs. 6 per acre, find how many acres of land he has. (*Ans.* 16 acres.)

8. A Gōdāvari jaggery pan costs Rs. 60. How many such pans can a ryot put in with Rs. 300 at his disposal? (*Ans.* 5 pans.)

9. A dairyman has Rs. 100 with him. At Rs. 5 a bag of cotton seed, how many bags can be bought with the money he has? (*Ans.* 20 bags.)

10. A cart-load of hay sells at Rs. 8. How many cart-loads must a cultivator sell so as to get Rs. 128 for erecting a good thatched cattle-shed and manure pit as recommended by the Agricultural Department?

(*Ans.* 16 cart-loads.)

11. I have Rs. 180 with me. Gingelly sells at Rs. 18 a putti. How many puttis of gingelly can I buy with the money? (*Ans.* 10 puttis.)

12. A ryot requires 20,000 bricks for erecting a granary. At 400 a cartload, in how many cart-loads can he remove the bricks from the brick-kiln to the place of construction? (*Ans.* 50 cartloads.)

Division as sharing.

Propose:—

Sugarcane cultivation in a field fetched 20 barvas of jaggery one year. If this quantity be divided equally between two tenants, how much would each get?

The boys would be able to find the result to be 10 barvas. Point out to them that to get the quantity divided into 2 equal parts it should be divided by 2 and each tenant would get half of 20 barvas. Thus lead them to interpret 20 barvas ÷ 2 as half of 20 barvas, i.e., as one of the two equal parts into which 20 barvas may be divided. Lead them to deduce that 20 barvas ÷ 4 may similarly be interpreted as one of the four equal parts into which 20 barvas may be divided, i.e., one-fourth of 20 barvas.

Mark out a rectangular plot in the grounds adjoining the school and propose to pupils to divide it equally first between two brothers, next among 4 brothers, 8 brothers, etc. Ask them to mark out roughly one-third, one-fourth, one-fifth and one-tenth of the field. Thus give pupils an idea about the method of finding how much each part would be if a particular quantity be divided into a specified integral number of equal parts.

Propose exercises like the following:—

1. Five ryots bought together some tamarind logs in an auction sale. If by splitting them into pieces they got

400 bundles of fire-wood and divided the bundles equally among themselves, find how many bundles each would get. (*Ans.* 80 bundles.)

2. A "Stothrium" of 200 acres was divided into 40 equal parts and your family was given one part. How many acres did your family get? (*Ans.* 5 acres.)

3. A heap of 100 cartloads of farm-yard manure was distributed equally among 5 fields. How many cartloads would each field get? (*Ans.* 20 cartloads.)

4. A bundle of 800 plantain leaves (*Aratti Akkulu*) was divided equally among 10 houses. How many leaves would each house get? (*Ans.* 80 leaves.)

5. Thirty cultivators made a cheap co-operative purchase of 600 seers of gingelly oil for Rs. 360. If they divided the quantity of oil and the price equally, how much of oil would each family get, and how much of money would each family have to pay? (*Ans.* 20 seers and Rs. 12.)

At this stage, point out that $\frac{600}{30}$ seers means one-thirtieth ($\frac{1}{30}$) of 600 seers, and that the line "—" used in so writing stands for the division sign ($\div$).

Short division.

Propose :—At 8 seers of seed for an acre, find how many acres can be sown with 112 seers of green-gram.

Point out that the division $112 \div 8$ may be performed as follows :—

$$8\overline{)\,112}$$
$$14$$

and that in so dividing we assume that as there is only one hundred in the dividend no figure could be had in the hundreds' place in the division by 8, hence one hundred is converted into 10 tens and this with 1 ten added to it gives 11 tens; 11 tens divided by 8 gives 1 ten as quotient and leaves 3 tens as remainder; 3 tens are then converted into 30 units and this number with 2 units added gives 32 units; 32 units divided by 8 gives 4 units as quotient and thus a quotient of 10 plus 4 or 14 is got. Also lead pupils to the fact that in so dividing, 112 is replaced by 80 plus 32 and that as 8 is contained 10 times in 80 and 4 times in 32 it is inferred that it is contained 10 plus 4 or 14 times in 112.

Point out how in working the above question it is only the following written work which is done orally :—

```
8) 112 (10
    80=10 times 8.
   ---
    32 (4
    32=4 times 8.
    --
     0
    --
```

∴ the quotient is 10 plus 4 or 14.

Point out that the figure "1" in the quotient is suggested by dividing 11 by 8.

Checking the division.

After the above question is done, ask pupils to verify if 8 is contained 14 times in 112 by seeing if 14 times 8, i.e., the value of 14×8 would give 112. Similarly, let them infer that *in all the divisions* done already the correctness of the quotient may be verified by seeing if the quotient × the divisor equals the dividend.

Propose exercises like the following, choosing only those divisors for which the pupils already know the multiplication table and direct pupils to attempt them orally and check the division by testing if the divisor × the quotient = the dividend :—

1. Expecting a yield of 40 kunchams of *gingelly* (*tolakari*) from an acre, how many acres should be sown with gingelly to yield 680 kunchams on the whole?
(*Ans.* 17 acres.)

2. Each day a middle-aged coolie makes 10 baskets. How many days should such a coolie work so that he may present 1,560 baskets on the whole for sale during the market days of a year? (*Ans.* 156 days.)

3. At Rs. 12 for a bag of paddy seed how many bags must be sold to get Rs. 300? (*Ans.* 25 bags).

4. A plantain bunch (*gola*) contains 50 plantains. How many bunches should be bought with a view to get 750 plantains for a marriage? (*Ans.* 15 bunches.)

5. If a ton of fish guano costing Rs. 119 be indented for jointly by 7 tenants and divided equally among them how much would each pay? (*Ans.* Rs. 17.)

6. A cultivator got 560 pots of cured turmeric (*pasupu*) from his lands. If 40 pots of pasupu weigh a putti, how many puttis of pasupu would he be getting?
(*Ans.* 14 puttis.)

7. Messrs. P. & Co. took in lands for lease for sugarcane cultivation, paying rent at Rs. 200 for an acre. If Mr. Suryaprakasa Rao got Rs. 5,800 by leasing his lands, how many acres of lands did he lease out to Messrs. P. & Co.? (*Ans.* 29 acres.)

8. A Zemindar lets out lands for cultivation at a rent of Rs. 30 for an acre. If a tenant has only Rs. 510 at his disposal, how many acres of lands could he rent?
(*Ans.* 17 acres.)

9. A cultivator requires 25,000 bamboo poles (*vethiru*) for propping sugarcane in his lands. How many cart-loads should he buy at the market where each cart brings 200 poles? (*Ans.* 125 cart-loads.)

10. Thummapala Gopala Raju leases out his lands for rent at Rs. 40 cash for an acre. How many acres should he lease out if he wants Rs. 1,800 cash?
(*Ans.* 45 acres.)

11. Storing at 12 seers a tin, how many tins would be required to store 192 seers of oil? (*Ans.* 16 tins.)

12. A landlord tells his brother at the time of partition of the ancestral property that he may either take Rs. 20,000 cash or he would give away landed property worth an equal amount at Rs. 500 an acre. If his brother is in favour of the latter proposal, how many acres of lands does he expect? (*Ans.* 40 acres.)

Long division.

Propose:—Paddy has to be carried from the threshing ground adjoining the fields to a landlord's store-room in bustas holding 26 kunchams each. If there be 1,836 kunchams in all, how many bustasful have to be sent?

The pupils may, of their own accord, attempt this division. Point out to them that they write down the figures at each step as they cannot orally work out the result of multiplication and subtraction in as much as they have not learnt the multiplication table for 26, and that further it is not very easy to remember the comparatively

large numbers which such tables would involve and that only gradually they would become familiar with such numbers. Point out to them that the method they are using is the same as the detailed working shown under short division already.

One usual difficulty which pupils feel is about the method of finding the figures in the various places in the quotient. If any pupil is found to feel such a difficulty, bring out the fact that in the division 1,836 ÷ 26 or $\frac{1836}{26}$ the dividend lies between 10 times 26 and 100 times 26, that is, between 260 and 2,600, that hence the quotient should lie between 10 and 100 and therefore it should be a number of two places only, viz., of the units' and the tens' places. Next question them why at the first step they should take 183 tens for division by 26 instead of of 1 thousand or 18 hundreds. Point out that the division of neither 1 thousand nor 18 hundreds by 26 would give any hundreds or thousands in the quotient and as 183 tens ÷ 26 would give some tens in the quotient they try this division at the first step.

Next let them find the figure in the tens' place in the quotient by dividing 184 by 26. If the pupils are not sufficiently alert at the work, point out that as 26 is 20 and odd and as 20 is contained 9 times in 180 they may try 8, the number 9 being improbable. Thus let them try 8 tens in the quotient,

```
26) 1836 (8
    168
    ---
     156
    ---
```

and find out the remainder to be 15 tens plus 6 or 156. Similarly, as 26 is 20 and odd and as 20 is contained 7 times in 156, let them try 7 and finding it impossible next try 6 in the divisor and thus get 86 as the quotient.

```
26) 1836 (86
    168
    ---
     156
     156
    ---
```

Insist on the pupils' writing down separately that 86 bustasful of paddy would have to be sent from the threshing ground to the landlord's granary.

Next propose :—

There are 568 kunchams of paddy lying in the threshing ground. These have to be sent to a granary in *bustas* holding 21 kunchams each. How many bustas will have to be sent ?

Let pupils divide as shown below :—

```
21) 568 (27
    42
    ---
    148
    147
    ---
      1
    ---
```

Point out that the division in this case is not exact and that there is 1 kuncham remaining. Ask them to verify the correctness of the division by seeing if 27 times 21 with 1 added to the product would give 568.

If any of the pupils work the above exercise as follows :—

```
21) 568 (26
    42
    ---
    148
    126
    ---
     22
    ---
```

ask the pupils what the mistake is. Point out to them that **the remainder obtained at each stage in the process of division should be seen to be less than the divisor and that the correctness of the division at each stage may be verified from the fact that the divisor × the quotient plus the remainder should give the dividend.**

Approximate expression of the quotient.

Next question pupils how they are to express the result required in the exercise last given. If they say that

the quantity of paddy will be sent in 27 bustas and that 1 kuncham will remain, remind them that no one would leave a kuncham of paddy in the threshing ground without taking it home, that the quantity being small, only a kuncham when compared with what a busta could hold, it may at first be tried if it could be distributed in parts among the 27 bustas and if this be possible the result required in the question may be approximately expressed as 27 bustas only. If, instead of 1 kuncham, a considerably larger quantity should remain which would not allow of being distributed among the 27 bustas, e.g., a quantity like 13 kunchams, tell them that then an additional busta may be sent with only 13 kunchams put into it, and that they will thus require 28 bustas. Postpone to a later date the consideration of the principle that is conventionally followed in effecting approximations.

Change the numerical data in the above exercises; give plenty of drill in division by integers like 31, 37, 52, 47, 61, 51, etc., and see that pupils pick up rapidity and accuracy in their work.

Propose :—

1. There are 6,182 kunchams of *Doosi Sannallu* to be sent from a place to Anakapalle. If these are sent in bags each holding 22 kunchams how many bags should be sent? (*Ans.* 281 bags.)

2. A bag holds 24 kunchams of *pesalu*. If 1,152 kunchams have to be removed from the threshing floor to the market, how many such bags will be required? (*Ans.* 48 bags.)

3. A busta of *Akullu* sells at Rs. 19. How many bustas should be sold by a ryot to get Rs. 798 for purchasing 2 pairs of bullocks and 2 carts? (*Ans.* 42 bustas.)

4. When the price of Samalkota sugar was Rs. 31 a busta, Messrs. P. & Co. sold Rs. 1,612 worth of sugar in the course of a day. How many bustas of sugar did they sell on that day? (*Ans.* 52 bustas.)

5. "*Senagalu*" sells at Rs. 18 a busta. How many bustas of "*Senagalu*" should be sold to get Rs. 378 for cultivation purposes? (*Ans.* 21 bustas.)

6. "*Krishna Kattukallu*" sells at Rs. 17 a busta whereas *Akullu* sells at Rs. 19 a busta. How many bustas of *Kattukallu* may be exchanged for 34 bustas of *Akullu* at the above rate? (*Ans.* 38 bustas.)

7. At Rs. 150 a ploughing pair, how many ploughing pairs can be bought with Rs. 1,650? (*Ans.* 11 pairs.)

8. A profit of Rs. 25,680 obtained one year by a *pindi* factory was divided equally among 240 share-holders. How much will each get? (*Ans.* Rs. 107.)

9. A Co-operative society bought 2,860 bags of castor-cake to be equally distributed among 44 tenants. How many bags may each tenant take? (*Ans.* 65 bags.)

10. A busta of rice weighs 180 lb. How many bustas will be carried in a cart which can bear a weight of 750 lb. only? (*Ans.* 4 bustas.)

11. It is found that a busta of *Akullu* weighs 220 lb. and one of *Doosi Sannellu* 190 lb. A landlord lets out his 3 carts for hire. Find approximately how many bustas of each variety of rice can be carried in them, each being able to carry 745 lb.
(*Ans.* 10 bustas of Akullu; 12 bustas of Doosi Sannellu.)

12. At Rs. 725 for an acre, find approximately how many acres of lands can be secured for Rs. 16,000.
(*Ans.* 22 acres.)

13. A metalled road costs Rs. 2,560 to be opened from a high road to a village. If 58 landlords who are to be immediately benefited by the opening of the road are to share the expenditure equally, find how much per head the contribution comes to. (*Ans.* Rs. 44.)

14. A broker gets Rs. 45 as commission for every pair of bullocks he manages to get sold for over Rs. 250. If by such business in the weekly markets he earns Rs. 2,475 in the course of a year, find how many pairs he has got sold as mentioned above during the year. (*Ans.* 55 pairs.)

15. A cart could hold 746 lb. A tin of groundnut oil weighs 79 lb. Find approximately how many tins of oil can be carried in the cart. (*Ans.* 9 tins.)

16. The cartage for carrying 18 kantlams of jaggery to market was Rs. 9 and the owner sold the jaggery fo

Rs. 387. How much does a kantlam fetch him after the total cart-fare is deducted from the sale price?

(*Ans.* Rs. 21.)

17. A cultivator realizes Rs. 845 by selling jaggery at Rs. 65 per barva. Find how many barvas he sells.

(*Ans.* 13 barvas.)

18. A ryot gets about 19 kantlams of jaggery from an acre of sugarcane crop. B. Hanumanta Rao has to secure 1,600 kantlams of jaggery. Find how many acres of sugarcane he should secure so that he might get the required quantity. (*Ans.* 84 acres nearly.)

Exact divisibility, factors, and measures.

I have 160 kunchams of ragi in the threshing ground near my house. These are carried in baskets holding 2 kunchams. How many basketsful may I send and have none over?

Get pupils to suggest 3, 4, 5, 6, 7, 8, etc., and find in which of the cases an exact number of basketsful may be sent and there be none over. Point out that when there is a remainder in the division by 2 an exact number could not be sent and when there is no remainder an exact number could be sent.

Propose :—

I buy paddy seed and tie it up in bags of 5 addas each. How much of paddy seed may I buy so that I may have none over when the seed is tied in bagsful as above? Could I say that 24 addas could be so distributed?

Get pupils to note that the quantities forming the answers to these questions should measure 5 addas an exact number of times. Hence the number of addas in them should be multiples of 5. Tell them that such numbers are said to have 5 as "*a measure*" or as a "*factor*."

Let pupils suggest what may be taken as the measures of 12 kunchams, 6 addas, 20 seers, etc. Let them next find out whether the same measure would suit two of the quantities at a time, e.g., 4 addas as measuring out both 16 addas and 20 addas without remainder. Tell them that in those cases "4 addas" is said to be a **common**

measure of 16 addas and 20 addas or 4 is said to be a **common factor** of 16 and 20. Get pupils to find the common measures of :—

12 kunchams,	16 kunchams ;
25 addas,	30 addas,
40 puttis,	50 puttis ; and
6 visams,	8 visams ;

Let them then write down the common factors of 25, 40 ; 40, 50 ; 35, 49 ; 6, 7 ; 3, 6 ; 4, 6 ; 15, 20 ; etc.

Get pupils to express each of the numbers above given as the product of a series of the smallest integers possible, e.g., expressing 45 as 5 × 3 × 3. Show them how this may be done by division as follows, provided they are not able to guess the factors. Get them to realize that the free use of the multiplication table as a division table would help them in the matter.

```
5) 45
  3) 9
     3
```

Repeat the above exercises with a series of other integers. Get them incidentally to note that numbers like 7 and 12 have no common factors other than unity.

TOPIC 6.—SOME FUNDAMENTAL GEOMETRIC NOTIONS.

(**Scope of the treatment.**—*Elementary space notions may at first be given, for example, each object occupies a portion of the vast space around us; objects are classified in geometry according to resemblance in form irrespective of difference in colour, material, weight, etc. Ideas about the nature of surfaces, edges and corners may next be imparted as well as instruction in the methods of detecting curvature in edges and faces apparently not curved. The physical representation of lines and points may then be taken up and the property that the straight line joining two points represents the shortest distance between them may be well impressed. The imparting of notions of the type that a surface has* no *thickness, that a line has* no *breadth, that a point has* no *length, breadth nor thickness, etc., need not be taken up for direct, teaching in a school of this type and at this stage.)*

Solids.

1. Ask pupils whether within a particular "*Gadhi, Puri* or *Kottu*" they could store all the paddy, ragi or ganti which could be got from the neighbouring fields. Bring out the idea that the space within the "*Gadhi, Puri* or *Kottu*" is limited and any quantity of corn thrown into the "*Gadhi, Puri* or *Kottu*" occupies a certain amount of space and it could not allow any other quantity of corn to occupy the same space unless it is removed. Question them as to where and how people store large quantities of jaggery, groundnut, etc., when they are brought for sale in the market. Refer them to the heaps made in the open yard and point out that, while the space within a "*Gadhi, Puri* or even a *Kottu*" is quite limited and can therefore hold only limited quantities, the space outside is vast and unlimited.

2. Place before the pupils half a dozen cuboids of equal as well as of varying sizes, half a dozen similar cylinders, spheres and cones mingled together and ask them to select from them and group together what they

consider to be of the same shape. (If there be no supply of models or of a sufficient number of them, clay models may be made by the teacher for the purpose.) After they have so classified, question on what basis they form the groups and elicit simple answers, e.g., tapering to a point and circular face for cones, roundness for spherical bodies, ruler-like forms for cylinders, etc.

Next let them compare the several solids, even of one and the same group, e.g., some cylinders of different sizes. *Elicit that though they are of the same shape they may* differ in size. Let them draw similar inferences about the *cuboids*, spheres, *cones, etc.*

Place before pupils cylinders differing in colour, weight, material, etc. Let pupils handle them and express in what respects the solids differ. Lead them to note the difference in colour, weight, material, size, etc. Tell them that these solids are grouped together and named cylinders because of their resemblance in shape. *Ask* pupils to mention names of objects which are cylindrical in form and with which they are familiar, e.g., *gadhi* for storing grain, cart-wheels, stone-rollers, pestle, coconut and palmyra stems, sugarcane sets, drums, tumblers and other utensils, logs of wood, pillars, wells, pots for boiling cane-juice, etc.

Repeat this exercise with different cuboids, cones, spheres, etc., and let pupils derive that the basis of classification is upon resemblance in shape and not upon anything else.

Tell pupils that the solids exhibited before them are not the only types that exist in nature, that there are several other types too, e.g., those like mangoes, eggs, jaggery cakes, baskets, etc., and all that they need remember about them is that they require some space to occupy and hence have a definite size, shape and position.

Surfaces.

Place some solids not necessarily the geometrical models and ask pupils to place their hands upon and show one of them. Tell them that what they show is only the outer portion of the solid and that it is called the *surface* of the solid.

Ask pupils to place one solid, e.g., a cube over another, e.g., a cuboid. Question which parts of these solids are in

contact with each other. Impress that when the objects are placed one above the other, their surfaces come into contact.

Next refer the pupils to the geometric models, to a clod of earth, to the trunk of a tree, to fruits like mangoes, plantains, guavas, vegetables like elephant yam, colocasia, sema, pendalam, kanda, anappakaya, pirikaya. Ask them to mention the differences they notice between the surface of a mango fruit and that of an elephant yam, between the surface of an egg and that of colocasia, etc. Bring out the fact that some solids have smooth surfaces, while others have rugged ones.

Place before pupils a "*Billa Penku*" (a flat tile) and also a *Netha Penku* (a curved tile) and let them compare the two and specify the difference in their surfaces. Elicit the fact that a large portion of the surface of the "*Netha Penku*' is curved, whereas the surface of a "*Billa Penku*" is flat. Next let them compare the surface of a cuboid with that of a sphere or of an egg or of an anappakaya or a papaya fruit. Lead them to infer that the faces of the cuboid are flat whereas the other solids mentioned above have a curved surface.

Ask pupils to show which faces in a cylinder are flat and which are curved. Let them note that in a cube and a cuboid all the faces are flat, in a cylinder two faces are flat and one is curved, in a sphere the surface is wholly curved, in a cone one face is flat and the other is curved, etc.

Edges and corners.

Let the class show the faces of a cuboid which meet and the place where they meet. Tell the class that the meeting place is called '*an edge*.' Ask pupils to point out all the edges in a cuboid, cube, prism, cylinder and cone. Elicit from them that a sphere has no edge, that the edges in a cube and a cuboid are straight, those in a cylinder are curved; a cone has only one edge and that, a curved edge.

Next let them show such two edges of a cube as meet. Intimate to them that the place where the edges meet is called '*a corner*.' Make them point out all the corners of a cuboid, cube, cone and elicit that there is no corner in a sphere or a cylinder.

Detecting curvature in edges that are apparently straight.

Let pupils examine what sort of edges the black-board, the desk, the doorway, beams, rafter, etc., have and question them how the carpenter makes them straight. (A visit to a carpenter's shop may be useful in this connection.) Point out that he holds a string or a piece of thread tight between the two ends of an edge to test whether the edge falls in with the string right through and that he concludes that there is curvature in the edge somewhere, in case the string does not coincide with the edge. Refer also to the fact how he may attempt to sight one end of the edge from the other end and examine if every part of the edge falls along the line of his sight. In this connexion show pupils how the mason attaches tight strings to pegs fixed in walls, how the carpenter stretches tight a string covered over with chalk dust along the length of wooden logs with a view to shaping them to have straight edges, how the sawer of wood holds a string tight to get straight planing, how the gardener in laying out the nursery beds makes their ridges straight with ropes held tight.

Test of a plane.

Talk to pupils as to how a carpenter planes surfaces of doors, planks, etc. Question how he tests that the plank he is planing has been well planed. Mention how the carpenter places a straight edge or a flat stick over the plank in various positions and in various directions and examines if any light is visible between the straight edge or flat stick and the surface he is attempting to plane. Tell pupils that he assumes that the plank is well planed or not, according as the light is invisible or not. Refer also as to how in plastering a wall a mason manages to get a smooth plane in the outer surface by moving a flat stick right over it in different directions.

Lines and points.

Ask pupils to represent on the surface of the floor in the school-room or in the yard in front how much of floor space would be covered by a plank or a a Cuddapah slab. Lead pupils to note that the space may be marked out by placing the plank or slab on the floor and

drawing lines along the edges. Point out that the lines thus drawn mark out on the floor that portion of its space which is occupied by the plank or slab in question from the adjoining portion.

Propose to pupils that they should *roughly* mark out on paper or on sand a sketch of the field under Cambodia cotton. After they do it roughly with pencil on paper or with their fingers on sand, point out to them that to make the sketch they have only drawn lines and that the lines so drawn mark out a particular portion of the surface of the paper or of the floor as representing the field and thus distinguish it from the remaining part of the surface.

Next, ask pupils to draw, on the blackboard or on paper, lines, using straight and curved edges. Point out that every one of the lines drawn marks out the portions of the board surface lying on either side of it. Lead pupils to infer that the lines drawn with a straight edge are straight and those drawn with a curved edge are curved. Impress on them that the lines they draw with pencil on paper should be very fine and they should hence maintain a sharp point for their pencil or a chisel edge. Point out in this connection how the carpenter draws lines on planks with a chisel and a straight edge and how the pupils draw lines on planks in the Manual Training Class. Refer also to the fact that it is in place of lines they have ridges, etc., separating one plot from another in fields.

Show a map of the country and ask pupils to point out definitely on the map the place where they live. If they show it with a stick, point out that the end of the stick occupies quite a considerable space on the map surface and that by using a pin they may mark the spot more accurately.

Next, show on the map a line representing a road and point out a place separating the road into two, e.g., so far is Bengal-Nagpur Railway, so far is Madras and Southern Maharatta Railway, or so far is metalled and so far is not metalled.

Point out that, when the line is to be divided into two parts, what shows the dividing spot is a point. Let them infer that any number of points could be marked along a line, that the ends of lines drawn are also points, that the ends of edges in the solid are also points in the above

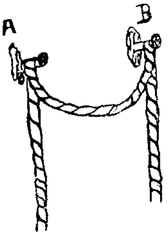
sense, and that to mark out parts of a line they only mark points in it. Point out that for the accurate representation of a point, a cross-mark (×) is serviceable, the place of intersection in the cross marking off the point.

Propose the following exercises :—

1. Mark a point on paper. Draw a straight line to pass through it. Draw another straight line, a third, a fourth, etc. How many straight lines could be drawn through it?

2. Mark two points on paper. Use a straight edge and draw a straight line so as to pass through the two points. Try if you could draw quite another different straight line to pass through the two points.

3. Drive two nails A and B on a wall. Place a string so as to pass over the nails A and B in different positions. Compare the lengths of the parts of the string lying between A and B in those positions. In what position is the length of the part shortest?



4. Draw two straight lines on paper appearing to be equal. Use the dividers or a string or tracing paper and find if they are exactly equal.

While the above exercises are being done, tell pupils that it is convenient to indicate points by means of letters of the alphabet like A, B, etc. and that if this is done the lines joining such points can be referred to by the letters indicating the points, e.g., AB, XY, ZW, etc. Insist on the fact that such letters are merely a convenience for the sake of reference and that AB is not a line but only indicates a line. Point out that the lengths of lines are compared by tracing and superposition or by using the dividers. Also get them to note that *only a single straight line can be drawn so as to pass through two points and the shortest distance between two points is along the straight line joining them.*

TOPIC 7.—MEASUREMENT OF LENGTH.

(**Points to be dealt with.**—*The necessity for the adoption of a fixed length like the yard as a standard for measurement. Sub-multiples of the yard—the foot and the inch in measuring smaller lengths. Measurement, with foot-rule, yard-stick or tape of the dimensions of buildings, girths of logs, heights of doors, etc. Approximate expression of lengths in terms of an appropriate unit. Rapid estimation at sight of lengths and distances to the nearest unit. Estimation of lengths by stepping or striding. Larger units of length—the chain, the furlong, the mile. Measurement with chain of lengths on level ground. The tables of linear measure—British and local.*)

Necessity for the adoption of the yard as a standard length in measurement.

Question pupils as to how the dimensions of a cloth are expressed when the people buy or sell it in the market. Point out that they express the dimensions as so many *muras* long and so many *muras* wide. Next refer to how smaller lengths like the width of a plank are measured and expressed in "*jans*" and finger-breadths and how larger lengths like the length of a rope and that of the mud-wall raised in the rear-yard are expressed in *muras* and *baras*.

Distribute pupils in four batches, each batch containing tall as well as short pupils. Set up each member of one batch to measure the length of a desk or a table or a board in "*jans*" using the length of his span, each member of the second batch to measure a particular length of cloth in *muras* using the length of his cubit, each member of the third batch to measure very carefully the width of the class room in feet as measured with his foot and each member of the fourth batch to measure very carefully in strides or steps the distance of a particular tree or well from the farm-gate. Remind pupils of the fact that when they measure they ought to measure along a straight line and that otherwise they would be giving incorrect measurements. Batch after batch, direct the members to write out the results of the measurement on the blackboard, say,

under the headings, Batch I, Batch II, Batch III and Batch IV and question whether the members of the same batch get the same result and why they do not all get the same result though they measure the same distance.

Ask a member of Batch I to place the length of his span along that of another member's and compare the lengths of the two spans. Thus point out that the length of the span varies with different members. Let pupils next compare their measurement with the cubit, foot or stride in a similar manner and find out that the length of the cubit, foot or stride varies with individuals. Impress the fact that as the members of each batch used a different length for measurement, their results did not agree.

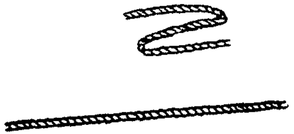
Propose :—Appayya desired to dig pits 40 feet apart for planting mango grafts in his gardens, using his foot as the standard of measurement. But when the servant dug the pits the latter made the measurements with his own foot. Would the distance be the same as that which Appayya wanted ? When would the distance be the same and when would it not be so ?

Point out that unless the servant knows exactly what the length of Appayya's foot is, he would be unable to carry out the order exactly. Hence intimate to the pupils the desirability of fixing some particular length as a standard to be adopted by every one so that lengths and distances expressed in terms of the unit by any one would be correctly understood by any other. Show the pupils a yard-stick and tell them that it has been prescribed as such a unit for the measurement of length by the Government and that, therefore, people adopt it as the unit of measurement. Ask pupils to mark places in the farm-yard which are 2 yards apart, 3 yards apart, 4 yards apart, etc. Give exercises to pupils in measuring the length and breadth of the class-room, the school verandah, etc., in yards with a yard-stick. If in the course of such measurements the pupils ask what they ought to do with the extra lengths they get which are smaller than a yard, inform them that these lengths may be left out of account for the present and that they will soon learn how to measure such lengths.

The sub-multiples of the yard.

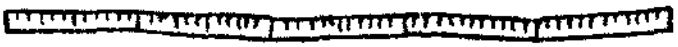
Draw lines on the floor smaller than a yard and also point out to pupils lengths shorter than a yard like

the width of a window, of a board, of a plank, of a table, etc. Let them attempt to measure their lengths in yards and find out that a yard is very long for measuring such lengths and a length shorter than a yard is, therefore, convenient and desirable. Tell them that with a view to help them measure such lengths a yard is sub-divided into three equal parts each of which is called a foot. Let each pupil cut out a stick a yard in length and mark in it three foot-lengths by taking a thread as long as the stick and folding it as shown in the accompanying figure to get three equal parts. Direct pupils to cut a *foot-length* from a stick and measure with it the lengths of the desk, blackboard, table, doorway, window, etc. Now ask them to handle a foot-rule and in measuring with it caution them not to assume that the entire length of the foot-rule is one foot. Let them note with care that it is the distance between two marks found on it which is one foot in length. If, while measuring, pupils get extra lengths shorter than a foot and ask what they ought to do with them, tell them that they may let them alone at present and that they would soon learn how to measure such lengths.



Next, point out to them that, to enable them to measure lengths shorter than a foot, a foot is further sub-divided into 12 equal parts called inches. Direct them to graduate their foot-stick into inches with a graduated foot-rule. After marking out certain points on the board, ask them to show how far from them it is 4 inches, etc. Let them next measure in feet and in inches the width of a door-way, the height of a pillar, the length of a cane, the distance between two plants in a field, in feet and in inches. Intimate to the pupils that when they have to record distances in feet and in inches it is usual to use special signs to denote feet and inches, for example, a distance like 10 feet 9 inches is written as 10′ 9″, the sign (′) being used to denote *feet* and the sign (″) to denote *inches*. If the pupils do not measure along a straight line, remind them that when the distance between two places is to be found, it is the shortest distance which is meant, that such dis-

tance is along the straight line passing through the two places and hence that if they place their foot-rule as shown in the figure below and measure, they would be making mistakes.

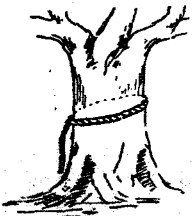


The use of the tape in measuring.

Ask pupils to measure in feet the length and breadth of the store-room and of the threshing floor in the farm. Point out to them that measuring with the foot-rule is tiresome, and that they may therefore conveniently use the tape of 50 feet. Insist on their always holding the tape tight and never allowing it to assume a curvilinear posi-

tion as shown in the above figure and that, if in placing the tape they find it occupying such a position, direct them to hold the further end high, give it a swing from one end and stretch it forth along the ground. Point out to pupils where exactly the first inch length begins in the tape.

Next, propose to pupils that they must measure the girth of a tree or of a pillar. First, let them try to measure it by means of a foot-rule and note for themselves that a tape is more convenient. If they measure in a wrong way, e.g., by placing the tape round the tree or pillar as shown in the accompanying figure point out to them how they ought to put the tape right. In this connection point out also how the length of a curved line or edge can be measured by placing the tape along it and that the measurement of such a line with a straight edge of a foot-rule is highly inconvenient.



Approximation in measurement.

Set different pairs of pupils to measure with a foot-rule or tape the length of the school-building or its breadth, the length of the store-room or that of the threshing floor in the farm. Let different pairs measure the same distance and record the results. If there is variation as there is likely to be, question why they do not obtain the same result and lead them to infer that there must have been errors in measurement. Bring out the fact that the measurement expressed by one pair, say, if it be something like 31 feet 2 inches, cannot be depended on as quite accurate. Elicit from them that the measurement is liable to be incorrect owing to the wrong placing of the foot-rule or of the tape as has already been shown, that the tape may not be measuring out exactly the lengths which its readings indicate if there is a sag in the middle or if its ends are worn out, and so on. Thus remind them of the various errors in measurement.

Next refer them to the difficulties they met with in the way of exact measurement. Recall to their minds the questions that they asked in regard to what they should do, with the extra length smaller than a yard which they got when they used the yard measure, with that smaller than a foot when using the foot-stick, and with that smaller than an inch when measuring out lengths in inches. Point out to them that the unit length chosen as the basis for measurement need not be contained an exact number of times in all the lengths required to be measured and hence however small the unit is chosen a fraction of it may still remain which cannot be exactly measured out. Thus impress on them that an absolutely exact measurement is something that is impossible. Lead them hence to note that a measurement of length expressed to have been obtained as 31 feet 2 inches is subject to errors characteristic of each person engaged in measuring and that if one states that he has quite accurately measured the length to be 31 feet 2 inches it cannot be conceded that he has measured with absolute accuracy without any mistake even in the inches' column. Tell them that in such cases it is enough if the measurements are expressed approximately in feet as 31 feet, a length of 2 inches being considered very small and negligible when compared with a length expressed in feet as 31 feet. Tell them also that, in so

expressing, the error of neglecting 2 inches may be overlooked as it does not seriously affect the measurement given out and as a measurement like 31 feet 2 inches to the exact inch is improbable and hence unreliable. In this connexion point out to them that in expressing approximately in feet a length of 31 feet 2 inches it is possible to express it either as 31 feet or as 32 feet, that the error committed is 2 inches if it be expressed as 31 feet and 10 inches if it be expressed as 32 feet, that as the error is less in the former case the length may be approximately expressed as 31 feet and that it is therefore usual to say that the length to the nearest foot is 31 feet.

Next propose to them that a boy says he gets a length to be 52 feet 11 inches on measuring. Ask them what the error comes to in assuming the distance as 52 feet and as 53 feet, respectively. Point out that the error is less in the latter assumption than in the former and that they may express the result to the nearest foot as 53 feet.

Next inform them that in so effecting approximations the following rule is conveniently adopted :—

"With reference to any unit of length, e.g., the foot in the above case, count as '1' parts of it equal to, or greater than, half, but neglect parts less than half."

Propose to them to express to the nearest foot a length of 100 feet 6 inches. Point out to them that arguing as before, the error would be the same if the length be expressed as 100 feet or as 101 feet, but still the convention may be adopted and the length expressed as 101 feet to the nearest foot. Also, tell pupils that, in expressing distances approximately the units chosen for approximate expression vary with the length of the distance. Point out to them that a length like an inch or two may be considered negligible in measuring the dimensions of a plot of ground but in measuring a precious article like gold thread it is not so. Also, inform them that the distance from the farm office to the remotest well in the farm may be expressed to the nearest yard and not to the nearest inch, the length of the threshing floor to the nearest foot, that of the blackboard or table to the nearest inch and so on.

Propose :—

1. Express 100 feet 5 inches, 114 feet 8 inches, 117 feet 7 inches, 119 feet 2 inches to the nearest foot.

(*Ans.* 100 feet, 115 feet, 118 feet and 119 feet.)

2. Express 15 yards 2 feet, 17 yards 1 foot, 24 yards 2 feet, 14 yards 1 foot, 16 yards 2 inches, 14 yards 1 foot 2 inches, 24 yards 1 foot, and 24 yards 2 feet, to the nearest yard.

(*Ans.* 16 yards, 17 yards, 25 yards. 14 yards, 16 yards, 14 yards, 24 yards, 25 yards.)

3. Measure the length and breadth of a field, e.g., under sugarcane or sweet potatoes to the nearest yard.

4. Measure, to the nearest foot, the breadth of a school-room, the length of a log cut out, the distance between two trees, etc.

Rapid estimation at sight of lengths in feet.

Let each pupil measure different lengths in feet and remember what length in space would come to 12 feet—the breadth of a room or the distance between two places in his house with which he is familiar. Mark out places with different distances apart and let pupils estimate the distance in feet. In making such estimates, tell pupils that they may step off mentally a length of 12 feet which they remember along the distances marked out and thereby get at the probable lengths. After the distances are estimated let them record their results and verify the correctness by actual measurement with a foot-rule or a tape. If the pupils show signs of diffidence, encourage them and tell them that it is only by such repeated measuring and checking that they will be able to estimate fairly approximately the distances. (The teacher should have also had practice in such exercises and be able to rapidly estimate *at sight* distances marked out ; otherwise he may not be in a position to decide rapidly whether a pupil's estimate is probable or far from probable. If he has not had sufficient practice previously, he will do well to record his estimates simultaneously with the pupils and should not feel shy to get himself corrected along with his pupils.)

Estimation of lengths by stepping or striding.

Lay out lengths of 29 yards, 30 yards, 40 yards, etc., along a straight foot-path, field-bund or cart-track. Ask pupils to walk with uniform pace along the lengths and find out how many of their steps would cover the distance. Thus, let them find, for instance, how many

of their steps would cover 10 yards and, remembering the equivalent, find approximately the lengths of bunds, fields, etc., by walking with uniform pace along them.

Longer measures.

Speak to the pupils about measuring the distance from the farm to Mulagapaka or Thummapala. Point out that the measurement in feet with a foot-rule or in yards with a yard-stick would be tiresome and inconvenient and even if such measurement be undertaken the distance will have to be expressed by big numbers as in 74,928 feet, 10,568 feet, etc. Thus lead them to infer about the necessity for the adoption of larger units in terms of which those distances can be expressed. Take pupils along the adjoining road. Show them the furlong stones and the mile-stones. Tell them that in expressing larger lengths, e.g., the length of a road or the distance between two remote villages, the mile and the furlong are used as convenient units. When the pupils go on an excursion to visit the Thummapala market, ask them to note how many furlongs they travel before they go from one mile-stone to the next. Ask them also to walk from one furlong stone to the next adjusting their paces equal to about one yard so that they may easily remember that

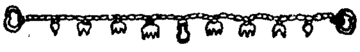
1 *furlong equals* 220 *yards.*
1 *mile equals* 8 *furlongs.*

Examination of a Gunter's chain and its parts.

Propose to pupils that the dimensions of the sugarcane plantations in the farm should be measured. If the pupils attempt to measure with a foot-rule or a yard-stick, point out that the measurement would be tiresome and inconvenient. If they suggest that a tape of 50 feet may be used, point out that it will very soon get soiled and unreadable. Tell them that in such cases a chain made of strong iron or steel wire is used. Show pupils a Gunter's chain of 22 yards, ask them to measure its length with a yard-stick and lead them to infer that it contains 22 yards or 66 feet. Tell pupils that the karnams use the above chain for field measurement.

Next refer to the fact that when the karnam uses the chain for field measurement he cannot always expect to get

an exact number of chains as the result of his measurement and that he does not carry a foot-rule, etc., for measuring smaller lengths. Let pupils examine a chain and note that it contains a series of pieces of wire with one or more small rings attached. Show pupils a link and ask them to count the number of links beginning from the tip of one of the two brass handles of the chain. Point out that, at distances of ten links from the tip of each brass handle, brass discs are attached along the chain, that thus there are 100 links in a chain and that lengths smaller than a chain are measured in links.



Let pupils examine the chain, note that the brass discs are attached at the end of every tenth link in the chain, the first disc from either end of the chain has a single point, that at the end of the twentieth link has two points, that at the end of the thirtieth link has three points, that at the end of the fortieth link has four points and that at the end of the fiftieth link, i.e., at the middle of the chain has a round form. Point out that the existence of these discs would enable them to give out the readings in links quickly if they note the discs and the number of extra links. Give exercises to pupils in reading the number of links from each end up to particular spots marked out. If the pupils make mistakes, e.g., in reading a length of 70 links to be 30 links by looking at the disc with three points, point out that they have passed the round disc in the middle, that the length they show is greater than half a chain, that hence their reading is wrong and that the correct reading is 70 links. Point out also that in measuring with a chain the brass handle at each end is included in the length of the link at each extremity. Tell them also that in case they get parts of a link extra in the measurement it is enough if they express the result of the measurement to the nearest link. Point out that as the chain used to get tangled, its length may be tested before measurement by means of a six-foot pole. (Take the off-set pole for the purpose) and that shortage of length may be made up by the addition of a requisite number of rings and excess of length cut short by the removal of

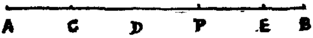
the necessary number of rings. Inform them that they should carefully scrutinize the defects in the chain used if on account of measurements given by it they are ever accused of *encroachments, etc.*

Measurement of distances on level ground with a Gunter's chain.

(i) Choose two guava trees near the school less than a chain apart and ask pupils to measure with a chain the distance between them. In stretching the chain, direct pupils to see that no knots are formed in the middle and that the chain is straight. Pointing to the brass discs, ask them to read rapidly the number of links from the brass handle at one end up to the disc. Set right pupils if they do not read correctly the number of links by a mere look at the points of the brass discs. Give some more examples of the above type.

(ii) Next point out two places more than a chain apart, e.g., two trees along a level road, two corners of a field, etc., and propose that the distance between them should be measured. Let pupils show roughly the line along which they must measure. Question them why they should measure along a straight line.

Suppose A and B are the extremities of the line.



Direct pupils to keep a flag over these points. Ask them to choose one of the two ends wherefrom they may begin to measure. If they choose the end A, tell them that A is then called the *starting point*. Tell them also that as the measurement should close at the other end B, B is called the *closing point*. Show pupils the ten arrows supplied along with the chain. Let them examine them and note that the ring at one end of each arrow would serve as a good handle; that the other end is pointed so as to allow of the arrow being easily thrust into soft ground or of cross-marks being drawn on hard ground to represent points. Tell them that the arrows would be used for counting the number of chain-lengths in a line. Ask two boys to hold the two extreme brass handles in a chain. Let one of them place the handle he holds exactly over A. Let the other go forward with the arrows and, holding the chain straight, stand roughly in a line with the flags at

A and B and *turn* towards his comrade at A. Tell them that the pupil dragging the chain forward will hereafter be called the *leader* and the other will be called the *follower*. Let the pupil standing by the side of the flag at A (*the follower*) caution the holder of the other end of the chain (*the leader*) to move to the right or to the left until he is in the straight line AB. Let both the pupils place the chain straight on the ground. At C, the end of the chain remote from A, let an arrow be stuck. Let the leader drag the chain forward without disturbing the position of the arrow. Let the other follow and come to C, place his handle at C and caution the leader to move to the right or to the left until he is in the straight line AB. Let them both place the chain straight on the ground. At the end of the chain remote from C let an arrow be stuck.

Let the leader move forward with the chain, the follower pick up the arrow at C and let them as before set up a chain length. In this way, let arrows be stuck at distances of one chain between A and B along the straight line AB, until at last a portion may be left like E B, which is less than a chain in length. Let the leader proceed to B and the follower come to E and both measure in links the distance EB. Inform the pupils that the arrows the follower will have picked up as soon as the leader proceeds, that is to say, the arrows that the leader stuck at C, D,.........E gives the number of chain-lengths and the distance EB gives the extra links in the distance between A and B. Point out that in some cases the distance to be measured may contain an exact number of chains and then an extra length like EB would not exist and hence there will be no length to be measured in links. Point out that if there are 5 chains and 28 links in the distance AB it is usual to express the distance only in links as 528 links.

Propose the following exercises :—

1. Express in links the total distance between A and B if the arrow was stuck 7 times and the distance between the last arrow and B was 85 links. (*Ans.* 785 links.)

2. Somewhere between A and B, place another flag so as to be collinear with A and B.

3. Starting from a spot A look steadily at an intermediate spot C in the line AB and practise walking in a straight line, that is, by always keeping yourself in a straight line with A and C.

4. By means of a chain, measure the distance between two furlong stones on the road and find how many chains there are in a furlong.

5. Measure with a chain the dimensions of the fields under sweet potatoes, *chodi, ganti, pasupu,* and *cheruku* in the farm.

6. Measure with a chain a distance of two chains along a road. Walk with uniform pace in a natural way, i.e., with pace neither too fast nor too slow along the distance and find how many of your strides would cover a chain.

7. With the help of the equivalent obtained in Ex. (6) above, find, by striding, the lengths in chains of different fields, bunds, roads, etc.

The Table of Linear Measure.

Recall the various units employed in linear measure and let pupils build up and memorize the following table of British Linear Measure.

12	inches	make	1 foot.
3	feet	„	1 yard.
22	yards	„	1 chain.
10	chains or 220 yards	make	1 furlong.
8	furlongs	make	1 mile.
1,760	yards	„	1 mile.

Point out that people who deal in timber and cloth require the lower part of the table, those who deal with house-plots and fields require the middle part and engineers, overseers, etc., require the upper part of the table.

Tell pupils that along with the above measures the old indigenous measures are also used as contained in the following table :—

1 angula = the breadth of a man's thumb (about ¾ of an inch)

12 angulas = 1 jana (or span)

3 janas = 1 mura (cubit)

4 muras = 1 bara (fathom)

(the distance between the tips of the fingers of the two hands when the arms are both stretched out horizontally to their greatest length).

2,000 baras = 1 koss (about 2½ miles.)

(the distance which it is possible to walk before the leaves of a green twig carried along with one will wither).

4 kosses = 1 amada (about 9 miles).

6 kosses = 1 majili (about 13 miles).

Tell pupils that besides the above, there are the *betta* or the breadth of four fingers placed together, the *lolitha* or half span made by extending the thumb and forefinger as far apart as possible, the *Niluvu*—the height of an average adult person, etc. Point out to them that the units of length in the indigenous system above specified vary with individuals and have hence been almost replaced by the units in the British system which have fixity of length.

Propose exercises like the following in order that the table of linear measure may be well remembered :—

1. In walking from the school to Mulagapaka I have passed a mile stone and 2 furlong stones extra. How many furlongs must I walk further in order to reach the next milestone ? (*Ans.* 6 furlongs.)

2. The distance round a plot of ground is 6 chains. I want to enclose the plot by putting in wire-netting to prevent the havoc done by jackals on the plantation. If wire-netting is sold by the yard, how many yards of netting should I buy ? (*Ans.* 132 yards.)

3. A person measures out the length of a rope and finds it to be 30 baras. If the bara as measured by him is 54 inches, find the length of the rope in yards. (*Ans.* 45 yards.)

4. A boy walks along the bunds of a field with even space and finds that the entire boundary covers 52 paces. If the length of a pace be 30 inches, express the length of the boundary in yards and feet. (*Ans.* 43 yards 1 foot.)

5. A piece of cloth measures 8 muras in length. If a mura is 18 inches long, express the length of the cloth in yards. (*Ans.* 4 yards.)

6. I count 64 bricks placed length-wise along a floor, just covering up its entire length. Reckoning a length of 8 inches for each brick-space, find the length of the floor. (*Ans.* 12 ft.)

TOPIC 8.—MEASUREMENT OF CAPACITY.

(**Scope of the treatment.**—*The pupils are not new to the capacity measures. Still, the comparative advantages and disadvantages of measurement by capacity may be slightly dealt with. The necessity for a fixed standard of capacity measure and the convenience in having larger measures may be pointed out. The pupils may be led to derive by actual measurement the table of equivalents between the several measures aad then memorize them. Plenty of exercises may be given in the rapid estimation of capacity at sight. Obsolete measures need not be taken up for treatment.*)

The different ways of measuring quantity and their comparative advantages and disadvantages.

Refer to how things are measured and sold in the bazaar and in the market. Point out that some articles are counted and sold, e.g., coconuts, plantains, etc., some are measured in vesselsful, e.g., rice, gram, etc., and some others are weighed and sold, e.g., jaggery, colocasia, onions, etc. Ask pupils to suggest why some articles are counted and sold, some are measured in vesselsful and some are weighed. Lead them to note that things like coconuts and plantains appear more or less equal in size and their being measured in vesselsful is not handy and convenient and hence they are counted, that things like oil, milk and rice, etc., admit of being easily measured in vesselsful and hence in some places are not weighed, that counting the number of grains in a heap, for example, would be tedious and hence may be replaced by a better method of measurement, that things like colocasia, yam and blocks of jaggery are irregular in shape and unequal in size and are hence better weighed than counted or measured in vesselsful, and that certain things like ghee, coffee-seed, etc., admit of being as easily measured in vesselsful as of being weighed and hence are weighed in some places and measured in vesselsful in other places. Tell them that they will presently have to study about the system of measuring in standard

The seer and its sub-multiples.

Propose to pupils that they may have to know how much paddy seed they must sow so that they may get seedlings just enough for being transplanted in a particular plot. Suggest that an experienced ryot says that he used to sow only ten "dosedus" (i.e., two handsful) of seed for a plot of ground of equal extent. Question if it is enough if each of them takes ten 'dosedus' of seed with his own hands. Point out that as in the case of the span, the cubit, the bara, etc., the quantity contained in a "dosedu" may vary with individuals. Hence impress the necessity for using a fixed standard of measurement, e.g., a prescribed vesselful known to every one as capable of containing a definite quantity and the superiority of such a measurement to the measurement with *Guppedu* or *Pidikedu* (the closed handful), with *charedu* (the open handful), etc.

Show pupils a seer measure and tell them that it is the standard vessel adopted in the locality for measuring grain, oil, etc. Intimate to them that the seer is also known as '*an adda*' in the locality, especially in *Pārvatipur*, and in some parts also as "*a manika.*" Point out that for measuring grain and liquids smaller in quantity than a seer, smaller standard vessels are necessary. Show pupils a "*tavva.*" Let pupils take a seer of rice and measure it in *tavvas*. Thus let them deduce that two *tavvas* make one seer and that a tavva is only half a seer (*Ardha Seru*). Point out that to measure quantities smaller than a *tavva* the following standard vessels are used. The *sola* or a quarter seer (*pavu seru*), the *ara-sola* or one eighth seer (*the artha pavu*) and the *gidda* or the one sixteenth seer. Taking two of the above units at a time, direct pupils to build up the table of equivalence between the seer and its sub-multiples by measuring from a pot of water. Make a diagrammatic representation on the graph board of the relative sizes of the different measures of capacity above mentioned. Let pupils record the following table and memorize.—

2 giddas make one ara-sola.
2 ara-solas make one sola.
2 solas make one tavva.
2 tavvas make one seer, adda, or manika.

Point out that whenever grain is measured with the above standard vessels the measurement is made in heaps and that such a heap measure is not possible when liquids like milk, ghee, oil are measured. Get pupils familiarized with *munthas*, tumblers, and other utensils of common use in their houses which would hold a seer, a tavva, two seers, a sola, and a gidda.

Larger units.—Refer to the quantity of paddy, ragi, etc., stored in the threshing ground in the harvest season and point out that it will be quite inconvenient to measure the paddy in seers or tavvas or solas and that the use of a larger standard vessel would be convenient for measurement. Show pupils a *Kuncham* measure, mention its name, and tell them that it is a standard vessel for measuring the quantity of grain in large heaps, e.g., the produce from a field, etc. Let pupils take a kuncham of sand or water and measure it in seers. Let them thus derive that

"a kuncham equals 4 seers."

Next refer to how a large quantity of grain, e.g., that got from a field or stored in a *Kottu*, *Gadhi* or *Puri* is expressed. Point out that quite a big number will have to be used to express the quantity in kunchams or seers and it will be difficult to remember the quantity as expressed by means of such numbers. Tell pupils that with a view to express such large quantities of grain, larger units are used as follows :—A quantity of 20 kunchams is called a "*Putti*" in the Vizagapatam district and a quantity of 30 puttis or 600 kunchams is called a "*Garisu.*" Let pupils note that if standard vessels were to be used which could hold a *putti* or a *garisu* they could not be easily handled by people in measuring, hence no such standard vessels are in vogue, that the measurement is made only with the kuncham measure and in some cases with baskets holding 5 kunchams and that it is only for expressing larger quantities the terms "*putti*" and "*garisu*" are used. Let pupils record the equivalents and memorize

**"4 seers make one kuncham.
20 kunchams make one putti.
20 puttis make one garisu."**

Make the above relation vivid by representing on the blackboard the relative sizes of the garisu, putti, kuncham,

seer, tavva, sola, and gidda by means of lines or rectangles drawn. The teacher must remember that the above measures are those found at and near Anakapalle and that there are several capacity measures in the Vizagapatam district and in the uplands of the Gōdāvari district for which the Agricultural Middle school is likely to be intended. If pupils be present who have come from those parts tell them that in some places like the uplands of the Gōdāvari district an adda contains 2 seers of the type mentioned already, 2 addas make one kuncham, 80 kunchams make one *palle putti*, half a putti is called a *pandamu*, a quarter of a putti a *yedumu* or *kāvadi* and seven and a half *palle puttis* a garisu of 600 kunchams, that in certain parts like Vizagapatam 3 tavvas make one seer and about 2¾ seers make one kuncham, in certain parts like Vizianagram a *pedda kuncham* consists of 4 seers and a *palle kuncham* 3 seers, in Parvatipur side the term "seer" is not used but the term "adda" is used instead, in the Gōdāvari district a putti means not the putti of 20 kunchams but the pedda putti of 80 kunchams. (Extracts from the Gazetteers of the Gōdāvari and the Vizagapatam districts regarding the weights and measures prevailing in the agency may be read if available.)

Relation between the Madras measure and the Anakapalle measure.

(As Mr. Wood's book of agricultural facts and figures may have to be consulted in regard to seed-rate, etc., tell pupils that the Madras measure referred to therein is slightly greater than 1½ seers and slightly less than 1¾ seers of Anakāpalle.

For the teacher's guidance the equivalents are given below :—

One seer holds 80 tolas weight of rice and one Madras measure (heap measure, i.e., not struck) holds 132 tolas weight of rice.)

Ask pupils to measure a seer of sand, add to it a tavva of sand and an ara-sola of sand. Point out that the quantity will be *roughly equal* to a Madras measure.

Rapid estimation of capacity at sight.

(*The ability to estimate capacity cannot be expected* **to be achieved in the course of a few exercises done in a**

day or two. The exercises should be repeated frequently and whenever heaps of grain are to be seen in the markets, threshing grounds, store-rooms, etc.)

(i) Let pupils take a seer of sand and empty it into a heap. Ask them to heap so as to contain 2, 3, 4 seers, a half seer, a quarter seer, one-eighth seer, one-sixteenth seer. Let them next make (i) seer heaps, and (ii) larger and smaller heaps and estimate the quantity in seers, in tavvas, in solas respectively and verify the quantity by actual measurement.

Let them make heaps so as to contain a sola, a gidda, a tavva, and a seer and verify by actual measurement whether the heaps are of the capacity proposed.

Repeat the above exercises till the pupils are fairly able to judge by the eye how much a seer heap, a tavva heap, a two-seer heap, a three-seer heap, a four-seer heap would be.

(ii) Let pupils measure a kuncham of sand and heap it up. Let them next make heaps of sand containing 2 kunchams, one and a half kunchams, two and a half kunchams, 4 kunchams, 5, 6, 7, 8, 9, 10 kunchams and gain an eye estimate of how much each heap is.

Make heaps of sand containing fairly one kuncham, 2 kunchams, 5 kunchams and 10 kunchams. Let pupils judge the quantity by the eye and verify by actual measurement. After the pupils have acquired sufficient skill in quickly heaping up sand, etc., up to 10 kunchams, take them for an evening walk to the Sarada river and let each of them make two heaps of sand side by side, each containing 10 kunchams (a kuncham or a muntha, or a basket holding one kuncham may be used by the pupils if necessary). Let them mix up into one the two heaps so obtained. Question them how much is contained in each new heap and in all the heaps together. Let them thus derive how much a putti would be, as also a garisa of grain.

(iii) Place before pupils such iron pots, buckets, etc., of different dimensions as are available in the farm premises, direct them to fill them with water in seers, bucketsful, etc., and lead them thus to note how much the capacity of each big pot or vessel or bucket would come to. Let them repeat the above exercise with the tins used for storing oil and come to know how many seers or kunchams each tin is likely to hold, say (4 seers), how many buckets the ordinary

household earthen or brass pots and other vessels would hold, etc.) Let them derive similar information with regard to the hostel vessels. Place some vessels of different sizes before pupils. Propose that 40 seers of milk have to be bought. Ask pupils to suggest what vessels may be chosen.

Let pupils choose the vessels, measure actually with water and examine if their choice is correct. Repeat the above exercises, choosing 15 seers of oil, 20 seers of ghee, 30 seers of oil, etc., instead of 40 seers of milk.

Propose the following exercises :—

1. Bapanna gets 25 kunchams of paddy from each of 12 tenants. How many puttis will he get on the whole?
(*Ans.* 15 puttis, buri puttis of 20 kunchams each).

2. Achayya bought a busta of rice containing 48 addas. How many kunchams of rice will he have?
(*Ans.* 12 kunchams).

3. One day Gourisam finds that the curd from his dairy was 44 vesselsful as measured with a particular vessel. If the vessel could hold 3 giddas, express in addas, solas, etc., the quantity of curd obtained.
(*Ans.* 16 addas, 1 sola.)

4. A person measures 500 kunchams of paddy on the threshing floor. How many puttis of paddy does he so measure? (*Ans.* 25 puttis.)

5. From a granary 3 garisalu of grain have to be measured and sold. How many kunchams will have to be measured? (*Ans.* 1,800 kunchams.)

6. At 2 addas of grain for every putti measured, how many kunchams of grain will the person measuring get?
(*Ans.* 15 kunchams.)

7. In Mr. S. Patrudu's house there are 5 gadis of paddy each holding 96 kunchams. How many puttis has he in all? (*Ans.* 24 puttis.)

TOPIC 9.—MEASURES OF WEIGHT.

(Scope of the treatment.—*It is a particular feature in the shops at Anakapalle that only the avoirdupois pound-weights and the Calcutta seer-weights are used for weighing articles and in many cases, it is after such weighing that the weights are expressed in terms of the indigenous units—the visam and the seer. Hence, the pupils must be made familiar with the three systems of weight measurement—the Imperial Indian or the Railway weights, the seer and maund, the weights in the British Avoirdupois system—the lb., the cwt., and the ton—and the local weights, the seer, the visam, manugu and putti, obsolete weight measures being avoided. Smaller units adopted in buying in retail may at first be taken up in each system and the larger units later. The relation between the smaller and the larger units in one and the same system, as well as that between the units in the different systems, must be well impressed. The correct methods of handling the balances, the common balance, the balance in the farm store-room, the railway-station balance and the country steel-yard may be explained. Plenty of exercises should be given so that the pupils may acquire ability in the rapid estimation of weights by direct handling. As an extension of the smaller units, goldsmith's weights may be taught. Finally the principle of "Volume-weights" may be introduced and therefrom the relation between the weight and the capacity measures.*)

Advantages of the weight measure over the capacity measure.

Referring to things that are weighed and sold in the bazaar, e.g., yam, colocasia, jaggery blocks, etc., question why they are weighed and sold instead of being measured by means of a capacity measure. Point out that if irregular solids be measured by the measures of capacity, they will not be found to occupy the space evenly and uniformly and hence measurement by weight will be a better way of expressing the quantity. Point out also that it is less tedious to weigh a busta of rice than to measure its contents in kunchams and that weighing equal quantities of

different kinds of rice, for example, will reveal difference in weight and hence, suggest some other points of difference too, e.g., difference in essence, etc.

The local units of weight-measurement.

Intimate to pupils that if each of them chose different stones with which to weigh articles, the difficulty may arise of one not being able to make out what weight another has taken. Thus point out the necessity for the adoption of *a standard weight* known to every one in the locality in terms of which the weights of articles may be expressed. Get ready beforehand a block of stone weighing a visam of the locality. Let pupils handle the weight, tell them that it is a standard weight in terms of which the weights of articles are expressed and that the weight is known as a "*Visam.*" Place at the disposal of the pupils a balance and a visam weight. Ask them to weigh out sand to the extent of a visam. Next ask them how they would distribute the sand so weighed equally between the scale pans of a balance and thus get half a visam of sand, how they can again distribute the half-visam of sand equally between the scale-pans of a balance and thus get a quarter visam of sand. Now tell them that weights smaller than a visam are expressed in terms of a (*padalam*) or "half visam" and "*yebalam*" or "quarter visam." Give exercises to pupils to get stones weighing a padalam and an yebalam, handle them and thus have an idea of feeling by direct handling how much a visam, a padalam and yebalam would come to. (In the course of handling the common balance in this connection refer incidentally to the various ways in which the bazaar man may deceive them in weighing articles in the balance and what precaution they should take to avoid being duped by him.)

Next show pupils a maund weight and tell them that it is in terms of the maund that the weights of big blocks of jaggery, tinsful of ghee, etc., are expressed, especially in wholesale transactions. Question pupils how with a visam weight and a suitable balance they can weigh out a visam weight of stone in one scale-pan with a visam weight counterpoising in the other scale-pan, how by emptying the contents of one scale-pan in the other, they can weigh out 2-visam weights and how by repeating a similar

process, they can weigh out 4-visam weights and 8-visam weights. Place an 8-visam weight of blocks of stone so obtained in *one* scale pan of a big balance and a maund weight in the other. Let pupils observe that the scale-pans counterpoise each other in that position and derive that

eight visams make one maund.

If there be stones, jaggery blocks, tins of oil, ghee or water, vessels holding grain or liquid, adjust their contents so as just to weigh one maund. Let pupils lift them up and thus derive an idea of a maund weight by direct handling.

Next, refer pupils to the convenience in having *terms* by means of which heavier weights may be expressed, just as they have the putti and the garisa in expressing capacity measure. Intimate to pupils that a weight of 8 manugus is called a "*kantlam*" and one of 20 manugus a *putti*, *barva* or *candy* and that these terms are used generally in wholesale purchases, yield, etc., when large quantities of articles have to be weighed. Let pupils record the table of equivalents and memorize :—

2	yebalams	make	one padalam.
2	padalams	„	one visam.
8	visams	„	one manugu.
8	manugus	„	one kantalam.
20	manugus	„	one candy.

The use of the pound-weight (avoirdupois) and the Calcutta seer as measures of weight and their equivalents in tolas.

Intimate to pupils previously that when they go to buy articles from the grocer or from the vegetable vendor, say, for the use of the hostel, they must examine what weights are used by the tradesman. Showing the weights that the bazaarmen use, point out to pupils that the weights used are not labelled as visams, yebalams, padalams, but, appear as 2 lb., ½ lb, etc., or as bazaar seer (India), half bazaar-seer (India), etc. Tell them that the term 'visam' is used to denote different weights in different places and even in one and the same place like Anakapalle

and hence in actual weighing the retailers use the fixed weights, viz., the Calcutta seer (known also as the bazaar seer, pucka seer, or the Imperial Indian seer) the two seer, the half seer, the quarter seer, etc., weights and also the avoirdupois pound, the two-pound, the four-pound, the half-pound, the quarter-pound, etc., weights.

Let pupils handle a rupee weight, weigh one rupee against another in a balance and note that all rupee-coins are expected to be of equal weight. Tell them that a rupee weight is called a "*tola.*"

Place before pupils a balance and some rupee-coins, say, five in number. Ask them how they would weigh out sand in the balance to the extent of 6, 7, 8, 9 and 10 tolas respectively with the help of the five rupee-coins only. Point out that a rupee weight of sand or stone may be weighed, put in one scale pan and five rupee-coins added to it and thus a weight of 6 tolas arranged to be placed in one scale-pan using only the five rupee-coins. Let pupils suggest how weights of 7, 8, 9, 10, etc., tolas may be secured in a similar manner. Let them handle the weights from 1 tola to 10 tolas and form an idea how much 1, 2, 3, etc., tolas would come to. Let them next handle letters, packets, spoons, etc., estimate their weights in tolas by direct handling and then check by actual weighing. Question pupils how with the help of the five rupee-coins given already, they would weigh out sand, stones, etc., to the extent of any number of tolas. Point out how three rupee-coins may be placed in one scale-pan and stones may be placed in the other to counterpoise and how the emptying of the contents of both the pans in one would give a weight of six tolas. Thus lead them to obtain stones placed in one scale-pan so as to weigh 10, 20, 30, 60, 80 tolas. Let them next weigh the 80 tolas weight thus obtained against the Calcutta seer, known also as the Imperial Indian seer or Pucka seer and infer from the counterpoising that

1 Calcutta seer weighs 80 tolas, i.e., as much as 80 rupee-coins.

Tell them that this is the seer weight in terms of which the weight of articles is expressed in railways and also the weight of common salt as sent from the salt pans. Question how many tolas half a seer, a quarter seer, an

one-eighth seer and one-sixteenth seer would weigh and let pupils note the equivalents. Thus see that they pick up the ability to express at mere sight of the labels how many tolas the two-seer, the quarter-seer, etc., would weigh. Let pupils handle these measures and also vessels, stones, etc., of equal weights. Next let them handle the various household utensils either empty or filled with water, and rapidly express by mere handling how many seers each would be weighing. Let them next check the weights by actually *weighing in* a balance. Repeat the above *exercises until* the pupils are able to quickly estimate weights by direct handling.

Ask pupils next to weigh a pound weight against a half seer weight of the type already described and note that they may be taken to be almost equal in weight though the scale pan containing the pound weight is slightly lighter. Let pupils derive that *a lb. weight is* 40 *tolas very nearly.* Direct pupils to see how much must be added to the scale-pan containing the pound weight in order that the scale-pans may counterpoise. Thus bring out the fact that about a rupee weight, i.e., a tola weight may have to be added, i.e., to say, a pound weight is 40 tolas less about one tola.

Let pupils note from this that a 2-lb. weight is 80 tolas, i.e., a Calcutta seer less 2 tolas, a half pound weight is 20 tolas less about half a tola, a quarter pound is 10 tolas less about a quarter tola.

Tell pupils that the tradesmen use the Calcutta seer weights and the pound weights and manage to express weights in visams according to the local usage; thus, in certain places a visam is taken as 120 tolas, then a visam is measured by a Calcutta seer and half seer, in some places a visam is taken as 110 tolas only and then it is measured by a Calcutta seer, a quarter seer and one-eighth seer weight. Inform pupils that weights smaller than a visam are expressed in terms of a seer called the katcha seer which is one-fifth of a visam. Let pupils derive that 1 *katcha seer* is 22 tolas when a visam is taken as 110 tolas and that it is 24 tolas when a visam is taken as 120 tolas.

Propose :—

1. A bazaarman in weighing a visam of an article puts in a two-pound weight, a half pound weight and a quarter-pound weight in one scale-pan, sufficient quantity of the

article in the other scale-pan to counterpoise and then says he has weighed out a visam of the article, reckoning a visam to be 110 tolas. If a pound weight is only 40 tolas less about a tola, examine if the above weighing is correct.

(Point out that the above weighing though in vogue is incorrect and has evidently been done under the wrong notion that a pound weighs just 40 tolas and that the pupils should avoid such incorrect weighings as far as possible.)

2. To weigh a visam of 120 tolas a tradesman uses only a 3-pound *weight. Explain whether there is shortage* in weight, or not.

(Point out that a three-pound weight is 120 tolas less about 3 tolas and so there is shortage in weight to an extent of 3 tolas in buying a visam. Tell pupils that the pound weight will hereafter be shortly expressed as "lb.")

3. A vessel weighs 1 bazaar Indian seer, a quarter of a lb. and one-sixteenth of a lb. How many katcha seers may it be taken to weigh? (*Ans.* Four and a quarter seers.)

4. A lead *ginni* weighs 4¼ lb. Express its weight roughly in local seers of 22 tolas.

(*Ans.* Seven and three-fourth seers.)

5. An "Annamthinni Ginni" weighs a Calcutta pucka seer, a half seer, a half pound weight and a quarter Calcutta seer. Express its weight in local seers of 22 tolas.

(*Ans.* Seven and a quarter seers nearly.)

Smaller and larger units in the British avoirdupois system.

Refer to the pound weight already explained and tell pupils that it is a standard weight used in weighing articles sold in the grocers' shops; that to weigh articles less than a pound, the half pound, quarter pound, one-eighth and one-sixteenth pound are used and that the weight which is known as one-sixteenth of a pound is usually called "an ounce" weight. Let pupils thus derive the equivalents.

1 pound contains 16 ounces.
A half pound contains 8 ounces.
A quarter pound contains 4 ounces.
An one-eighth pound contains 2 ounces.

Tell pupils that the term "ounce" is written in the short form "oz."

Direct pupils to examine the weights used in the local firewood depot while weighing firewood. Show them the *weight of 56 lb. and tell them that two such weights*, i.e., a weight of 112 lb. is called a "hundred-weight" and written shortly as "cwt." and that 20 such hundred-weights is called a "ton." Let pupils thus derive the table of equivalents and memorize

16	ounces make	1	lb.
112	lb. ,,	1	cwt.
20	cwt. ,,	1	ton.

Intimate to pupils that a quarter of a hundred-weight, i.e., a weight of 28 lb. is simply called a quarter by the British people. Intimate to them also that a weight of 56 lb. is known as a "Gundu" at Anakapalle, one of 28 lb. as an "Arthagundu" or half gundu, one of 14 lb. as "*pavu gundu*" or quarter *gundu*, and that where cast iron weights are not available, stones of equal weights are used instead.

Take pupils to the balance kept in the store-room in the farm, show them how to weigh articles with the balance and give exercises to them on weighing, in the balance, their own bodies, a busta of paddy or ragi, the amount of fodder given to the farm bullocks for the day, the quantity of produce got from a plot, e.g., pasupu, etc., in cwt.

The weights used in railway.

Refer pupils to the Calcutta seer of 80 *tolas* already mentioned. Intimate to them that the weights of parcels, etc., sent by railway are expressed in terms of that seer. Tell them also that to weigh luggage and goods far heavier than the seer, the term maund is used to indicate a weight of 40 seers. Take pupils to the railway station on a convenient day. Show them how the balance there is adjusted before weighing. Let them note the graduations and see how jaggery blocks, packages, sacks, basket-loads, etc., are weighed by means of the balance and their weights read off.

Inter-relation of the local, avoirdupois and railway weights.

Question pupils how many tolas are contained in an Imperial Indian or Railway seer and how many in an avoirdupois pound. Let them work out and record that a lb. is 40 tolas less about a tola, an Indian Imperial seer is 80 tolas, that if a visam be 120 tolas, a visam is slightly greater than 3 lb., or if it be 110 tolas, it is almost equal to 3 lb., that a manugu of 8 visams may hence come to about 24 or 25 lb., that a hundred-weight of 112 lb. is roughly equal to four and a half maunds, that a putti or candy or barva of 20 manugus weighs from 480 to 500 lb., that a ton of 2,240 lb. is roughly ninety-three and a quarter maunds, and that a railway maund of 40 Imperial seers is about eighty and a quarter lb., and hence, is about three and a quarter maunds (of the locality).

Goldsmiths' weights.

Set pupils to visit a goldsmith's and examine the weights he uses. Lead them to note that the tola or the weight of a rupee coin is a chief unit employed, that to weigh heavier jewels and vessels, e.g., those of silver, etc., the goldsmith has the 2-tola, 5-tola weights, etc., and to weigh lighter jewels, he has the half tola (*arathulam*), the one-eighth tola (*bedathu*), the quarter tola (*pavalam or pavuthulam*), the one-sixteenth tola (*annaethu*), and one thirty-second tola (*arthanna ethu*).

Next refer pupils to the fact that it is not in tolas that gold is weighed but that the weight of a sovereign is taken as the standard. Tell them that for smaller weights, the goldsmith has the "*pathikam*" which is roughly the weight of 4 paddy grains or 4 visams as the weight of a paddy grain is otherwise called, the *addiga* which weighs 2 pathikas, the *chinnam* which weighs 2 addigas.

Lead pupils to weigh with a balance and derive the equivalents :—

1 tola weighs 30 chinnamulu.

1 sovereign weighs about 20½ chinnamulu.

Tell pupils that when buying silver, etc., in large quantities by weight the quantity used to be expressed in seers

of 24 tolas each, that this is the kutcha seer prevalent in the locality and that in some places it is reckoned at only 22 tolas.

Additional weight terms used in the suburbs.

Tell pupils that there are a lot of weight terms used in particular localities and that such of the terms as are prevalent in their own villages may be remembered by them. Let pupils suggest any extra weight terms not mentioned already. In addition to what they suggest, enumerate to them the several terms current, e.g., in some places the term "*Yettedu*" is used to indicate 5 maunds, the term "*Padumu*" to denote 10 maunds, the term "*Navtak*" to denote one-eighth of the local katcha seer of 24 or 22 tolas, the term "*Chatak*" to denote one-sixteenth of the seer, that one-eighth visam is called a *pampu*, etc.

Incidentally refer also as to how in certain parts the people have something like a Danish steel-yard (Thunikka katta) with its peculiar graduations to indicate varying weights. Let pupils also handle one such steel-yard, lift it by the string when its loop fits into the balancing point (the *Yethugethu mark*) and put in weights at one end, arrange the position of the string accordingly so as to fit into the correct graduation and thus obtain the correct weight. Tell pupils that this steel-yard is gradually becoming extinct and is giving place to the common beam-balance and still they must be familiar with the weights indicated by the graduations in it, if it is in vogue in their locality.

Volume weights.

Let pupils measure a seer (capacity measure) of rice and a seer (capacity measure) of black-gram. Let them next weigh the same capacity of each cereal in lb., or tolas and note the difference in weights.

Let them next take the same weight of sugar and of coffee seed. Let them measure with the sola or gidda and note the difference in the holding or capacity. Let pupils next measure a seer, two seers, three seers, etc. (capacity), of rice and weigh them. Lead them to note that the weight of 2 seers of rice is double that of one seer, that of three seers

is treble that of one seer, that of 4 seers is four times that of a seer and so on. Let them repeat the above exercises with several other articles and arrive at the following facts with reference to certain articles like rice, ragi, jonna, mustard, etc., which are found in fine grains.

1. **Different articles of equal capacities need not weigh equally.**
2. **Different articles of equal weights need not be of equal capacities or holding.**
3. **The weights of different capacities of the same article are proportional to their capacities, that is to say, to what extent the capacity increases, to that extent the weight increases, and vice versa.**

Point out that it is because of item (3) above that little difference in quantity is noticeable when certain articles are weighed or measured with a capacity measure and that is why such articles are weighed by some and measured with a capacity measure by others.

(Point out also that item (3) above need not hold good in the case of articles like pendalam, yam, etc.)

Get pupils to realize that because of item (3) it has been rendered possible to remember *capacity by means of weight and vice versa.* Let pupils weigh a kuncham of the several kinds of produce available in the farm and record the weight in lb. as follows :—

A kuncham of	*pyru gingelly*	—Six and three quarter lb.
Do.	*pyru chodi*	—Seven and a quarter lb.
Do.	*Bangara Thiga*	—Six and a quarter lb.
Do.	*Navakoti Sannelu*	—Six and a half lb.
Do.	*Mypali*	—Six and a quarter lb.
Do.	*Konamani*	—Six and a half lb.
Do.	*Gunupur Sannelu*	—Six and a quarter lb.
Do.	*Bobbili Ganti*	—Six and a quarter lb.
Do.	*daincha seed*	—Nine lb.
Do.	*sunnhemp seed*	—Eight lb.
Do.	*ganti grain*	—Seven and a half lb.

Similarly, let them have an idea about the weight of a busta of paddy, *e.g.*, a busta weighs 166 lb. Tell them that they may eventually remember the equivalents.

Relation between the weight measures and the capacity measures.

Let pupils weigh a seer (capacity measure) of rice of different kinds and come to the conclusion that a seer weighs 80 tolas roughly and that the capacity of the seer vessel is so adjusted in its construction. Let pupils similarly find that a gidda of rice weighs about 5 tolas and hence work out that a kuncham holds 320 tolas, a garisa 192,000 tolas, etc.

Propose :—

1. A cart is loaded with 5 bags of rice, each weighing 166 lb. Another cart is loaded with 50 bundles of firewood, each weighing a pavu gundu. Express in cwt., qrs. and lb. the weight in each cart.

(*Ans.* 7 cwt. 1 qr. 18 lb., 6 cwt. 1 qr.).

2. It is known that a busta of Akullu weighs 220 lb., one of Doosi Sannelu 190 lb. and one of green-gram 164 lb Express in tons, cwt. and qrs., the weight of (*a*) 14 bustas of Akullu, (*b*) 42 bustas of Doosi Sannellu and (*c*) 40 bustas of green-gram. (*Ans.* (*a*) 1 ton 7 cwt. 2 qrs., (*b*) 3 tons, 11 cwt. 1 qr. (*c*) 10 tons 5 cwt.).

3. There are 80 blocks of jaggery in a cultivator's store-room. If each of the blocks weighs 10 viss, express in puttis the quantity of jaggery he has. (*Ans.* 5 puttis.)

4. A busta of paddy weighs 168 lb. at the time of the harvest but 166 lb. afterwards. A person bought and stored 1,000 bustas of paddy at the time of the harvest. What will be the loss in weight he may find afterwards? Express your answer in puttis at 500 lb. a putti.

(*Ans.* 4 puttis.)

5. When purchasing, a trader reckons 120 tolas for a visam weight and, when selling, he reckons 110 tolas for a visam weight. How many visams would he be selling if he buys 11 visams of an article as aforesaid?

(*Ans.* 12 visams.)

6. 1 putti = 500 lb. 1 maund = 25 lb. Use the equivalents to express an yield of 1,300 lb. of jaggery in a farm in puttis and maunds. (*Ans.* 2 puttis, 12 mds.)

7. A bag of castor-cake contains 160 lb. of the cake. I use the cake as manure, apply 15 bags to an acre of sugarcane and 2 bags to an acre of paddy cultivation. If I so serve with manure 14 acres of sugarcane cultivation and 7 acres of paddy cultivation, what weight of the fertilizer would I require? Express your answer in tons.

(*Ans.* 16 tons.)

8. Change the numerical data in the above and propose a series of exercises.

TOPIC 10.—OUR SYSTEM OF MONEY.

(**Scope of the treatment.**—*Reference may be made to the barter system and the use of money as a convenient means of exchange. The different coins in use, the necessity for their issue, their composition and shape may be alluded to and the relation between weight and value in the case of coins of the same composition pointed out. Currency notes and their use may next be briefly dealt with. How coins and currency notes should be examined before acceptance may be shown. The table of equivalents between coins may be derived, and certain indigenous money terms in vogue in the locality such as Dabbu, Ekani, etc., may be explained.*)

Money as a convenient means of exchange.

Refer pupils to the barter system prevalent in some places. Talk to them as to how the hill tribes bring hides and skins from the Agency tracts and exchange them for common salt and kerosine oil from the plains. Question them if with reference to every article they require they may resort to such a system of exchange. Point out that such a system of exchange would not be possible when one does not require the commodity another has.

Cite as instances (1) the case of a person owning only hides and skins finding it hard to secure jewels or ragi in exchange directly from a jeweller or a ryot, (2) the difficulties that a South Indian owning lands or timber at Burma would experience in securing, instead, lands in his native village if no landowner in his village likes the exchange being effected. Hence lead pupils to recognize the importance of the use of money in life in the convenience it gives in disposing of and in going in for commodities, property, etc. At the same time, emphasize the fact that "money" and "coins" and "notes" are in themselves merely token and of no specific value; they are of value only if they can be exchanged for needed commodities; and money on a desert island with a population of one man will be quite valueless.

The different coins in use.

Ask pupils to mention the names of the different coins in use. (Show the token coins if necessary.) Let them distinguish the shape of a coin from that of another wherever distinctions exist and also the metals of which the coins are made. Point out that in the case of some of the coins there is a difference in value corresponding to a *change in the metals of which they are made.*

Weigh a quarter-anna coin against three pie-coins. Let pupils note the equality in weight as well as the equality in value. Similarly, let them weigh a half-anna piece and six pie-coins or two quarter-anna pieces and note the equality in value as well as in weight. Thus, point out *that in the case of coins made of the same kind of material there is equality in value corresponding to equality in weight.* Illustrate this further by pointing out that a rupee would weigh against two silver half-rupee pieces or four silver four-anna pieces or eight silver two-anna pieces.

Next question them whether a rupee coin would weigh against 16 one-anna pieces and whether an one-anna coin would weigh against 12 pie-coins. Get them to note the obvious inequality in weight and explain why in spite of the inequality in weight they have equality in value. Point out that the metals used in the coins are different and hence that there is equality in value in spite of inequality in weight.

Next question why they should not have the rupee coin alone and why they have coppers, nickels, etc. Point out the difficulty that people would feel, especially the poor, in making small purchases, e.g., a match-box, a neck-rope, etc. Now question why they should not have all the dealings in coppers alone. Point out the inconvenience which would be caused in counting, and in carrying a large number of these coins, say 2,000, to make purchases in a market. Thus get them to note the convenience in having coins of large and small value. In this connection point out to them that a similar convenience is afforded by having coins of intermediate value, e.g., the half-rupee, quarter-rupee, three pies, the two-anna pieces, etc.

Arbitrary standardization in value.

Propose to pupils as to how silver is sold in the market. Point out to them that a rupee would often fetch them a

little more than a rupee weight of silver or more than 192 pies weight of copper. Thus get them to note that the value of a coin is not to be gauged by the value which the quantity of the metal used in its making would fetch in the market. Talk to them about the mixture of metals to form the alloys of which the coins in use are made, of the refining and polish that is given in the mint office and of the standard values arbitrarily assigned to the coins by state regulations so that the subjects may take them to be of fixed values, unaffected by the fluctuation in prices in the market of the metals used in their composition. Point out that that the state which assigns a standard value to coins has thereby actually to supply any number of these coins according to the demand, even if the prices of the composing metals should go high. Thus get pupils to realize that in our country the Government has hence to go in for large quantities of silver, nickel and copper to issue the several coins made of such metals. Incidentally refer to the fact that some countries have got gold coins standardized, e.g., the sovereign, the dollar, etc., and as the value of gold in terms of silver is not steady it is not possible to express the price of those coins in terms of a fixed number of the rupee coins current in our country.

Paper currency.

Next refer to the arbitrary value assigned to the currency notes issued for various sums of money by the Government. Point out the convenience in carrying and circulating a hundred-rupee note instead of using 100 rupee-coins for the purpose. Point out also what a vast quantity of silver and copper would have to be used in the shape of coins if people who own such currency notes use only coins instead. Refer to the fact that all Government treasuries and offices having money transactions will freely take in these currency notes and give change, if necessary, and, but for that, no one would attach any value to these notes.

Testing coins and currency notes before acceptance.

Talk to pupils as to why before accepting a coin especially of silver or of nickel, they should test whether it is a genuine coin. Get them actually to test coins by seeing if they have a good ringing sound, are not defaced

or of under-weight, bear the usual impressions on the obverse and reverse sides, etc. Refer to the fact that such a test is necessary as otherwise the shop-keepers may reject them, especially in the case of coppers of other countries, defaced coins, coins of under-weight, coins failing to produce a good ringing sound, etc. Next show them genuine currency notes, lead them to note down the serial numbers in the notes repeated in each half, the water colour, etc. Tell them how by reporting to the authorities they could trace out stolen currency notes for large sums provided their serial numbers are known. Refer to why when two halves of a note are pasted together in the middle they should examine if the numbers on either half tally with each other, etc. Incidentally, talk about the measures taken by Government against counterfeit coins and currency notes being manufactured and used.

The table of Indian money.

Even though different denominations have been devised for expressing different sums of money, e.g., the several coins and currency notes above referred to, question pupils how people enter sums of money, spent or received, in their account books. Point out that generally three columns are opened there, viz., for RS. A. P. Tell them that in making up totals there or in making other calculations with money which would be dealt with later on they will have to remember the following table of money :—

12 pies make	*1 anna.*
16 annas make	*1 rupee.*
1 rupee equals	*192 pies, or 64 quarter annas or 32 half annas.*

Some indigenous money terms in vogue in the locality.

Refer to what is known as "*a dabbu*" in the locality. Question pupils if there is a corresponding coin of that denomination. Point out that there is no such coin and still the people use the term "dabbu" to indicate "four pies." Allude also to the fact that the people in the locality refer to a three pie-coin as a copper or a kani, the two-anna piece as a "beda," a half anna piece as a "paraka," and that they make combinations of these, e.g., a "beda

paraka" to denote two and a half annas, "kani thakku beda" to denote a beda less a kani, i.e., 1 anna 9 pies, etc.

Tell pupils that in some parts of the Gōdāvari District they have peculiar money terms, e.g., the dabbu means in some places a kani and is generally characterized by the name "kotta dabbu" (new dabbu) as distinguished from a "pata dabbu" (old dabbu) of 4 pies. Refer them to various other money terms such as

a *pavala* or a *dulam* } to denote a quarter rupee
a *tankamu* to denote a third of a rupee.
a *chavulam* to denote half a rupee.

a *muppavala* for 12 annas, a *pavu* for a rupee, a *mada* for Rs. 2, a *varaka* (pagoda) or *punji* for Rs. 4, a *puli varaka* for three and a half rupees, a *vanda* for Rs. 100, etc.

Propose the following exercises :—

1. A post-card sells at 6 pies each. Express in RS. A. P. the cost of 300 post cards required for sending marriage invitation. (*Ans*. Rs. 9–6–0.)

2. At 5 dabbus a basket, express in RS. A. P. the price realized by selling 3 dozen baskets in a market day. (*Ans*. Rs. 3–13–0.)

3. At 5 pies in the rupee, express in RS. A. P. what income-tax a man should pay who gets an income of Rs 3,400 per year. (*Ans*. Rs. 88–8–8.)

4. A ryot sells 10,000 canes during the Gonri Amman festival days at 9 pies a cane. Express in RS. A. P. how much he would get. (*Ans*. Rs. 468–12–0.)

5. Express in RS. A. P. the cost of 25 kunchams of paddy husk at 8 pies a kuncham.

(In the course of working the above exercises point out how the remembrance of the relationship, "*A dabbu (pata) equals 4 pies, three dabbus make one anna, forty-eight such dabbus make 1 rupee*" *would facilitate the finding out* of the result, e.g., in exercise (5) 25 × 8 pies = 25 × 2 dabbus, i.e., Re. 1 plus 2 dabbus = Re. *1–0–8*.)

6. A person buys a neck rope for 5 as. 6 ps. He has coppers only (quarter anna pieces). How many coppers, must be counted and given? (*Ans.* 22 coppers.)

7. A person demands 15 dabbus as the price of a basket. The buyer has one anna coins only. How many such coins should be counted and given? (*Ans.* 5 anna coins.)

8. Sriramulu wants change for Rs. 10. He gets coppers (quarter anna coins) for Re. 1, nickel one anna coin for Rs. 2, arthanna (half anna) copper coins for Rs. 3 and four anna silver coins for the remaining sum. How many coins of each kind will he get?

(*Ans.* coppers 64; nickel 32; arthannas 96; pavala 16.)

9. If you want to give "*kani thakku beda*" and have pie coins alone with you, how many such coins should you give? (*Ans.* 21 coins.)

10. I bought a viss of yam for "*beda paraka.*" I have coppers (quarter anna pieces) alone with me. How many such coins should be given for the yam?

(*Ans.* 10 coins.)

TOPIC II.—THE FOUR FUNDAMENTAL RULES APPLIED TO MONEY, WEIGHTS AND MEASURES.

(**Scope of the treatment.**—*The work intended to be done here is mostly revision. The pupils are expected to be familiar with the tables of money, weights and measures and the four rules applied to them even in their past school-career. Still, it may be tested here if they have correctly memorized the tables and are able to apply them aright. Hence a few exercises may be given in expressing any number of units of one denomination in terms of units of another denomination of the same kind,* **unpractical examples being discarded.** *These may be followed by exercises on the application of the four rules to money, weights and measures. It may be pointed out that the application of the four rules in the case of quantities denoting lengths, money, weights and measures is similar to their application in the case of integers with the variation that while ten units of one order make up one unit of the immediately higher order in the case of integers, such a—10 to 1 relation is not found to exist between the units in our system of money, weights and measures. Short cuts should wherever possible be insisted on, e.g.:*—**If the price of a kantlam of jaggery be Rs. X, that of a viss is "X" coppers (quarter anna pieces); if the price of an article be "X" pies, that of a dozen is "X" annas, and vice versa; if the price of an article be "X" annas, that of 16 articles is Rs. "X", and vice versa; etc.** *Elaborate explanation of the method of applying the four rules is not necessary as the work is only mere revision. Too many examples on compound multiplication and division which require written work need not be taken up as in the business world most of the problems have to be attempted only orally.*

Extensive use of the tables for common fractions in making calculations may be made when vulgar fractions are taken up for study. The fundamental operations have been here restricted only to quantities expressing length, money, weights and capacity, as only these tables have been treated up till now in this book.)

Expressing quantities in terms of one or more units of the same kind.

Question pupils in terms of what units quantities by weight may be expressed. Point out familiar cases of expressing the quantities as 5 puttis 7 kantlams, (3 manugus 5 visams), (5 manugus 7 visams), (2 puttis 12 manugus), etc., in the indigenous system or as 4 lbs. 2 oz., (5 lb. 8 oz.), (5 cwt. 40 lb.), (7 ton 6 cwt.) etc., in the British Avoirdupois system or as 4 seers, 6 maunds, 5 maunds, 20 seers, etc., in the Imperial Railway measures, etc. Get pupils to see that the quantities may be expressed in terms of a single unit of weight or in terms of more than one unit of weight. Point out that the same system of expressing in terms of a single unit or in terms of more than one unit is adopted when quantities denoting capacity, length, money are expressed, e.g., in saying 2 feet 6 inches, 4 yards 2 feet, 5 miles, 5 miles 6 furlongs, 5 chains 57 links, 5 chains 70 links, 7 garisas 18 puttis, 10 kunchams, 2 puttis 10 kunchams, 7 puttis 2 addas, Rs. 5–6–6, etc. Impress the fact that *when a quantity is expressed in terms of more than one unit, the units must belong to one and the same class*, e.g., weight or capacity or length or money as the case may be and *may differ only in size and name.* Question pupils if it is usual to express a length as 4 miles 7 furlongs 2 inches or a quantity by capacity as 7 garisas 5 puttis 3 giddas or a quantity by weight as 8 baravas 7 manugus 12 tolas. Point out that it is not usual to express quantities in terms of extremely big and extremely small units as stated above and that such a combination only shows measurements not found in practice.

Conversion from one denomination to another.

Propose a question like the following :—

A sugar-cane is 19 feet 6 inches long. How many sets, each 1 foot 6 inches, could be cut from the cane ?

Let pupils work the question. Point out to them that one way of doing the question is to convert both the quantities 19 feet 6 inches and 1 foot 6 inches into inches, that is, express them as 234 inches and 18 inches respectively and then work out the division 234 ÷ 18. Lead pupils to note that when a quantity, given in terms of two

units of the same kind, is expressed in terms of the lower of the two units, there is only multiplication and addition and one can easily do the conversion, provided the table giving the inter-relation of the units is remembered correctly.

Next propose :—

It is suggested that 35 lbs. of sunnhemp seed sown over an acre would give manure sufficient for an acre. Find how much of sunnhemp seed will thus be required for the 32 acres of lands belonging to a landlord.

Let pupils work out by multiplication that the seed required is 35 × 32 or 1,120 lbs. Get pupils to realize that it is not usual to express the quantity in this case in terms of a smaller unit and *that*, by a big number and that it is usual to express the above quantity as 10 cwts., by using a larger unit and a smaller number. If the pupils are unable to see how 10 cwts. was got as an equivalent of 1,120 lbs., point out that 112 lbs. make one cwt., and hence the equivalence was obtained or let them work as follows, using the tables, " 28 lbs. make one quarter " and " 4 quarters make one hundredweight."

28) 1,120 lb.

4) 40 qrs.

10 cwt.

Thus point out that by a *process* of division a quantity expressed in terms of any unit may be expressed in terms of higher units of the same kind. Let the pupils note that the two kinds of conversion above stated help us to work out several operations involving money, weights and measures.

The four fundamental operations applied to money, weights and measures.

Propose :—

A tenant has Rs. 28–6–6 in his pocket. The sale of onions then fetches him Rs. 20–8–5, of yam Rs. 15–8–4, of guavas Rs. 20–4-8 and of jaggery Rs. 25–4–6. How much of money will he then have ?

Let the pupils adopt the following working :—

	RS.	A.	P.
	28	6	6
	20	8	5
	15	8	4
	20	4	8
	25	4	6
	108	30	29
or	108	32	5
or	110	0	5

If in adding the columns of pies and annas, the pupils adhere to the traditional method, let them alone. If they are not quick at the method, drill them to say (*a*) in adding the pies' column 6 pies, 14 pies, 1 anna 2 pies, 1 anna 6 pies, 1 anna 11 pies, 1 anna 17 pies, 2 annas 5 pies. Put 5 pies in the pies' column and (*b*) in adding the annas' column 6 annas, 10 annas, 18 annas, Re. 1–2 annas, Re. 1–10 annas, Re. 1–16 annas, Rs. 2, put " 0 " in the annas' column and carry " 2 " to the rupees' column.

Point out to pupils the resemblance between the above addition and the ordinary addition of integers, viz., in adding up the number of pies, next the number of annas and next the number of rupees just like adding up units first, tens next, then hundreds and so on with this difference that while in the latter case,

ten units make	1 *ten*
ten tens make	1 *hundred,*
ten hundreds make	1 *thousand,*

in the former case, 12 pies have to be reckoned as one anna, and 16 annas as one rupee. Lead pupils hence to understand that the four rules applied to money may be done in more or less the same way as in the case of ordinary integers and that similar methods would hold good in applying the four rules to quantities expressed in terms of the units of weight, capacity, length, etc. Only the tables giving the relation between the smaller and the larger units of the same kind must be well borne in mind.

Propose:—

A cultivator has Rs. 44–9–6 with him. He then buys paddy for Rs. 34–10–9. What balance is left with him ?

Let pupils work as follows :—

	RS.	A.	P.		RS.	A.	P.		RS.	A.	P.
	44	9	6	=	44	8	18	=	43	24	18
less	34	10	9	less	34	10	9	less	34	10	9
								i.e., equals ...	9	14	9

Insist on pupils verifying the correctness of the result by adding Rs. 34–10–9 and Rs. 9–14–9 and seeing if Rs. 44–9–6 is got. Practice may be given in the shop-method of complementary addition if that method is already familar to the pupils.

RS.	A.	P.
44	9	6
34	10	9
9	14	9

Working.—9 pies plus *3 pies* equals 12 pies or 1 anna, plus 6 pies equals 1 *anna 6 pies*; put 3 plus 6 or nine pies in the pies' column and carry 1 anna; 1 anna plus 10 annas equals 11 annas, *plus 5 annas* equals 16 annas or 1 rupee *plus 9 annas*, equals 1 rupee 9 annas, put *5 plus 9* or 14 in the annas' column, carry 1 rupee to the rupees' column; 1 rupee plus Rs. 34, Rs. 35, *plus Rs. 9* equals Rs. 44, *put Rs. 9* in the rupees' column.

If the pupils have a grasp of the method show how a question of the following type could be done in a single step :—

A ryot had Rs. 19–9–4. He bought seed for Rs. 6–6–6, chilly seedlings for Rs. 4–15–10 and sugarcane sets for Rs. 2–5–6. How much has he left with him ?

Let pupils adopt the method given below :—

	RS.	A.	P.
	19	9	4
less the sum of	(6	6	6,
	4	15	10
and	2	5	6)
equals	5	13	6

Working—

"*Pies*" *column.*—6 pies, 16 pies, 1 anna 4 pies, 1 anna 10 pies, *plus 2 pies, plus 4* pies equals 2 annas 4 pies. *Put 2 plus 4* or 6 pies in the pies' column, carry 2 annas to the annas' column.

"*Annas*" *column.*—2 annas. 7 annas, 22 annas, or 1 rupee 6 annas, 1 rupee 12 annas, *plus 4 annas, plus 9 annas* equals Rs. 2 annas 9. Put *4 plus* 9 or 13 in the annas' column, carry Rs. 2 *to the rupees' column.*

"*Rupees*" *column.*—Rs. 2, Rs. 4 Rs. 8, Rs. 14, plus Rs. 5, equals Rs. 19. Put Rs. 5 in the rupees' column. Direct pupils to add up Rs. 6–6–0, Rs. 4 15–10, Rs. 2–5–6 and Rs. 5–13–6 and test if a sum of Rs. 18–9–4 is obtained.

Next propose :—

Find the cost of sending 0 maunds of an article by goods train from one place to another at Re. 1–9–10 per maund.

(Let pupils adopt the working given below :—

	RS.	A.	P.
	1	9	10
		×	10
	10	90	100
i.e.,	10	98	4
i.e.,	16	2	4

If the cost of sending 24 maunds be required, ask pupils to find the cost of 4 maunds first and then obtain 6 times the result, i.e., multiply by 6 to get the cost of sending 24 maunds. Hence in questions involving multiplication by numbers greater than ten, let pupils adopt, wherever possible, multiplication by factors. Thus Rs. 15–5–10 × 20 equals (Rs. 15–5–10) × 4 × 5, i.e., equals (Rs. 61–7–4) × 5, i.e., equals Rs. 307–4–8 and Rs. 15–5–10 × 19 equals (Rs. 15–5–10) × 4 × 5 less Rs. 15–5–10, i.e., Rs 291–14–10.)

Next propose :—

A busta of Akullu actually contains 26 kunchams. I pay Rs. 19–8–0 *for it.* How much does this come to per kuncham ?

(Point out that Rs. 19–8–0/26 equals 312 annas/ 26, i.e., equals 12 annas. Point out that if the price of a busta had been Rs. 20–5 annas the price per kuncham may be found to be Rs. 20–5 annas/26, i.e., 325 annas/26, i.e., 12 annas plus 13 annas/26, i.e., 12 annas 6 pies. Point out also that the same method may be adopted in the case of division by larger numbers. If in any such division a fractional result occurs in the pies column, lead pupils to note that coins smaller than a pie do not exist and hence that a part of a pie greater than, or equal to, half a pie may be counted as one pie and a part less than half a pie may be neglected and thus the result may be expressed to the nearest pie. Associate this method of approximation with that effected for the expression of lengths to the nearest foot or inch. Illustrate this further by showing that the cost of a guava at 6 annas a pathika comes to 2 pies nearly and that that of sugarcane at 5 for an anna is 2 pies nearly.)

Propose the following :—

1. The following particulars give details about the cost of cultivating sugarcane in a plot :—

	RS.	A.	P.
Preparatory cultivation ...	2	8	3
Manure	27	1	5
Seed	44	5	7
After cultivation	52	6	10
Harvest and preparation for market	32	1	3

What is the total cost of cultivation ?

(*Ans.* Rs. 178–0–6.)

2. The cost of cultivating "*Pasupu*" over an acre has been found to be Rs. 100–9–9 during one year and the value of the produce obtained Rs. 500. Calculate the net profit in the cultivation. (*Ans.* Rs. 399–6–3.)

3. Find the net profit from chilly cultivation over an acre, given that the cost of cultivation is Rs. 88–8–6 and the value of the produce Rs. 200. (*Ans.* Rs. 113–7–6.)

4. In a garden there are 10,000 canes which would fetch locally 6 pies each. If the gardener would wait for a month, a company would offer for them Rs. 500. Find the gain that would be secured by waiting and selling to the company. (*Ans.* Rs. 187–8–0.)

5. The following details refer to the cost of raising Vazzanam paddy in a plot :—

	RS.	A.	P.
Preparatory cultivation ...	16	9	3
Manure	0	6	6
Seed	1	10	8
After cultivation ...	0	14	0
Irrigation	0	15	0
Harvesting, threshing ...	11	8	3

If the value of the produce obtained be Rs 81–0–5, calculate the net profit in the cultivation.

(*Ans.* Rs. 49–0–9.)

6. For pulling out "GOGU" a tenant engages 16 coolies at *Bedaparaka* each for a day. Find how much would be spent as wages for them. (*Ans.* Rs. 2–8–0.)

7. A coconut sells at *kani thakku beda*. Find how much should be given for a dozen coconuts.

(*Ans.* Rs. 1–5–0.)

8. The following details relate to the cost of raising a particular variety and the value of the produce obtained from sugarcane crop in our farm :—

Value of produce.

	RS.	A.	P.
Jaggery	1,359	1	4
Value of cane sets ...	206	5	2
Canes sold	6	3	6

Cost of production.

	RS.	A.	P.
Preparatory cultivation ...	127	2	0
Manure	182	4	0
Cost of sets and charges for planting	133	0	0
After cultivation	319	14	5
Harvesting and preparing jaggery	178	3	7

Calculate the net profit of raising the crop.

(*Ans.* Rs. 559–10–9.)

9. The following details relate to pyru ragi raised in a plot :—

Value of produce.

	RS.	A.	P.
Grain	103	1	7
Straw	10	1	3

Cost of cultivation.

	RS.	A.	P.
Preparatory cultivation	26	15	9
Manure	5	3	4
Seed and seed-sowing ...	6	15	5
After cultivation ...	5	1	7
Irrigation	16	13	11
Harvesting, threshing ...	10	12	3

Calculate the net profit in raising the crop.

(*Ans.* Rs. 41–4–7.)

10. For the bare maintenance of a ryot's ploughing pair the following cost is incurred :—

For each month from April to August and for each of the months January and February, straw Rs. 3-12-0; watching and cleaning sheds, Re. 0-15-0; and for each month from September to January, straw Rs. 2-13-0; watching and cleaning sheds, Re. 0-15-0. Find at this rate the total cost of maintaining five pairs for a year.

(*Ans.* Rs. 257-13-0.)

11. If a viss of jaggery costs 6 annas, find how much of jaggery can be got for Rs. 6.

(Point out short cuts like the following :—

Rs. 6 = 6 × 16 annas, i.e., 16 six annas. For 6 annas, a viss of jaggery can be got. Therefore for 16 six annas, 16 viss or 2 maunds can be got.)

12. If a viss of jaggery costs 9 annas, find how much of jaggery could be got for Rs. 4-8-0. (*Ans.* 1 maund.)

13. A shepherd finds that to maintain 32 sheep it costs Rs. 100 per year. How much does it cost per head on the average? (*Ans.* Rs. 3–2–0.)

(Point out the use of the following short cut in working the question :—

Rs. 100 equals 100×32 half annas ;
therefore the cost of maintaining 32 sheep is 32 " hundred half annas"; therefore the cost per head is "hundred half annas," i.e., 50 annas or Rs. 3-2-0.)

14. If the price of 2 maunds of jagerry is Rs. 7, how much does this come to per viss ? (*Ans.* 7 annas.)

(Teach short cuts like the following :—

(1) If the price of an article is " X " pies, that of a dozen is " X " annas.

(2) If the price of an article is " X " annas, that of 16 articles is Rs. " X."

(3) If the price of an article is Rs. 15-15-10, that of X articles is Rs. " 16 X " less " 2 X " pies.)

15. A cultivator requires 25 bustas of rice for his family-one year. A busta costs Rs. 17-10-0. How much will the rice he requires cost him ? (*Ans.* Rs. 440-10-0.)

16. A yard of cloth costs Re. 0-15-6. How much should I pay for 12 yards ? (*Ans.* Rs. 11-10-0.)

17. A cooly earns 6 annas a day. Find his wages for 32 days. (*Ans.* Rs. 12.)

18. The price for a kantlam of certain articles has been as follows one day in the Vizianagram market :—

Tamarind, Rs. 6 ; Jaggery, Rs. 20 ; Onions, Rs. 8, and Turmeric, Rs. 12. What is the price of a visam of each article at the above rate ?

(*Ans.* Tamarind 6 coppers or 1 an. 6 ps. ; Jaggery 20 coppers or 5 as. ; Onions 8 coppers or 2 as. ; Turmeric 12 coppers or 3 as.)

(Let pupils use the undermentioned artifice :—

1 kantlam equals 8 mds., i.e., 64 viss., Re 1 equals 64 quarter-anna pieces or 64 coppers or 64 kanis.

So, if the price of a kantlam is Rs. 6, the price of 64 visams is six " 64 kanis."

Therefore the price of a visam of the article is 6 kanis or "coppers," i.e., 1 a. 6 ps.)

19. In the local market the price of a visam of jaggery is 6 annas, of tamarind 3 annas and of turmeric 8 annas. What is the price of a kantlam of each article ?

(*Ans.* Jaggery Rs. 6 × 4 or Rs. 24 ; Tamarind Rs. 3 × 4 or Rs. 12 ; Turmeric Rs. 8 × 4 or Rs. 32.)

(Lead pupils to deduce a short cut in each case above. Point out that the price of a visam of jaggery is 6 annas or 24 kanis and therefore that of a kantlam is 24 kanis × 64 or Rs. 24. Lead them to apply the short cut in the other cases too.)

20. Achayya's two goats and four cows grazed in Bapannah's paddy lands under cultivation. If they were shut up in a distant cattle pound, how much should Achayya remit to the Village Munsif to redeem them at 4 annas a cow and 1 anna a goat ? (*Ans.* Re 1-2-0.)

21. If a busta of rice contains 60 addas and costs Rs. 17-8-0, how much will the man buying it pay for an adda ? (*Ans.* Re. 0-4-8.)

22. If a busta of rice contains 50 addas and costs Rs. 14-2-0 and if the dealer sells the rice in retail gaining Rs. 2 per busta, how much should be charged for an adda of rice ? (*Ans.* Re. 0-4-6 to the nearest pie.)

23. I bought cattle manure at Rs. 1-8-0 a cart-load including cart-fare. I thereby spent Rs. 22-8-0 for manure. How many cart-loads did I buy ? (*Ans.* 15.)

24. A smith buys wood and iron for Rs. 10 and makes 7 ploughs out of them. If he must get Rs. 2-4-0 for his own labours, for how much must he sell a plough ?

(*Ans.* Re. 1-12-0.)

25. A *golla* keeps 3 she-buffaloes each yielding 2 seers of milk in the morning as well as in the evening and 4 cows each yielding 4 seers of milk in the morning as well as in evening. Find how much he can realize in a month of 30 days by selling the milk at 3 annas a seer.

(*Ans.* Rs. 247-8-0.)

If he has to spend 12 annas a day for feeding each animal, how much would remain after meeting this expenditure from the sale proceeds of milk ? (*Ans.* Rs. 90.)

26. A field is bounded by four bunds which are respectively 250 links, 200 links, 300 links and 400 links long. Express in chains and links the distance round the field.

(*Ans.* 11 chains 50 links.)

27. In a plot 70 yards long and 70 yards wide paddy seedlings are planted 4 inches apart in rows and in columns. Find (*a*) the number which can be planted in a row lengthwise, (*b*) the number of rows that would be so planted and (*c*) the total number so planted in the plot?

(*Ans.* (*a*) 630, (*b*) 630 and (*c*) 396,900.)

28. Answer the above question with the seedlings planted 6 inches apart. Find the saving under seedlings by adopting such a method of planting.

(*Ans.* (*a*) 421, (*b*) 421, (*c*) 177, 241, 220, 920 seedlings.)

29. In a field 100 yards long and 80 yards wide tobacco seedlings are planted 2 feet apart each way. Find the number that can be planted in a row, the number of rows that can be planted and the number of seedlings so planted in the plot.

(*Ans.* (*a*) 150, (*b*) 120 and (*c*) 18,000.)

30. In a road 4 furlongs long leading through our farm it is proposed to plant on one side trees 32 feet apart. Find the number of trees that could be so planted.

(*Ans.* 83 trees.)

31. Work the above exercise with guava grafts 20 feet apart substituted in place of coconut trees 32 feet apart. (*Ans.* 133 guava grafts.)

32. It is proposed to sow '*gogu*' over 280 acres of lands near Nilanagaram. At 4 kunchams per acre how much of seed would be required? Express the quantity in garisas and puttis. (*Ans.* 1 garce 26 puttis.)

33. Expecting a yield of 35 maunds of fibre from an acre, express in puttis the weight of the yield from the lands referred to in the last exercise. (*Ans.* 448 puttis.)

34. A busta of '*Doosi Sannellu*' weighs 190 lb. Express the weight of 200 bustas in tons, cwt., to the nearest cwt. (*Ans.* 16 tons, 19 cwt., to the nearest cwt.)

35. If a brinjal costs 5 pies, how many could be got for Rs. 1-4-0? (*Ans.* 48 brinjals.)

36. The weights of certain loads of palm fibre supplied by Kothandam Pantulu to the local fibre factory during a month have been found to be as follows :—

10 cwt. 0 qr. 20 lb.	7 cwt. 2 qrs. 20 lb.
5 cwt. 0 qr. 8 lb.	5 cwt. 1 qr. 22 lb.

Find the total weight of the fibre supplied.

(*Ans.* 1 ton, 8 cwt., 42 lb.)

37. Out of 10 cwt. 0 qrs. 20 lb. of fibre got from a customer a bale was packed and sent weighing 8 cwt. 0 qr. 30 lb. What weight would remain ?

(*Ans.* 1 cwt. 3 qrs. 18 lb.)

38. From a mixed crop of jonna and pesalu a cultivator gets 5 puttis of jonna and 2 puttis of pesalu. Estimate the value of the produce at 5 kunchams of jonna a rupee and 3 kunchams of pesalu a rupee. (*Ans.* Rs. 33–5–4.)

39. A gundu of casuarina costs 7 annas 6 pies and one of country wood 5 annas 6 pies. If two gundus make 1 cwt., find the cost of a ton of casuarina and a ton of country wood at the above rate. (*Ans.* Rs. 32–8–0.)

40. For making '*Annam Thinne Ginnalu*' lead costs Rs. 7–4–0 an 'ethidu' (= 2 viss) and the making charges 5 dabbus extra for a seer. A cultivator wants to buy '*Annam Thinne Ginnalu*' weighing 12 seers. How much will he be charged at the above rate ? (*Ans.* Rs. 9–15–2.)

41. Find the cost at the rates mentioned in the last exercises of '*Ginne*' weighing 18 seers.

(*Ans.* Rs. 1–14–10.)

42. Find the total cost of turmeric cultivation in a plot from the particulars given below by a ryot at Tenali :—

Three ploughings at Rs. 4–8–0 for each ploughing, 2 candies of rhizomes at Rs. 20 a candy.

Twenty cart-loads of farm-yard manure at Rs. 1–8–0 a cart-load.

Wages given for 12 coolies for planting for each of two days at 5 annas a cooly for each day.

Weeding Rs. 5.

Water-tax Rs. 17.

Harvesting and Transport charges.—Two ploughings per day for each of three days at Rs. 3 per ploughing per day.

Twenty coolies for each of 3 days at 5 annas per cooly per day, Rs. 2 for carting home. (*Ans.* Rs. 151–10–0.)

If 12 puttis of turmeric (pasupu) were obtained and sold at Rs. 50 a putti, find the net profit in the cultivation.

(*Ans.* Rs. 448–4–0.)

43. In a rice-mill the charges for pounding a busta of paddy is 4 annas during ordinary seasons and 5 annas during harvest seasons. A family has to pound 12 bustas during ordinary seasons and 4 bustas during harvest seasons. How much does the family give as pounding charges during the year? (*Ans*. Rs. 4–4–0.)

44. A pair of '*Kadayamulu*' of solid silver weighs 80 tolas. The goldsmith demands 15 annas as the cost of metal for a tola weight and Rs. 5 extra as making charges. What does he demand for the pair? (*Ans*. Rs. 80.)

45. A pair of 'Thodalu' weighs 25 tolas. The goldsmith demands 15 annas as the cost of metal for a tola weight and 4 annas per tola as making charges. How much does he thus demand for the pair? (*Ans*. Rs. 29–11–0.)

46. During Vari Panta a cultivator gets 12 puttis of Sunki Sennelu. If the cost of paddy be Rs. 12 for a garisa of 600 kunchams, estimate the value of the paddy obtained. (*Ans*. Rs. 50.)

47. A barva of jaggery sells at Rs. 65. If 20 manugulu make one barva, find the price per manugu. (*Ans*. Rs. 3–4–0.)

48. If guavas sell at 11 annas per 100, find how much could be got for Rs. 5–8–0. (*Ans*. 800.)

49. The boundaries of a field are 40 yards 2 feet, 24 yards 1 foot, 30 yards 2 feet, and 39 yards 2 feet, respectively. Find the cost of enclosing it with fence at Rs. 2 a yard length. (*Ans*. Rs. 270–10–8.)

50. A tenant had undertaken to give his landlord 2 puttis of paddy, 3 maunds of jaggery and Rs. 300 as rent for cultivating his lands and availing of the produce. Estimate the value of the rent when paddy sells at Rs. 60–8–0 a putti and jaggery at Rs. 3–2–0 a maund. (*Ans*. Rs. 430–6–0.)

TOPIC 12.—ON TIME-MEASUREMENT.

(**Scope of the treatment** :—*This topic is intended here mostly for revision. The several units adopted in the indigenous and the British systems of time-measurement may be dealt with as well as their inter-relations. The study of the calendar, the Telugu calendar in particular, may next be taken up and along with it any agricultural rhymes current in the locality which suggest the important agricultural operations peculiar to the several 'Karthulu.' Digression from the essentials, e.g., too much dilating on the reasons why there are leap years and 366 days for them, dwelling in detail on the relation between time and space, etc., may be avoided. The pupil should be made familiar with reading time from the clock-face as well as reckoning time of day by indigenous methods like shadow-measurement, if they continue to be adopted in the locality. Sufficient exercise may be given so as to enable pupils to judge of the time of day by mere sight, say of the sun or of the shadow cast by a wall, etc.*)

The units adopted in time-measurement and their inter-relation.

Question pupils as to how people in the locality express how long a crop lasts, how long it is since they ploughed a field, or watered a garden, or planted seedling in a plot, purchased a cattle pair, borrowed a sum of money or repaid it, when the train from Waltair reaches Anakapalle, what time after sunrise or sunset a child was born and so on. Thus, remind pupils of the several units in terms of which time is expressed, the year, month, week, day, hour, minute, second, the ghadiya, jamu, etc. Point out that of the various units adopted for expressing time the chief from which the others are derived are the '*day*' and the '*year*,' the day being the time taken by the earth to make one rotation and the year being the time taken by the earth to go once round the sun along its orbit. Thus, lead pupils to note that in the case of other measures, e.g., of capacity or of length, the standards of measurement have been only arbitrarily chosen by man, e.g., the adda or

manika as the capacity of a particular vessel, the yard as the distance between two points arbitrarily chosen but the units of time-measurement are based on the movements of the earth through space. Point out that no one can dictate a change in the duration of the day or of the year, but it is easy to rule out a change in the length of the yard or in the capacity of a manika.

Next question pupils how many days generally make one year, one month and one week, respectively, how many hours there are in a day, how many minutes in an hour and how many seconds in a minute. In this connection bring out the following facts :—(i) As the earth rotates about 365 *times* in the course of a single revolution there are *about* 365 *days* in the year. (ii) As the number of times the earth rotates in the course of a revolution is not exactly 365 but is somewhat greater than 365 and less than 366, some years are taken as *leap* years containing 366 days and the others as ordinary years of 365 days. (iii) As periods of shorter duration than a year but longer than a day they have the months of which 12 are reckoned for the year and the weeks of 7 days each. (iv) For expressing shorter intervals of time than a day, the day is divided into 24 hours, the hour into 60 minutes and the minute into 60 seconds. Get pupils to realize that in the indigenous system also they have the day, the week, the month and the year as units of time. Tell them that a whole day is there taken to contain 60 'Ghadiyas', that periods shorter than a 'Ghadiya' are generally expressed as fractions of a 'Ghadiya', that such periods are at times expressed in 'Vighadiyas' of which 60 are taken to make a ghadiya and that as a period longer than a ghadiya but shorter than a day they have the 'Jamu' which means a period of seven and a half ghadiyas. Lead pupils to work that as 24 hours make 1 day as well as 60 ghadiyas, an hour may be taken as two and a half ghadiyas, and a jamu as denoting a period of 3 hours, that for the same reason 60 ghadiyas may be taken as the equivalent of 24 × 60 minutes and hence 24 minutes may be reckoned for a 'Ghadiya' and that as 60 Vighadiyas may be taken as equivalent to 24 × 60 seconds, 24 seconds may be reckoned for a vighadiya. Let them then record the tables expressing the relation among the units of time-measurement and memorize.

60 seconds make 1 minute.
60 minutes make 1 hour.
24 hours make 1 day.
365 days make 1 common year.
366 days make 1 leap year.
60 vighadiyas make 1 ghadiya.

60 vighadiyas make 1 day.
24 seconds make 1 vighadiya.
24 minutes make 1 ghadiya.
2½ ghadiyas make 1 hour.
3 hours make 1 jamu.
8 jamus make 1 day.

The Calendar.

Place a calendar at the disposal of the pupils. Ask them how the current year is named in the English and the Telugu systems. Point out that in the English system the year is marked out only by the serial number in the Christian Era while in the case of the Telugu systems not only the serial number according to the Telugu Era (*Salivahana and Vikramaditya Eras) is given but also a* definite name is assigned to the year. Tell pupils that sixty years are so named and form a cycle of constant repetition. Incidentally point out that the numbering of the year is only arbitrary as it indicates the lapse of a definite number of years after a particular event, e.g., the birth of Christ in the case of the Christian Era, etc.

Ask pupils to find out from the Telugu and the English Calendars the commencement of the Telugu and the English year. Let them note that the Telugu New Year's Day occurs about three months after the English New Year's Day. Get them next to give out, in order, consulting the calendar if necessary, the names of the months in the English year as well as in the Telugu year, e.g., January, February, March, April, May, June, July, August, September, October. November and December, for the English year and Chitram, Vaisakam, Jeshtam, Ashadam, Sravanam, Bhadrapadha, Asweejam, Kartigam, Margasiram, Pushyam, Magham and Palgunam for the Telugu year. (If the pupils are not familiar with the above order in which the months occur they may be directed to get it memorized.) Ask pupils to examine a Telugu Calendar for the year and find out to what periods according to the English months the Telugu months correspond. Lead them to gather the above information by consulting the Telugu Calendar for three consecutive years.

The English year.

Let pupils next examine the number of days assigned to each month and if there is an equal number of days in all the months. Point out that in the English system, the distribution of 365 or 366 days among the 12 months has resulted in the number of days varying with different months and that the distribution is effected as expressed in the following rhyme:—

Thirty days hath September,
April, June and November,
All the rest have thirty-one,
Except February alone,
Which has but twenty-eight days clear,
And twenty-nine each leap year.

Direct pupils to memorize the rhyme in English or its substance in the vernacular. Point out that in the English system, February should ordinarily be assigned 28 days, that if the serial number of the year according to the Christian Era is exactly divisible by 4, February may be taken to contain 29 days except in the case of Century Years, and that in the case of Century Years, the serial number of the year should be divisible by 400 in order *that February may be assigned 29 days.*

The months of the year in the Chandramana System, the Pakshas and Thithis.

Ask pupils to examine the Telugu Calendar carefully and *find on what "Thithis" the months begin* or *end.* Lead them to note that they begin on the Sudda Paddimi Day and end on the New Moon Day. Tell them that the moon is restlessly moving round the earth and the time it takes for a single period of revolution is that from New Moon to New *Moon* and that they reckon *this period as* their month—the Lunar or Chandramana month of thirty thithis. Tell them also that they have the New Moon Day when the moon lies between the sun and the earth and the full moon day when the moon is on the exactly opposite side of the earth away from the sun.

(Illustrate by means of diagrams of the following type.)

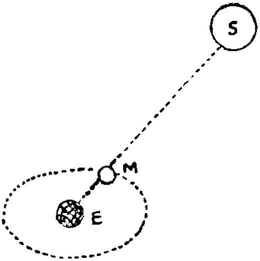


Fig. (1)

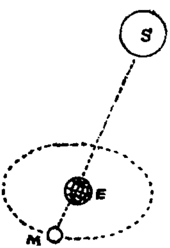


Fig. (2)

Get them to note that the lunar-month, i.e., the period from New Moon to New Moon contains about 29½ days only. Let them compare the period in days of 12 such months and the year of 365 days. Point out that 12 such months fall short of the year of 365 days and that to make up the shortage they have once in three years an extra month called '*Adhikamasam*' occuring somewhere between Chitram and Bhadrapadam. (The teacher may note that a lunar month should, strictly speaking, contain 29·53059 days and the year 365·2564 days.) Incidentally point out that, in Adhikamasam, there are 30 days generally and that the number varies according to alteration in thithis, i.e., increase or decrease of thithi.

The Thithis.

Direct pupils to examine the Telugu Calendar for a month and note down the thithis in serial order during a month. Get them to note that there is a period of 15 thithis, beginning with Paddimi, Vidhiya, Thathiya, etc., etc. ending with Full Moon, alternating with another beginning likewise, but ending with the New Moon. Tell them that these periods are called *Pakshas* and are distinguished as white and dark pakshas. Point out that the white paksha means a period of the waxing moon, i.e., refers to the period taken by the moon to make half a revolution from the position shown in *Fig.* (1) to that shown in *Fig.* (2) above and that the dark paksha is the period from Full Moon to New Moon.

Next tell pupils that, because of the earth's rotation, the moon appears to move round the earth once in about a day and the time taken by the moon for such an apparent movement round the earth is known as a 'Thithi.' Let pupils next examine a Telugu Calendar and verify if the thithis cover a day each and are of equal duration. Point out to them that they do not cover a day each and are not of equal duration. Intimate to them that this is due to some peculiarities in the movement of the moon. Point out to them also that instead of assigning dates to the several days of the Telugu month it has been the custom to name each day after the prominent thithi, though, now-a-days, the *custom is gradually being given up in preference to the* dates of the English months.

The 27 Stars.

Refer pupils again to the Telugu Calendar and let them examine the names of the stars mentioned against the days of the month. Get them to note that the names of 27 stars are found to occur repeatedly in the following order:—Asvani, Bharani, Krithika, Rohini, Mrigasira, Arudra, Punarvasu, Pushyami, Aslesha, Makha, Pubba, Uttara, Hastha, Chitra, Swati, Visakha, Anoradha, Jeshta, Mula, Purvashada, Uttarashada, Sravanam, Dhanishta, Satabisha, Purvabhadhra, Uttarabhadra and Revati. Tell them that it is usual to say that the moon spends about a day in the house of each star in the above order. Point out to them that the significance of this is that the moon's path round the earth has been *divided into* 27 parts and as the parts appear to lie in order near the stars above specified they are named after them. Thus get pupils to note why the several days of the month are said to be ruled by one or other of the above stars. Let them examine a Telugu Calendar once again and infer that the periods of influence said to be exercised by the stars are not all equal. Intimate to them that the difference in the periods of duration are due to the parts of the moon's path above referred to being unequal.

The Karthulu and their Padams.

Refer pupils to how people in rural areas make mention of Karthulu and padams in regard to beginning the cultivation of a crop, the commencement of the monsoonal rains,

etc. Tell them that as the sun's apparent path during the year is only slightly inclined to the path of the moon, the sun's path is also divided into 27 parts and the parts named successively after the 27 stars above mentioned which are found to lie in the neighbourhood. Intimate to them that the sun is also said to be in the house of a star or Karthu when it passes by it and the period it spends in the jurisdiction of a star or karthu is named after the star or Karthu. *Thus get them to note how the 27 karthulu are obtained.* Next describe to them how the 27 karthulu beginning with Aswani are divided among the 12 months of the year beginning with Chaitra and how to represent fractions of a karthu each of the karthulu is divided into 4 quarters or padams. Get them to remember the following distribution of the Karthulu among the several months consequent on such apportioning :—

Month.	**Karthulu.**
Chitra ...	... Aswani and Bharani, the first padam of Krithika.
Vaisakam	... Second, third and fourth padams of Krithika, Rohini, first and second padams of Mrigasira.
Jeshtam ...	... Third and fourth padams of Mrigasira, Arudra, first, second and third padams of Punarvasu.
Ashadam	... Fourth padam of Punarvasu, Pushyami and Ashlesha.
Sravanam	... Makha, Pubba, first padam of Uttara.
Bhadrapada	... Second, third and fourth padams of Uttara, Hasta, first and second padams of Chitra.
Asweejam	... Third and fourth padams of Chitra, Swati, first, second and third padams of Visaka.
Karthikam	... Fourth padam of Visakha, Anoradha and Jeshta.
Margasiram	... Mula, Purvashada, first padam of Uttarashada.
Pushyam	... Second, third and fourth padams of Uttarashada, Sravanam, first and second padams of Dhanishta.

Makham ...	...	Third and fourth padams of Dhanishta, Satabisha, first, second and third padams of Purvabhadra.
Palgunam	...	Fourth padam of Purvabhadra, Uttara bhadra and Revati.

Point out that thus nearly two karthulu are assigned to each of the twelve calendar months from Chaitram to Palgunam.

Let pupils distribute the 365 days of the year among the 27 karthulu and note that the division would roughly give about 13 days for each of the Karthulu. Tell them that from Chaitram to Bhadrapadam one Karthu is very nearly of 15 days' duration and from Asweegam to Palgunam it is of 13 days' duration. Tell them also that the Karthulu which bring in rain in their parts are from Mrigasira to Anoradha, i.e., from the full moon day of Jeshtam to that of Margasiram and to the agriculturist these 13 karthulu are very important. If any agricultural rhymes are current in the locality giving the agricultural operations suited to the Karthulu, allude to them and let pupils memorize.

Judging time from the clock and by measurement of shadow.

Show pupils a clock-face and the minute and the hour hands in it. Question how many minute-spaces the moving ends of the minute and the hour hands move in the course of an hour and where they should be pointing at stated hours. Thus lead pupils to the correct reading of time from the clock-face.

Next refer to how the time of the day is reckoned by the villagers in the absence of a clock. Point out that during the day-time the villagers reckon time by measurement of shadow. If the pupils admit the existence of the method in the parts to which they belong, inform them that the following is one of the chief methods peculiar to the locality.

Suppose the length of one's shadow is 8 feet on the forenoon of a certain day as measured by one's own foot. Suppose also that according to the calendar the period of duration of day-light to be 27 ghadiyas, 40 vighadiyas.

Subtract from this a constant 26 ghadiyas, 30 vighadiyas, add 8 ghadiyas, the number 8 being suggested by the number of feet in the length of the shadow measured. Thus get a total of 9 ghadiyas, 10 vighadiyas. Keep this result separately. Multiply the period in ghadiyas of the duration of day-light for the day as obtained from the Telugu Calendar, viz., 27 ghadiyas, 40 vigadhiyas for the particular day specified above by a constant "8" and thus get the product "83 *ghadiyas*." Divide this 83 ghadiyas by 9 ghadiyas, 10 vighadiyas which has already been got and find the quotient to be 9 approximately in the above case. Assume this 9 to represent so many ghadiyas and convert this 9 ghadiyas into hours and get, for example, three and a half hours nearly in the above case. Then infer that it is three and a half hours after sunrise. If the measurement had been taken afternoon, infer that it is three and a half hours to sunset. The teacher may note that the above rule may be remembered by memorizing the following slokam :—

Ahasvarthanga Neithrynam,
Thaschesham Padasmyutham,
Divamanam Thrigunitham,
Vibhajethe ghatinirnayah.

In the course of different days ask pupils to guess the time at stated instants, in hours after sunrise or to sunset, and verify the correctness of the result by calculation from shadow or by means of a clock. *Thus see that the pupils attain the power to judge fairly correctly the time of day corresponding to any instant.*

Propose the following exercises :—

1. How many days are there from 11th January 1922 to 15th April 1923?

(Point out that only one of the dates either 11th January 1922 or 15th April 1923 should be counted and not both. Let the pupils work as follows :—

In January there are 21 days including the 11th
In February „ 28 „
In March „ 31 „
In April „ 14 days, the 15th of April being omitted in view of the 11th of January being included.

Altogether, there are 94 days during the period specified.)

2. For a journey by rail it takes from 3-10 one afternoon to 6-20 next morning to reach from Anakapalle to Madras. What time does the railway journey take?

(*Ans.* 15 hrs. 10 minutes.)

3. If the above train was late and reached Madras only at 8-40 next morning, by what time was it late?

(*Ans.* 2 hrs. 20 minutes.)

4. There was a solar eclipse lasting from 7-45 to 10-15 one morning. How long did the eclipse last?

(*Ans.* 2 hrs. 30 minutes.)

5. A sugar-mill is rented from the 4th December one year to the 20th February next year. For how many days is it rented? (*Ans.* 78 days.)

6. The following details refer to crops grown in our farm during certain years.

Crop.	Date of sowing.	Date of harvesting.
Sugarcane	7-5-1915	10-4-1916
Punasa-chodi ...	3-6-1915	30-11-1915
Maharaja-Bogam ... Paddy.	13-7-1915	3-12-1915
Vazzanam Paddy...	21-7-1915	14-12-1915
Bungaratheegam Paddy.	16-9-1918	15-12-1918
Pyru ragi ...	18-1-1920	30-3-1920
Pyru gingelly ...	4-2-1920	22-4-1920
Pyru chodi ...	19-1-1920	23-9-1920

Find the number of days in the period between the dates of sowing and harvesting in each case above.

(*Ans.* 339 days, 180 days, 144 days, 147 days, 60 days, 72 days, 78 days, 70 days.)

7. A man was born on the 7th January 1882. How old was he on the 6th January 1922? (*Ans.* 40 years.)

8. The *farm clock* is 10 minutes slow. A labourer who ought to be at *6 o'clock in the forenoon came* ten minutes late for his work in the farm. What was the correct time when he came? (*Ans.* 6-20 a.m.)

9. Answer the above question taking the clock to be 10 minutes fast. (*Ans.* 6 a.m.)

10. A house-holder lent Rs. 100 to B. Hanumanta Rao on 8th January 1920 on the strength of a pro-note to that effect. If the pro-note should be renewed for legal purposes before the end of a period of three years, before what date should he get it renewed if the money is not paid in the meanwhile? (*Ans.* 8-1-1923.)

11. A contractor undertook to finish up clearing silt from a channel within three months from 14th April 1922, the date of contract. If he required 15 days' extension find by what date he expects to finish the work.

(*Ans.* 29-6-1922.)

12. A train leaves Waltair at 6-40 one morning and reaches Anakāpalle at 9-15 the same morning. How long does it take to reach Anakāpalle from Waltair?

(*Ans.* 2 hrs. 35 minutes.)

TOPIC 13.—APPROXIMATE ESTIMATION.

(**Scope of the treatment.**—*The pupils may at first be given some notions about the inexact nature of all our measurements and about approximate expression being hence a necessity. With reference to quantities like the worth of a landlord, the value of a garden, that of the exports and imports of a town, etc., it may be pointed out why exact evaluation is impossible and approximate estimation of values a necessity. The convention regarding the approximate expression of values may be explained. The use of rough estimates and rough checks in working questions may be shown. Plenty of drill in approximate estimation may be given in regard to value of lands, cost of cultivation, quantity and value of yield, so far as only the four fundamental rules are involved. The estimation may also extend to finding the number of plants which could be raised in plots of specified dimensions at stated distances apart, etc.*)

The necessity for approximate expression.

(*a*) *Imperfections in measurement.*

Refer pupils to what they have already experienced while measuring lengths about their measurements being inexact. (See paragraph with the heading "Approximation in measurement" in the treatment of the topic " Measurement of length ".) Remind them of the causes giving rise to a large number of petty and unavoidable errors, e.g., of not measuring along a perfect straight line, of the sag occurring unnoticed while stretching the tape or chain, of the difficulty in marking precisely the place at which the ends of the measuring rod or chain fall when laid out successively, of the chain wearing out at the joints and getting longer, of the tape or string measuring out varying lengths because of its elastic nature, etc. Point out that similar defects exist in the measurement of capacity, weight, etc. Cite also the following :—(1) If a man measures out 200 kunchams of ragi or paddy or dry chillies from a heap another may be measuring out a little more or little less ; (ii) if the same quantity of yam or firewood be weighed in

visams or gundus, there will be noticed a difference in the total weight obtained as weighed by different persons or on different occasions, and the balance used for weighing could not be relied on as weighing out with perfect accuracy; (iii) when they express the time of day at a particular instant, say, 10 o'clock by reading from a clock face they seldom care to read minutely to the last second and, besides, the time they give out cannot be relied on as quite accurate as the correctness of the time indicated by the clock is also open to question. By illustrations of the above type, bring home the fact that an absolutely exact measurement is impossible and get them to note that therefore they have to be satisfied with approximate measurements. Point out also that once the measuring unit is fixed, the quantity to be measured cannot be expected to contain it an exact number of times and that therefore approximation in measurement has to be resorted to.

(b) Difficulties in the way of exact evaluation.

Next get any pupil to express how much his father or any rich person in his native place is worth, for how much a familiar garden is valued, how much his father or anybody else has spent in erecting a house, in digging a well in the garden, in buying a trotting pair of bullocks, in celebrating a marriage, how much of paddy or ganti a ryot gets from the crop in his fields. Set before pupils statements of the following type made in the above connection :—

"My father is worth Rs. 4,000.

The neighbouring gardens are worth Rs. 8,000.

Ramesam spent Rs. 600 for celebrating his daughter's marriage.

R. Devayya got 600 kunchams of paddy from his lands.

A well cost Rs. 350.

A tiled house cost Rs. 3,000 to build.

A trotting pair of bullocks is worth Rs. 400.

etc. etc. etc."

Get pupils to examine the correctness of the figures arrived at in the above cases and let them suggest a good number of petty errors entering into the valuation. For example, with reference to a statement of the type "My

father is worth four thousand rupees" made by a pupil question him what items he took into account to get at the above statement. If he should give out that he took into account the extent of lands possessed, lead other pupils to suggest that a good deal of other items he has yet to reckon —the value of his house, the bullocks, the implements, the jewels, vessels, etc., and that hence his valuation is inaccurate. Let him next take into account all such items as mentioned above and once again express more accurately that his father is worth, say, six thousand rupees. Point out to pupils that even then his valuation is imperfect. Get them to note that the pupil who so expressed his father's wealth has actually over-valued his belongings or undervalued them a little when compared with what they would actually fetch and that it is altogether impossible to express beforehand what exactly the property, etc., would fetch when sold.

With reference to a statement of the type "the neighbouring gardens are worth Rs. 3,000". get pupils to argue similarly and conclude that quite an accurate valuation is an impossibility. Point out, if necessary, that with more competitors, nearness to roads and markets, the gardens may fetch a higher price, that the valuation rests upon chance, that the charges incidental to the sale, etc., e.g., advertising, brokerage, etc., have also to be reckoned and that all these items are only the result of circumstances and could hence never be foretold with certainty.

In a like manner, lead pupils to suggest a lot of petty items not included in the calculation when people give out the values of houses, the value or quantity of produce obtained, the expenditure incurred in digging a well, buying a carting pair, celebrating a marriage, etc. Thus make them feel that in expressing values it is almost very difficult to be absolutely precise and they have hence to be content with an approximate expression of the values.

(c) *Approximate expression as giving a fair and a handy estimate.*

Next propose to pupils to examine the correctness of the figures contained in a statement like the following made by a person: "The exports of jaggery from Anakapalle came to 25,687 manugus during a particular month." Lead

pupils to note that the figures could not be claimed to be quite accurate as it is almost an impossible task to reckon each and every manugu of jaggery sent from the town. Explain to them, if necessary, that however keen and sharp the person reckoning the quantity of exports might have been to get at the correct figures, there is the possibility of his figures going wrong, e.g., because of the smuggling taking place without his knowledge, of the quantity sent from a market being eaten or used on the way to the railway station, or of undetected theft taking place meanwhile, of excess weight or of under-weight occurring in the course of weighing, etc.

Next question how these defects in the reckoning would tell upon the total quantity above expressed. Ask them which of the places—the units', tens', hundreds' or the thousands', etc., in the number 25,687—they are likley to affect more seriously. Get them to grant that the possibility of the defects affecting the ten-thousands' or thousands' place is remoter than that of their affecting the units' or tens' place and that as the person engaged in the reckoning would have been keen about arriving at the actuals as far as possible it would be giving a fair estimate of the actuals if one should express and remember it as 26 thousand manugus roughly.

Approximation: How effected.

(*a*) *Neglecting petty errors.*

Refer pupils to the various cases when they effected approximations in the past. Get them to note that they expressed sums of money involving fractions of a pie to the nearest pie, weights, e.g., of lead vessels, etc., involving fractions of a tola to the nearest tola, lengths smaller than a foot to the nearest foot, etc. Question them whether they were justified in so effecting approximations. Point out that in the case of money there were no coins smaller in value than a pie and hence they had to express results to the nearest pie and in the case of lengths, weights, etc., the errors arising from effecting approximations were only of a petty nature when compared with the actual values of the quantities taken up for approximate expression, e.g., a tola or two in the case of a weight of 4 or 5 lb., an inch or two in the case of a length of 18 or 20 feet, etc., and hence they considered them negligible.

(b) Effecting approximation by a consideration of errors.

Next tell pupils that some one took the trouble of ascertaining as far correctly as possible how much M. Thathayya is worth and found out that the entire property is worth Rs. 3,25,684. Ask them to express the value approximately in lacs. Point out that as Rs. 3,25,684 lies between Rs. 3 lacs and Rs. 4 lacs, the approximate value in lacs should be one of the two—Rs. 3 lacs or Rs. 4 lacs. Question pupils what the error comes to in expressing the value as Rs. 3 lacs and as Rs. 4 lacs, respectively. Get them to note that the error committed in expressing the value as Rs. 3 lacs consists in under-estimating the actual value by Rs. 25,684 and that committed in expressing the value as 4 lacs consists in over-estimating the actual value by Rs. 74,316. Question pupils which of the two involves less error. Point out that as the error is less in the former case than in the latter the value is expressed approximately as Rs. 3 lacs, thus choosing an approximate value which involves less error. Tell pupils that the value is then said *to be expressed to the nearest lac*, as Rs. 3,25,684 is nearer to Rs. 3 lacs than to Rs. 4 lacs. Next question them whether even the committing of a less error such as neglecting a sum of Rs. 25,684 is excusable. Point out that the error may be considered as negligible not only because the figures 25,684 are likely to be incorrect, but also because the sum forms but a petty part of so many lacs of rupees.

(c) The use of different units in approximate expression of quantities of the same kind.

Next propose that one is not prepared to neglect so many figures in expressing approximately the value of M. Thathayya's property, but is eager to be more accurate and if he chooses to express the value to the nearest thousand rupees question what he should take the value to be. Let pupils argue as before, find out that Rs. 3,25,684 is near to Rs. 3,26,000 than to Rs. 3,25,000 that the error is less in the former case than in the latter and that the value may therefore be expressed as Rs. 3,26,000 to the nearest thousand rupees. Here again get pupils to feel that the error has been committed of assuming the value to be Rs. 316 too much and that this sum may be considered negligible when compared with the total amount of Rs. 3

lacs and odd. Incidentally point out that the same sum may be expressed to the nearest lac or thousand or hundred or ten thousands as one may choose it necessary or desirable for approximate expression. In this connexion lead pupils to note that it is only from the standpoint of convenience and the degree of accuracy that is desirable and possible that the length of a road is measured to the nearest mile or furlong and not to the nearest foot, e.g., in statements of the type:—" Mulakapakka is 3 miles from Anakapalle," " Varadarajapettah is 2 furlongs from our farm," "the length of a field is measured to the nearest link and not to the nearest inch, the quantity of grain in a big heap on the threshing floor to the nearest kuncham and not to the nearest gidda or sola, the weight of jaggery obtained from sugarcane cultivation in a field to the nearest maund and not to the *nearest tola, etc.*" *Thus impress the fact that different* units may be choosen for approximate expression even of quantities of the same kind and that the choice of the units may be decided by the accuracy that may be sought or may be found possible and by the size of the quantities taken up for approximate expression.

The convention for effecting approximation.

Propose :—

"The quantity of selected paddy seed available for sale in a farm is given as 216 kunchams. Express to the nearest 100 kunchams what the available quantity is."

Let pupils argue as before that 216 lies between 2 hundreds and 3 hundreds, that it is nearer to 2 hundreds than to 3 hundreds and hence that the quantity to the nearest hundred kunchams may be expressed as 200 kunchams. Also, question what the error is in assuming the quantity as 200 kunchams and 300 kunchams respectively, and in which case the error is less. Point out here again that the quantity is only *expressed approximately* and that, in so expressing, a quantity like 16 kunchams is considered negligible when compared with so many hundreds of kunchams.

Next intimate to pupils that an argument like the above in each and every one of the several cases to be dealt with will take up a good deal of time and hence with a view to secure uniformity of result and rapidity in approximate expression the following convention is adopted.

First ascertain in terms of what unit the result is required to be approximately expressed, e.g., hundreds, thousands or lacs, etc., in rupees or hundred rupees or thousand rupees, in lb., or visams or seers or manugus, etc. *Find by division how many of the units are contained in the result. If the division leaves a remainder, compare the remainder with the unit chosen. If it be equal to or greater than half a unit, count it as one unit and if it be less than half a unit, neglect it.*

Propose to pupils to express in the above manner 216 kunchams approximately to the nearest hundred kunchams. Ask them to first find how many hundred kunchams there are in 216 kunchams and then find whether the remaining 16 kunchams is greater than, equal to or less than half of 100 kunchams. Point out that, as 16 kunchams is less than half of 100 kunchams it may be neglected and the quantity expressed as 200 kunchams to the nearest 100 kunchams. Similarly, let pupils express 1,818 manugus as 2,000 manugus to the nearest 1,000 manugus and Rs. 1,85,684 as Rs. 2 lacs to the nearest lac. Point out that Rs. 3,500 may, by the method of error consideration, be expressed either as Rs. 4,000 or Rs. 3,000 for in each case the error is only Rs. 500 and still it is expressed approximately as Rs. 4,000 according to the convention aforesaid. (The teacher will do well to note that the above method should not be replaced *at present* by another modified form of it, namely, of increasing by "1" the figure in the lowest place required if the figure to the right of it is 5 or greater than 5 and of leaving that figure unaltered if the figure to the right of it is less than 5. Good grounding may be given at first in the above method of arguing and the alternate method allowed to be deduced by the pupils themselves after working several exercises.)

Propose :—

Express the value of the following approximately as required :—

(1) Rs. 3,58,965 to the nearest lac of rupees.
(*Ans.* Rs. 4 lacs.)

(2) Rs. 26,895 to the nearest thousand rupees.
(*Ans.* Rs. 27,000.)

(3) 6,476 acres to the nearest thousand acres.
(*Ans.* 6,000 acres.)

(4) 9 manugus 7 visams to the nearest manugu.
(*Ans.* 10 manugus.)

(5) 65 kunchams 1 adda to the nearest kuncham.
(*Ans.* 65 kunchams.)

(6) Rs. 17-14 annas to the nearest rupee.
(*Ans.* Rs. 18.)

Point out in this connexion that the convention above used to effect approximations may be applied also in expressing quotients to the nearest unit. (Vide exercise given in pages 38, 40.) Lead pupils to note that if after dividing in full a remainder is left which is less than half the divisor it may be neglected and the quotient so far found may be expressed as it is and that if the remainder is equal to, or greater than, half the remainder, the quotient so far found may be increased by "1" and thus expressed to the nearest unit. Explain this fact by means of examples like the following :—

Suppose 600 kunchams of paddy are to be sent in bustas each holding 22 kunchams. The number of bustas required will be found by division to be 27. It will be found that 6 kunchams will remain. Point out that the result above has to be expressed in bustas, the extra 6 kunchams is less than half of what a busta is to hold, viz., 22 kunchams, and that hence the result to the nearest busta is 27 bustas. Point out also that if instead of 600 kunchams the quantity to be sent be 610 kunchams, the division by 22 will give a quotient 27 and remainder 16, that as 16 kunchams is greater than half of what a busta is to hold, viz., 22 kunchams, the result will be expressed as 28 bustas to the nearest busta and that if the quantity to be sent be 605 or 606, 607, 608, 609.........616, kunchams, the result to the nearest busta will be 28 bustas only.

```
22)600(27
   44
   ---
   160
   154
   ---
     6
   ---
```

Ask pupils to apply the convention stated above and express the quotient in the exercises given in pages 40, 41 to the nearest unit.

Rough Estimate of Results.

Propose :—

(i) A ryot finds that by sugarcane cultivation he got an income of Rs. 580-10-6 from one field, Rs. 624-9-6 from a second field, Rs. 284-10-5 from a third field and Rs. 785-10-6 from a fourth field. Express roughly without working out the addition in full how much he got from the fields on the whole.

Point out to pupils that instead of working out the accurate total in full a rough idea about it can be got by taking in mentally the approximate values Rs. 600, Rs. 600, Rs. 300, and Rs. 800 and getting the total to be about Rs. 2,300. Let pupils work the actual total to be Rs. 2,273-9-0 and find out that the rough estimate is not far wrong.

RS.	A.	P.
580	10	6
624	9	6
284	10	6
785	10	6
2,275	9	0

Propose :—

(ii) A tenant had 576 kunchams of ragi in his granary. If he sold away 258 kunchams, find roughly how many kunchams will remain.

Let pupils take in mentally the approximate values 600 kunchams and 300 kunchams and find the balance to be about 300 kunchams roughly. Lead pupils to work out the accurate result to be 318 kunchams and infer that the approximate estimate is not far wrong.

Propose :—

(iii) A landlord has 27 acres of dry lands worth about Rs. 525 an acre and 22 acres of wet lands worth about Rs. 1,775 an acre. Find how much he is worth on the whole.

Let pupils take in mentally the approximate values and work out the estimate roughly to be Rs. 30 × 500 plus Rs. 20 × 2,000 or Rs. (15 plus 40) thousands, i.e., Rs. 55 thousands. Ask *pupils to work out in full the accurate* result and derive that the approximate estimation is not far wrong.

Propose :—

(iv) Mattapili Thathayya has Rs. 15,725 cash with him. He desires to secure the produce from the sugarcane cultivation in the adjoining lands by giving advances to cultivators. Estimate roughly how many acres of sugarcane he can secure at Rs. 486 an acre.

Lead pupils to take in mentally the approximate values, e.g., Rs. 16,000, Rs. 500 and find out roughly that the produce from about 32 acres of sugarcane could be secured. Let pupils work out mentally the division in full and find out that the result of approximate estimation is not far wrong.

The use of rough checks.

Point out that if one is able to make approximate estimation as above, he would not be committing gross mistakes, e.g., in *saying, for example, that the produce from* 3,000 acres could be secured in Exercise (iv) above nor would he be giving out absurd and ridiculous answers like saying that the produce from 3 acres would be secured in Exercise (iv) above. Point out also that if, in performing the fundamental operations, absurd, ridiculous and erroneous results be obtained they could easily be detected and checked by the above method of approximate estimation.

Exercises.

Propose :—

1. The value of the produce obtained from paddy cultivation in five different fields has been as follows:—

Field	I	II	III	IV	V
	RS. A. P.	RS. A. P.	RS. A. P.	RS. A. P.	RS. A. P.
Paddy	216 6 8	225 10 6	114 6 8	170 5 6	114 6 8
Straw	44 10 9	46 2 8	19 6 8	38 13 6	38 8 8

Find roughly how much has been realized from all the fields (*a*) from paddy alone ; (*b*) from straw alone ; and (*c*) from both paddy and straw together.

(*Ans.* (*a*) about Rs. 800 ; (*b*) about Rs. 190 ; (*c*) about Rs. 1,000.)

2. There are 39 acres of sugarcane cultivation in Sanyasi Razu's lands. At about Rs. 490 for the produce of an acre, estimate roughly the value of the produce.

(*Ans.* About Rs. 40 × 500 or Rs. 20,000.)

3. About Rs. 440 worth of turmeric may be expected from an acre. At this rate estimate how much a ryot may expect to get from 21 acres of pasupu.

(*Ans.* About Rs. 400 × 20 or Rs. 8,000.)

(The teacher should note that uniformity of results could not be expected in pupils while they work out the above exercises on approximate estimation. For example they are likely to express the result in the above exercise as Rs. 440 × 20 or Rs. 8,800. The teacher must satisfy himself if the principle of approximate expression is understood as well as its application.)

4. In a store-room there are 88 maunds of jaggery worth Rs. 3-4-0 a maund and 26 puttis of paddy worth Rs. 56 a putti. Estimate roughly how much the contents are worth.

(*Ans.* Rs. 90 × 30 plus 30 × 60 or Rs. 300 plus 1,800 or Rs. 2,100.)

5. A cultivator finds that he has bought manure for his lands as follows one year :—

28 cartloads of farm-yard manure worth Rs. 1-2-0 a cartload, 24 bags of fish guano worth Rs. 6-4-0 a bag and 18 bags of castor cake worth Rs. 8-12-0 a bag. Estimate at this rate the value of the manure he may require for his lands for the forthcoming year. (*Ans.* Rs. 360.)

6. An acre of sugarcane cultivation is expected to fetch normally 19 barvas of jaggery. If a barva of jaggery be expected to sell at Rs. 66-14-0 estimate the value of jaggery that may be expected from 21 acres.

(*Ans.* 19 × 21 × Rs. 66-14-0 or Rs. 20 × 20 × 70 or Rs. 28,000.)

7. In a herd there are 97 buffaloes and 76 cows. The cows are valued at Rs. 74 each and the buffaloes at Rs. 93 each. Estimate how much the whole herd is worth.

(*Ans.* About Rs. 15,000.)

8. A cultivator finds that chilly cultivation in an acre cost him one year Rs. 90-14-0 and the produce fetched him Rs. 200-12-6. Assuming that the cost of cultivation as well as the value of produce increases by one-sixteenth in value, estimate what net outturn he may expect from chilly cultivation in five acres during the forthcoming year.

(*Ans.* 200 − 90 = 110; 110 + 7 = 117; 117 × 5 gives 600. . . . Rs. 600 is the probable outturn.)

9. There are 387 guava trees in the gardens adjoining our school. Each tree may be expected to produce 2,596 guavas during a year in two seasons. If the trees are taken on lease for Rs. 3,000, estimate how much the lessee may expect to gain. Assume that 100 guavas would fetch ½ a rupee.) (*Ans.* Rs. 2,000.)

10. Year before last a tenant found that *gogu* cultivation in a plot cost him Rs. 25-14-6 and fetched him a produce worth Rs. 65. Cambodia cotton in a second plot cost him Rs. 40-4-0 and fetched him produce worth Rs. 148, turmeric cultivation in a third plot cost him Rs. 98-14-0 and fetched him produce worth Rs. 510-14-0. If this year he has *gogu* cultivation over twice the former area, Cambodia Cotton over four times the former area and turmeric over twice the former area, estimate the total profit he may expect from the cultivation at the above rates. (*Ans.* About Rs. 2 (65-25) plus 4 (150–40) plus 2 (500–100) or Rs. 1,300.)

11. A cart can hold 400 bricks. How many carts have to be engaged to remove 29,000 bricks?

(*Ans.* 73 carts.)

12. The area under sugarcane in Palkonda taluk one year was 1,993 acres. Each acre was expected to produce 19 puttis of jaggery. At Rs. 66 a putti, estimate the value of the entire outturn of jaggery.

(*Ans.* Rs. 2,000 × 20 × 70 or Rs. 28,00,000.)

13. A plot is 70 yds. long and 70 yds. wide. *Mocca Jonna* is planted 2 ft. apart in rows 2 ft. apart. Estimate the number of jonna plants in the plot.

(*Ans.* About 10,000 plants.)

14. A garden is 1 furlong long and half a furlong wide. Mango grafts are planted 40 ft. apart in rows 40 ft. apart. Estimate the number of grafts planted.

(*Ans.* About 130 grafts.)

15. A plot is 7 yds. long and 7 yds. wide. Tobacco seedlings are planted 2 ft. apart in rows 2 ft. apart. Estimate the number of tobacco plants in the plot.

(*Ans.* About 1,100.)

16. A sovereign sells at Rs. 18-12-6. A gold chain weighing 22 sovereigns has to be made of sovereign gold. The cost of making is at Re. 1 for a sovereign weight. Estimate the cost of making the chain.

(*Ans.* About Rs. 400.)

TOPIC 14.—THE ARITHMETIC MEAN.

(**Scope of the treatment.**—*Notions of the Arithmetic mean may be given and its place in approximate estimation pointed out. The principle involved in its calculation may next be explained. Exercises may next be set on the mean cost of cultivation, quantity of yield, etc., in regard to several crops. Short cuts like the Excess and Defect method in the calculation of the arithmetic mean need not be dealt with unless there is time and an extraordinarily brilliant set is had.*)

The Arithmetic Mean.

Propose to pupils that they should have an idea as to how much a sugarcane of a particular variety as grown in a garden is likely to weigh. Suggest that one particular cane weighs 3 lb. 2 oz. Question if that may be taken to be the probable weight of a cane grown in the garden. Lead pupils to note that the particular cane whose weight is given may be of exceptional growth or weight and that its weight could not hence be completely relied on as an estimate giving the weight of other canes also. Intimate to pupils that to get at a fair approximation of the weight of a cane grown in the garden the following method used to be adopted :—

Suggest to them that some canes may be selected from different spots in the garden and then they may be weighed. Get them to grant, say, that on weighing eight canes, the total weight is 24 lb. Question how much the weight of a cane may then be guessed to be. Point out how the result of dividing 24 lb. by 8 would in the above case give an idea of the probable weight of a cane. Let pupils work out the division and find the weight of a cane. Lead pupils to infer that the actual weight of each cane will not be 3 lb., some canes may weigh more and some may weigh less, that the weight 3 lb. is only what is probable, suggesting as it does, that, if each cane were to weigh so much, the total of 24 lb. would be made up. Tell them that the probable weight so assigned to a cane is called the *average* or *mean* weight of a cane. Get them to note that *this*

average is got by first finding the total weight of the canes and then dividing the weight by the number of canes.

Propose :—

Twelve fully grown canes in B. Devaiyya's gardens were found to weigh respectively 2 lb. 11 oz., 2 lb. 4 oz., 2 lb. 15 oz., 2 lb. 11 oz. 3 lb., 3 lb. 2 oz., 2 lb. 15 oz., 3 lb. 1 oz., 2 lb. 13 oz., 3 lb., 1 oz., 3 lb. and 3 lb. 1 oz. What may be taken to be the average weight of a cane ?

By a glance at the figures lead pupils to derive that the average should be between 2 lb. and 3 lb. Question them how the total weight of the canes could be got in the above case and after they find the total, let them divide it by the number of canes and thus find the average weight of a cane to be 2 lb. 15 oz.

Lead pupils to realize that some difference exists between the actual weight of some of the canes and the average weight as found above. Question them which canes are of the average weight, which are below the average and which are above the average.

Here too impress the fact that the average weight of a cane need not be the actual weight of any cane and though it differs from the actual weight it gives a general idea of the weight of a cane out of a group in the garden.

Propose :—

Ten guava trees of normal produce growing in the neighbouring gardens have been found to give 1,652, 1,702, 1,680, 1,690, 1,710, 1,668, 1,695, 1,694, 1,698 and 1,710 guavas, respectively, during the season. What may be taken to be the average yield of a tree in the gardens during the season ?

Let pupils proceed as before and find the average number yielded by a tree to be 16,899/ 10 or 1,690 nearly. Point out that if the division involved in finding the average is not exact, i.e., leaves a remainder, the quotient may be expressed to the nearest unit, adopting the convention already mentioned in regard to approximate expression. [See page 117.] Let pupils note that by a glance at the figures in the above case 1,700 may be derived to be the normal produce from a tree during the season, also that the yield from some trees must necessarily be above the average and hence is so actually and that the yield from some other trees must necessarily be below the average and is so actually.

Exercises.

Propose :—

1. The yield of paddy from a plot of wet lands about an acre in extent has been 295 kunchams, 300 kunchams, 290 kunchams, 298 kunchams and 300 kunchams during five successive *varipantas*. What is the average yield from the plot during a season ? (*Ans.* 297 kunchams.)

2. The cost of cultivating ragi (dry) in a plot during three successive years has been found to be Rs. 22, Rs. 21 and Rs. 23 respectively. Find at this rate the average cost of the cultivation. (*Ans.* Rs. 22.)

3. The following particulars have been taken from the cultivation accounts of a landlord and they give the cost of cultivation of particular crops in an acre of land during three years. Estimate to the nearest rupee the average cost of cultivation of each crop in an acre of land during a season.

	RS.	A.	P.	RS.	A.	P.
1. Sugarcane (annual) ...	252	1	2			
	245	8	3			
	248	1	4			
		(*Answer*		249	0	0)
2. Tobacco	100	12	0			
	92	8	0			
	82	4	0			
		(*Answer*		92	0	0)
3. Chillies	94	14	0			
	90	8	0			
	85	15	0			
		(*Answer*		90	0	0)
4. Turmeric	102	13	6			
	99	10	8			
	96	6	1			
		(*Answer*		100	0	0)
5. Gogu	26	0	0			
	24	14	6			
	23	13	0			
		(*Answer*		25	0	0)

	RS. A. P.	RS. A. P.
6. Groundnut	34 4 7	
	32 4 3	
	30 4 11	
	(Answer	32 0 0)
7. Gingelly (Tolakari) ...	16 6 3	
	15 10 6	
	14 14 9	
	(Answer	16 0 0)
8. Cambodia cotton ...	42 4 0	
	39 9 0	
	35 14 0	
	(Answer	39 0 0)
9. Castor	19 15 0	
	19 0 0	
	18 2 6	
	(Answer	19 0 0)
10. Greengram	6 14 9	
	6 9 6	
	7 4 3	
	(Answer	7 0 0)
11. Onions	71 1 6	
	68 7 0	
	65 12 6	
	(Answer	68 0 0)
12. Jonna	21 5 9	
	20 8 0	
	19 7 3	
	(Answer	20 0 0)
13. Ragi (dry)	22 8 0	
	21 3 9	
	19 14 6	
	(Answer	21 0 0)
14. Daincha	13 15 0	
	13 6 0	
	12 13 0	
	(Answer	13 0 0)
15. Ragi (irrigated) ...	35 1 7	
	33 2 4	
	34 7 6	
	(Answer	34 0 0)

4. A family consumes 13, 14, 10, 13, 12, 11, 13, 15, 14, 12, 13, 14 kunchams of rice, respectively, during the 12 months of a year. Find to the nearest kuncham the average monthly consumption of rice in the family.

(*Ans.* 13 kunchams.)

5. Different blocks of jaggery are heaped up in the market yard as brought by the villagers. Some blocks were taken at random from a heap, weighed and found to be 7 visams. 3 seers, 8 visams, 7 visams 4 seers, 8 visams 1 seer, 8 visams 2 seers, 7 visams 2 seers and 7 visams 4 seers, respectively. Find the average weight of a block to the nearest visam. (5 seers = 1 visam.)

(*Ans.* 8 visams or a manugu.)

6. From six acres of onions an yield of 70 puttis was got. Find the average yield per acre to the nearest putti.

(*Ans.* 12 puttis.)

7. To have an idea of the yield ot groundnut from some lands the yield got from the lands in a village was ascertained and found to be as follows :—

From a plot 1 acre in extent 320 kunchams were obtained, from an adjacent plot 2 acres in extent 600 kunchams, from a third plot 4 acres in extent 1,100 kunchams and from a fourth plot 3 acres in extent 1,200 kunchams. Find the average yield per acre, judged from the entire yield of the lands. (*Ans.* 322 kunchams.)

8. The cost of plantain cultivation over an acre has been found to be

Rs.	45-14-0	during the	first year.
"	79-5-6	"	second year.
"	80-11-6	"	third year.
"	81-1-6	"	fourth year.

The value of the produce during the first year has been found to be Rs. 160 and that during each of the next three years about Rs. 216. Estimate the average yearly net profit per acre after calculating the corresponding profit for all the four years. (*Ans.* Rs. 130.)

9. The cost of cassava (tapioca) cultivation in 5 acres is found to be Rs. 499-6-6 while the value of the yield is Rs. 899-7-6. Estimate the average net profit per acre.

(*Ans.* Rs. 80.)

TOPIC 15.—DECIMALS (INTRODUCTORY).

(**Scope of the treatment.**—*At first the pupils may be made familiar with the method of expressing fractions in the decimal system. They must know how to read decimal fractions up to a hundredth or a thousandth and how to perform addition and subtraction with such numbers. The similarity between the addition and subtraction of integers and those of decimal fractions may be pointed out. As a knowledge of the decimals is required for the pupils of this school only in connexion with surveying, mensuration, measurement of lines in figures, in problems on percentage and in preparing figures for the cultivation sheets as adopted in the Anakapalle Agricultural station and even there decimals are used only for the sake of convenience and not as an unavoidable necessity, practical problems are hard to frame. The teacher will do well to note that the fundamental operations as applied to decimals need not be dealt with in detail. Approximation may be effected where a required decimal result does not terminate, figures beyond the hundredths' or the thousandths' place being considered unnecessary. Further, as this is only an introductory Chapter, decimal multiplication and division may be confined to cases where the multipliers and divisors are integers whose multiples may easily be given out by remembering the multiplication table. The artifices for multiplying and dividing by* 10, 100, 1,000, *etc., may be well impressed. From the standpoint of utility a knowledge of the decimal notation as extended to fractions is alone essential. Hence too much of example—grinding need not be attempted. It is enough if the teacher sees that the above object is secured. No objection need be raised to the free use of the terms "acre" and "cent" as the pupils are expected to be familiar with them as terms denoting extent of land. No doubt, clear space-notions about them are reserved for later treatment.*)

1. The decimal system of expressing the unit and its subdivisions.

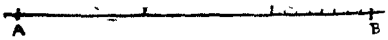
(a) Subdivision of the unit into tenths.

Question pupils how lengths shorter than a yard are measured and expressed. Get them to realize that smaller

units of the same kind such as the foot and the inch are used for the purpose. Question them how lengths shorter than an inch could be measured and expressed. Lead them to suggest that the inch—length should be further subdivided into a certain number of equal parts say 4, 8, 10 or 16, etc., so that lengths shorter than an inch may be measured. Let them examine a foot-rule graduated into inches and tenths of an inch and find the number of equal parts into which an inch is divided. Tell them that each part is '*a tenth of an inch*' and that such a graduation along the straight edge of the foot-rule would help measurement being made in inches and tenths of an inch.

Let pupils note that this is only one of the many ways in which lengths shorter than an inch may be measured and expressed.

Next ask them to draw a straight line and along it step off a segment AB 2 inches and eight-tenths of an inch long. (Show how the foot-rule and the dividers may



be used for the purpose.) Let them express what its length is. Tell them that the length " 2 inches and eight-tenths of an inch " may be written shortly as 2·8″ (read as two point eight inches). Point out that, in so writing, 2 stands for 2 inches and eight for eight-tenths of an inch. Starting from the '0' graduation in the straight edge, direct them to mark out lengths of 2 inches and four-tenths of an inch, 5 inches and six-tenths of an inch, 4 inches and five-tenths of an inch, etc. Ask them to write the length in each case in the decimal system as above shown, that is, as 2·4″, 5·6″, 4·5″, etc. Let them next read the lengths so written. Pointing out any inch mark in the straight edge of the foot-rule, ask them to show how far from it the distance would come to 2·5″, 7·8″, 5·4″, 2·8″, etc., respectively.

Propose to them the following :—

If 2·8″ means 2 inches and 8-tenths of an inch what may 2·8 furlongs be taken to mean ? 2·7 furlongs ? 3·8 acres ? Rs. 4·5 ?

Point out that 2·8 furlongs may be read as two point eight furlongs and that it denotes 2 furlongs and 8

tenths of a furlong. Question how they are to get 8 tenths of a furlong. Get them to realize that a furlong may be divided into 10 equal parts and 8 parts taken to get 8 tenths of a furlong and that as a chain is one-tenth of a furlong, 8 tenths of a furlong simply means 8 chains. Similarly lead them to see how 8·8 acres, Rs. 4·5, etc., may be read and what they indicate. Thus lead them to note the following :—

(i) *If any unit—an inch, a furlong, an acre, a rupee, a ton, or a putti be divided into ten equal parts each part is a tenth of the unit,* (ii) *A dot called the decimal point is used as the dividing point between units and their subdivision into tenths. Immediately to the left of the dot is the units' place and immediately to the right of it is the figure denoting the number of tenths. The place where the number of tenths is written is called the tenths' place.*

Propose to pupils to set down the following in figures in the decimal system of notation :—

(*a*) 7 tenths of a chain.

(Point out if necessary that 7 tenths of a chain may be written as ·7 chain.)

(*b*) 5 tenths of a chain.
(*c*) 1 tenth of a visam.
(*d*) 2 tenths of a seer.
(*e*) 6 tenths of a garisa.
(*f*) 3 tenths of a maund.
(*g*) 9 tenths of a ton.
(*h*) 10 tenths of a visam.
(*i*) 2 tons and 8 tenths of a ton.
(*j*) 8 visams and 7 tenths of a visam.

If in expressing the result in (*h*) above, any of the pupils should write by mistake '·10' to represent 10 tenths, point out that the '1' after the decimal point indicates only 1 tenth, and that as 10 tenths make one unit the pupil must correct what he has written into '1' or 1·0, the '0' in the latter method of representation indicating zero tenths and the '1' one unit. Tell them also that as the sub-division of the units into tenths is devised only to represent parts less than a unit, one or other of the figures 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, could alone appear in the tenths' place in a number.

Incidentally, point out also that as a visam is 110 tolas at Anakapalle, a tenth of a visam would come to 11

tolas, that a tenth of a chain is the distance between two consecutive brass discs in it, that a tenth of a ton is 2 cwt. and 8 tenths of a ton would hence come to 16 cwt., a garisa is 600 kunchams, a tenth of a garisa would hence come to 60 kunchams and 6 tenths of a garisa to 360 kunchams.

(b) Subdivision of the tenth into hundredths.

Now propose to pupils how they are to express parts of a unit smaller than a tenth. Tell them that, to express these smaller divisions of the unit, a tenth of a unit is further subdivided into ten equal parts. Question them how many such parts there would be if each of the ten tenths be considered as so sub-divided. Lead them to note that there would be 100 such parts in the unit. Tell them that each part is hence called '*a hundredth of the unit.*' Taking a chain, for example, point out that a hundredth of a chain is a link and that a tenth is ten links. Tell them that a length of 4 chains 5 tenths of a chain 8 hundredths of a chain may be written shortly as 4·58 chains (read as four point five eight chains), the figure 5 representing 5 tenths of a chain and the figure 8 representing 8 hundredths of a chain. Point out that the figure denoting tenths is written immediately to the right of the figure denoting units with a dot put in as the dividing mark between the two and that the figure denoting hundredths is put in immediately to the right of the figure denoting tenths, i.e., in the second place to the right of the decimal point. Get the pupils to understand why 4·58 is read as four point five eight chains and not as four point fifty-eight chains. Lead them to note that if so read 5 would be representing fifty but that it actually represents 5 tenths. Ask pupils to write in the decimal notation as above shown 8 tenths of a chain, 3 hundredths of a chain, 8 chains 7 hundredths of a chain, etc. If they commit mistakes, point out that 3 hundredths of a chain may be written as ·03 chain and that, in so writing, the 'o' is put in to indicate that there are no tenths and also to make it quite clear that '3' is put in the hundredths' place.

Ask pupils to represent in a similar manner 1 hundredth, 2 hundredths, 4 hundredths, 5 hundredths, 6 hundredths, 7 hundredths, 8 hundredths and 9 hundredths, respectively. Impress the fact that the number of hund-

redths is put in not *immediately to the right of the* decimal point but *one place further* to the right.

Ask them also to read the following :—

2·24 acres, 3·35 chains 4·45 tons, 7·84 maunds.

Question how many tenths there are in 1 unit and how many hundredths in 1 tenth. Let pupils note that ten hundredths make one tenth and ten tenths make one unit

(Illustrate the above by using the chain.)

Propose the following exercises :—

1. Write solely in figures the number of chains in 8 chains 8 tenths of a chain 8 hundredths of a chain.

(*Ans.* 8·88 chains.)

2. Express in chains :—6 links, 16 links ; 4 chains ; 16 links ; 8 chains, 38 links.

(*Ans.* ·06 ; ·16 ch ; 4·16 chs. 8·38 chs.)

3 Express 14 acres 7 tenths of an acre 3 hundredths of an acre in acres and decimal of an acre.

(*Ans.* 14·73 acres.)

4. Read the following :—

2·25 tons, 39·37 inches ; ·52 furlong.

5. 1 garisa = 600 kunchams. How many kunchams are there in ·1 garisa, ·05 garisa ?

(*Ans.* 60 kunchams, 6 kunchams, 120 kunchams, 12 kunchams, 180 kunchams, 18 kunchams, 300 kunchams, 30 kunchams.)

6. Express in chains and decimal of a chain :—

564 links, 804 links, 740 links, 44 links, 80 links, 8 links, 704 links.

(*Ans* 5·64 chains, 8·04 chains, 7·4 chains, ·44 chain, ·8 chain, ·08 chain, 7·04 chains.)

(Lead pupils to see how 564 links may be expressed at sight as 5·64 chains. Let them also work as follows :—

564 links =		564 hundredths of a chain.
10)564		Therefore 564 links = 56 tenths of a
56	–4	chain and 4 hundredths of a chain.
10)56		56 tenths of a chain = 5 chains and
5	–6	6 tenths of a chain.

Therefore 564 links = 5 chains 6 tenths of a chain and 4 hundredths of a chain = 5·64 chains.)

Point out in this connexion that it is far easier to express 5 chains 64 links in chains and decimal of a chain than to express 4 yards 2 inches in yards and decimal of a yard. *Let the pupils explain why it is so. Lead them to see* that the facility lies in the use of a decimal scale of tables.

7. 1 putti = 500 lb. Express in lb. ·1 of a putti, ·01 of a putti, ·3 of a putti, ·03 of a putti.

(*Ans.* 50 lb., 150 lb ; 15 lb.)

8. If 1 kantlam = 200 lb. express how many lb. would be represented by 2·11 kantlams, 3·14 kantlams and 11·44 kantlams. (*Ans.* 422 lb., 628 lb., 2,288 lb.)

9. One cent is one hundredth of an acre. Express in acres and decimal of an acre 88 cents, 41 cents, 9 cents. 408 cents, 480 cents, 625 cents, 555 cents, 120 cents.

(*Ans.* ·88 ac. ·41 ac.; ·09 ac. 4·08 acs.) 4·8 acs., ; 6·25 acs. ; 5·55 acs. ; 1·2 acs.)

(c) *Sub-divisions of the unit smaller than a hundredth.*

Now propose how they are to express parts of a unit smaller than a hundredth. Point out to them that to express those smaller divisions of the unit a hundredth of a unit may be further sub-divided into ten equal parts. Question how many such parts there would be if each of the hundred hundredths be so divided. Lead them to note that there would then be thousand such parts in the unit. Tell them that each such part is therefore '*a thousandth.*' Taking a furlong, for example, explain to pupils that there are 1,000 links in it, and hence a link is a thousandth of a furlong. Showing a rectangle 5″ by 2″ on squared paper, illustrate that a square-inch of space is one tenth of it, a rectangle 1″ by ·1″ is one hundredth of it and a square ·1″ by ·1″ is one thousandth of it. (The amount of space alone need here be pointed out. The terms "square inch", "rectangle" and "square" need not be used.) Referring to a plot of 1 acre in extent, point out how much one-tenth of it comes to, one-hundredth of it by showing a cent, and one-thousandth of it by showing one-tenth of a cent.

Next intimate to pupils that a quantity like 2 puttis 4 tenths of a putti 7 hundredths of a putti and 9 thousandths of a putti may be expressed in a shorter form as

2,479 puttis (read as two point four, seven, nine, puttis). Point out that the figures 4 and 7 respectively represent 4 tenths and 7 hundredths of a putti and the figure 9 represents 9 thousandths of a putti, i.e., to say, the number of thousandths is written immediately to the right of the hundredth's place. Propose to pupils to express 4 thousandths of a putti as the decimal of a putti. If they do not write it correctly as ·004 putti, point out that the number of thousandths should be written in the third place to the right of the decimal point and that the '0's are put in not only to indicate the absence of any tenths and hundredths but also to make it quite clear that '4' is written in the hundredths' place.

Propose to pupils to express the following in a shorter form, by using the decimal system of notation :—

1. 2 furlongs 5 tenths of a furlong 7 hundredths of a furlong 9 thousandths of a furlong.

(*Ans.* 2·579 furlongs.)

2. 7 tons 5 tenths of a ton 7 hundredths of a ton 9 thousandths of a ton. (*Ans.* 7·579 tons.)

3. 4 puttis 3 tenths of a putti 7 hundredths of a putti 9 thousandths of a putti. (*Ans.* 4·379 puttis.)

4. 3 thousandths of a rupee. (*Ans.* Rs. 003.)

5. 3 thousandths of an acre. (*Ans.* ·03 acre).

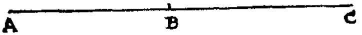
6. 118 thousandths of a furlong.

(*Ans.* ·118 furlong.)

Next intimate to pupils that to express parts of a unit smaller than a thousandth, a thousandth may as has hereto fore been specified, be subdivided into ten tenthousandths, and so on. Point out to them that such very small sub-divisions of the unit are not generally required for ordinary calculations. Tell them, for example, that in measuring in inches the lengths are expressed in inches and tenths of an inch and as a hundredth of an inch is very small hundredths of an inch are not generally marked out along the straight edge and lengths lying between 2·6″ and 2·7″, for example, are therefore expressed to the nearest tenth of an inch, e.g., as 2·6″ or 2·7″; similarly, in expressing lengths in chains, the results are generally given only up to the hundredth of a chain; i.e., a link; in expressing weight in tons results are generally given only up to the thousandth of a ton which comes to about 2 lb. Point out how approximations are effected.

2. Decimal Addition.

Propose :—(1) Draw any straight line. Along it step off a segment AB 2·8″ long and in continuation of it a segment BC 3·4″ long. Find the length of AC.

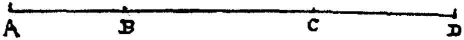


(Point out that the length of AC may be found either by actual measurement or by adding the lengths of AB and BC, namely, 2·8″ and 3·4″ as follows :—

AB contains 2 inches and 8 tenths of an inch.
BC „ 3 „ 4 „ „
AC must contain 5 inches and 12 tenths of an inch.

12 tenths of an inch is 1 inch and 2 tenths of an inch. The length of AC may hence be expressed as 6 inches and 2 tenths of an inch or as 6·2 inches. Ask pupils to measure the length of AC and verify if the length is 6·2″

Propose :—(**2**) Along a straight line successive segments



AB, BC, CD are marked which are 2·8″, 4·5″, 2·9″ long. Calculate the length of AD. (*Ans.* 10·2″.)

(**3**) Find the length of AD referred to in the last question, given that AB = 10·9″
BC = 2·8″
CD = ·9″

(After the pupils work the above exercises as shown in (1) above lead them to adopt the following quicker method of writing the figures and adding :—

Set down the numbers one below the other so that their decimal points shall all appear in the same vertical line and that as a consequence tenths shall appear below tenths, units below units, etc., as shown in the accompanying working :—

$$\begin{array}{r} 10{\cdot}9 \\ 2{\cdot}8 \\ {\cdot}9 \\ \hline 14{\cdot}6 \\ \hline \end{array}$$

Beginning at the right hand side, add up the figures in the first column, tenths in the case above; when in this exercise 26 tenths are so obtained, express it as 2 units and 6 tenths. Set down 6 in the tenths' place. Mark it out by putting in the decimal point to the left of it. Carry 2 units in the units' column. Add the figures in that column. Next add the figures in the tens' column and set down the total '1' in this case in the tens' place. Thus obtain a total 14·6. Hence express the length of AD to be 14·6 ".)

Next propose :—

(**4**) Five trees stand along a straight hedge, T_1, T_2, T_3, T_4, T_5. (Read 'T_1 as the first tree T_2 as the second tree, and so on.). The distance from T_1 to T_2 is 1·78 chains, that from T_2 to T_3, 1·08 chains, that from T_3 to T_4, 2·4 chains, and that from T_4 to T_5, 1·44 chains. Calculate the distance from T_1 to T_5.

Let pupils write the distances at first, if necessary, as

1 chain	7 tenths of a chain	8 hundredths of a chain.	
1 ,,	0 ,, ,,	8 ,,	,,
2 ,,	4 ,, ,,	8 ,,	,,
1 ,,	4 ,, ,,	4 ,,	,,

Let them add up the hundredths of a chain. When they get 28 hundredths of a chain, ask them to express this as 2 tenths of a chain and 8 hundredths of a chain, set down 8 hundredths of a chain and carry 2 tenths to be added to the tenths written above. Let them next add up the number of tenths of a chain. When they get 17 tenths of a chain, let them express this as 1 chain and 7 tenths of a chain, set down 7 tenths of a chain and carry 1 chain to be added to the number of chains. Let them next add the number of chains and get 6 chains. Let them hence express the distance required to be 6 chains 7 tenths of a chain 8 hundredths of a chain and then express it shortly as 6·78 chains.

Get them to write the numbers as shown in the margin i.e., with the decimal points all appearing in a vertical line, add and see if the addition could thereby be done quicker.

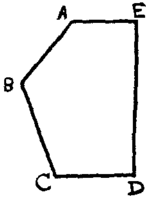
```
1·78
1·08
2·48
1·44
----
6·78
```

The first column to the right, i.e., *the hundredths column* :—4, 12, 20, 28. Set down 8, carry 2.

The second or the tenths' column:—6, 10, 17. Set down 7, carry 1.

The units' column:—2, 4, 5, 6. (Set down 6.)

Next propose:—(**5**) ABCDE is a garden requiring fencing along AB, BC, CD, DE, EA.



AB is 2·08 chains, BC is 1·5 chains, CD is 1·04 chains, DE is 3·09 chains and EA is ·89 chain. Find the length of the total fencing required.

(*Ans.* 8·6 chains.)

(**6**) Four coolies are engaged in a day to work in your gardens. One gets ·125 rupee, another ·25 rupee, a third ·5 rupee and the fourth ·625 rupee. How much on the whole have you to give as wages for the day?

(*Ans.* Rs. 1·5.)

(**7**) S. Patrudu has four plots of *pallamu* whose extents are as follows:—

Plot I ·27 acre.
Plot II 1·05 acres.
Plot III ·63 acre.
Plot IV 3·78 acres.

Find the total extent of the lands in acres.

(*Ans.* 5·73 acres.)

(**8**) V. Gopal Razu owns *mettubhumi* in four different villages and their extents are as follows:—

Mettu in	Yellamanchili	17·8 acres.
„	Kasimkota	8·09 „
„	Chodavaram	17·97 „
„	Narasingapalli	15·44 „

Find the total extent of the *mettubhumi* in acres.

(*Ans.* 59·3 acres.)

(9) From 1st April 1920 to 31st March 1921 the rainfall at Anakapalle has been as follows :—

April and May 2·93″ June, July, August and September 12·63″, October, November and December 7·55′ January, February and March 3·52.

Find the total rainfall during the period of twelve months above mentioned. (*Ans.* 26·68 inches.)

(Take the report of the Agricultural station, Anakapalle, for 1920–21 and from the particulars found therein propose some more exercises of the above type.)

(10) Kothandam has 8·75 acres of lands. Pattabhiramayya has 2·28 acres more than Kothandam. How many acres of land has Pattabhiramayya ?

(*Ans.* 11·03 acres.)

(11) In the Anakapalle Agricultural Farm there are 9·45 acres of dry land, 11·34 acres of garden land, 16·91 acres of wet land and 3·25 acres covered by roads, buildings, etc. What is the total extent of the lands in the Farm ?

(*Ans.* 40·95 acres.)

3. Decimal subtraction.

Propose :—A fence is 3·49 chains long. Another fence is 2·21 chains long. How much longer is the first fence than the second ?

Let pupils note that here 2·21 should be subtracted from 3·49 and that the subtraction may be performed as follows :—

	Units.	Tenths.	Hundredths.
3·49	3	4	9
—2·21	2	2	1
= =1·28	1	2	8

Therefore the first fence is 1·28 chains longer than the second fence.

Point out to them that the *subtrahend* is written below the *minuend* in such a way that their *decimal points* appear in the same vertical line and consequently figures of the same place-value appear likewise, i.e., units below units, tenths below tenths, etc., and that the subtraction is done as shown below :

Beginning at the hundredths' place which appears in the extreme right, 9–1 gives 8. Set down 8 in that place. Then proceed to the tenths' place, 4—2 gives 2. Set down 2 in that place. To indicate the tenths' place, put in the decimal point to the left of it. Next proceed to the units' place. 3—2 gives 1. Set down 1 in that place. Thus get that the first fence is 1·28 chains longer than the second.

3·49
2·21
1·28

Let the pupils realize that the operation of subtraction is done in the same way as in the case of integers.

Propose :—

Pattabhiramayya has 27·45 acres of lands while Rama Rao has 18·89 acres of lands. How much more of lands has Pattabhiramayya than Rama Rao ?

Let the pupils first set down the numbers thus :—

27·45
18·89

and then work out the subtraction as stated below :

Proceeding as before, 9 hundredths should be subtracted from 5 hundredths. This is not possible. *From 4 tenths* in the minuend, take away 1 tenth, express *it* as ten hundredths and add it to 5 hundredths. Then take away 9 hundredths from 15 hundredths. 6 hundredths remain. Set down '6' in the hundredths' column.

Proceeding to the tenths' place, *8* tenths *could* not be taken from 3 tenths. From *7* units in the minuend take away *1* unit. *Express it* as *ten tenths. 10 tenths* and 3 tenths make 13 tenths. 13 tenths – 8 tenths would give 5 tenths. Set down 5 in the tenths' place. Put in a decimal point to the left of the '2.'

Similarly, find the remainder to be *8* units which should be set down in the units' place.

27·45
18·89
8·56

Thus obtain 8·56 as the remainder and that Pattabhiramayya has 8·56 acres of lands more than Rama Rao.

Propose :—

1. A plot is 1·68 chains long and another is ·89 chain long. How much longer is the first field than the second?

(*Ans.* ·79 chain.)

2. One field is ·58 acre in extent and another is ·49 of an acre in extent. Which is the larger field and by how much?

(*Ans.* The first field is the larger by ·09 of an acre.)

3. A tenant got 3·7 puttis of paddy one year, while another got only 1·9 puttis. How much more did the former tenant get than the latter? (*Ans.* 1·8 puttis.)

4. Sanyasi Razu has 88·75 acres of lands. Gopala Razu has 15·76 acres of lands less than Sanyasi Razu. What extent of lands has Gopala Razu? (*Ans.* 72·99 acres.)

5. A lb. weight is equivalent to 38·88 tolas. The bazaarman asks you to take a lb. weight as equivalent to 40 tolas. How many tolas in excess of the true weight does he want you to reckon a lb. weight? (*Ans.* 1·12 tolas.)

6. The total rainfall at the Anakapalle Agricultural Station during the year 1920–21 was 26·68 inches as against 44·21 inches in the year previous and an average of 39·66 inches for the last eight years. How much less of rainfall was there during the year than during the year previous? How much was it less than the average?

(*Ans.* 17·58 inches ; 13·03 inches.)

4. Multiplication of decimals by integers whose multiples are given by the multiplication table.

Propose :—

There are 4 fences surrounding a field which are all equal. If each fence is 2·58 chains, find the total length of the fences.

Lead pupils to note that there are two ways of working this exercise :—(1) by finding the value of 2·58 plus 2·58 plus 2·58 plus 2·58 chains, or shorter still, (2) by multiplying 2·58 by 4. Point out to them that the multiplication may be done as follows :—

	units.	tenths.		hundredths.
2·58 × 4	2	5		8
			×	4
=	8	20		32
=	8	23		2
=	10	3		2
=	10·32			

herefore the length of the fencing required =10·32 hains. Tell them that the above multiplication may be orked more quickly as shown below :—

2·58	(1) Hundredths 8 × 4, 32 Set down 2. Carry 3.
4	
10·32	(2) Tenths 5 × 4, 20, 3, 23. Set down 3 carry 2.

Put in the decimal point to the left of 3 to denote that is in the tenths' place.

(3) *Units* 2 × 4, 8. 2, 10. Set down 10.

The product required = 10·32

Ask pupils to work the following exercises in a similar ianner :—

1. The weight of a kuncham of gingelly is 6·7 lb. ind the weight in lb. of 8 kunchams, 10 kunchams, 11 unchams, 12 kunchams and 16 kunchams of gingelly.

(*Ans.* 53·6 lb. 67 lb. 73·7 lb. 80·4 lb. and 107·2 lb.)

(Point out how 6·7 × 10 gives 67 and if any pupils put a 67·0 or 67· lead them to note that the presence of '0' or of '.' is unnecessary and '0' as well as '.' may hence be mitted.)

2. An acre of "*metta*" lands costs Rs. 100 at Kasim-ota. D. Gopalakrishnayya wants to purchase 5·78 acres of he lands. How much would he have to pay ?

(*Ans.* Rs. 578.)

(Point out to the pupils that there are two ways of vorking this question :—

(1) 5·78 × 10 = 5 units 7 tenths 8 hundredths × 10

= 5 tens 7 units 8 tenths

Therefore 5·78 × 100 = 5 tens 7 units 8 tenths × 10
= 5 hundreds 7 tens 8 units
= 578.

Therefore the cost of 5·78 acres = Rs. 578 or

(2) 5·78 acres = 5 acres, 7 tenths of an acre and 8 hundredths of an acre.

At Rs. 100 an acre, a tenth of an acre costs Rs. 10 and a hundredth of an acre costs Re. 1.

Therefore 5 acres cost Rs. 500, 7 tenths of an acre Rs. 70 and 8 hundredths of an acre Rs. 8.

Therefore 5·78 acres cost Rs. 578.

Change the number of acres in the above example to 6·84, 7·45, 5·49, etc., and lead pupils to generalize that in multiplying a decimal by 100 the product may be obtained by shifting the decimal point two places to the right.

Similarly, *get pupils to the artific· that multiplication by 10 may be effected by shifting the decimal point one place to the right, multiplication by 1,000 by shifting the decimal point three places to the right, etc.*)

3. A kuncham of *Gunupur Sannellu* weighs 6·25 lb. Find how much 10 kunchams of the paddy would weigh.

(*Ans.* 62·5 lb.)

4. An acre of "Pallamu" costs Rs. 1,000. Find how much 2·85 acres may be valued at this rate.

(*Ans.* Rs. 2,850.)

5. At the rate given in question (4) above, find the value of 2·75 acres, 2·43 acres and 2·25 acres of lands.

(*Ans.* Rs. 2,750, Rs. 2,430 and Rs. 2,250.)

6. The weight of 1 kuncham of gingelly is 6·8 lb. Find the weight in lb. of 8 kunchams, 12 kunchams and 20 kunchams of gingelly respectively.

(*Ans.* 54·4 lb. 81·6 lb. 108·8 lb. and 136 lb.)

7. The weight of one kuncham of "chodi" is 7·25 lb.

"	"	"mypali"	" 6·3	"
"	"	"konamani"	" 6·5	"
"	"	"daincha seed"	" 9·2	"

Calculate the weight in lb. of 20 kunchams, 15 kunchams, 12 kunchams, 8 kunchams, 6 kunchams and 11 kunchams of each respectively.

(*Ans.*

kunchams	(1) Chodi.	(2) Mypali	(3) Konamani.	(4) Daincha seed.
20.	145 lb.	126 lb.	130 lb.	184 lb.
" 15	108·75 "	94·5 "	97·5 "	138 "
" 12	87 "	75·6 "	78 "	119·4 "
" 8	58 "	50·4 "	52 "	73·6 "
" 6	43·5 "	37·8 "	39 "	55·2 "
" 11	79·75 "	69·3 "	71·5 "	101·2 " .)

Division of decimals by integers whose multiples are given by the multiplication tables.

Propose—

A property of 33·58 acres has to be divided equally between two brothers. How much will each get?

Question pupils how they are to find the result and lead them to work the division as follows :—

16·79	
2)33·58	
2	=2 tens.
13	=13 units.
12	=12 units.
1·5	=15 tenths.
1·4	=14 „
·18	=18 hundredths.
·18	=18 „

First divide 3 tens by 2. Put down 1 in the tens' place in the quotient. Convert the remainder 1 ten into 10 units, add 3 units and get 13 units. Divide 13 units by 2. Put down 6 in the units' place in the quotient. Convert the remainder 1 unit into ten tenths. Add 5 tenths and get 15 tenths. Divide 15 tenths by 2. Put down 7 in the tenths' place in the quotient. Convert the remainder 1 tenth into ten hundredths, add 8 hundredths and get 18-hundredths. Divide 18 hundredths by 2. Put down 9 in the hundredths' place in the quotient. Thus obtain 16·79 as the quotient.

Therefore the brothers will each get 16·79 acres of land.

Point out that the division is carried on in the same way as in the case of integers.

Let pupils work in a similar way the following exercises on division:—

(1) Five fully grown canes in a field have been together found to weigh 16·5 lb. What is the average weight of a cane? (*Ans.* 3·3 lb.)

(2) Eight fully grown canes of another variety have been found to weigh 25 lb. What is the average weight of a cane? (*Ans.* 3·125 lb.)

(Point out in the course of this division how '1' unit is converted into 10 tenths, 2 tenths into 20 hundredths, 4 hundredths into 40 thousandths and so on by affixing a '0' to the right of the remainder in each case.)

```
  3·125
8)25
  24
 ----
  1·0
   ·8
 ----
   ·20
   ·16
 -----
   ·040
   ·040
 -----
```

(3) A cane 12 ft. long is cut into 7 sets of equal length. What is the length of a set? (*Ans.* 1·7 ft.)

(Point out that here the division may be continued to as many places as we like but as figures beyond the tenths' or hundredths' are of little importance they may be neglected in practice and the result expressed to the nearest tenth or hundredth as the case may be. For example, in the above division, 12 ft. divided by 7 gives 1·714285 ft., i.e., 1·7 ft. and something less than 5 hundredths of a foot, i.e., half a tenth of a foot. Applying the convention for approximations, the result is 1·7 ft. to the nearest tenth of a foot.)

(4) A cart could hold ·325 ton of fire-wood. If a cart-load of fire-wood is divided equally among 5 cultivators, how much of a ton will each get? (*Ans.* ·065 ton.)

(5) One manugu of turmeric rhizomes has been found to give 10 manugulu of turmeric in the harvest. How many manugulu of turmeric rhizomes should be shown to give 32·5 manugulu of turmeric in the harvest?

(*Ans.* 3·25 manugulu.)

(6) 100 visams of green chillies when dried yield 25 visams dry. How many visams dry could be got from 1 visam green? (*Ans.* 25 lb.)

(7) 10 kunchams of a particular variety of paddy has been found to weigh 62·5 lb. Find the weight of a kuncham of the paddy. (*Ans.* 6·25 lb.

(8) 100 addas of chodi has been found to weigh) 181·5 lb. Find the weight of an adda of chodi. (*Ans.* 1·815 lb.)

(Point out in the course of working the last four exercises *that division by 10 could easily be done by shifting the decimal point one place to the left, that by 100 by shifting the decimal point two places to the left and that by 1000 by shifting the decimal point three places to the left and so on.*

(9) S. Patrudu has given Rs. 275 for 10 cents of land. How much does this come to for a cent? (*Ans.* Rs. 27·5).

(10) The *cost of an acre of* 'Pallamu' is Rs. 2,775 at Samalkota. Taking 100 cents for an acre, find the cost of a cent of Pallamu at that rate. (*Ans.* Rs. 2,775.)

(11) There are 88 deadhearts for every 1,000 ear-heads in a paddy field. What portion of the plants in the field are thus attacked? (*Ans.* ·088.)

(12) There are 8 deadhearts for every 100 ear-heads in another paddy field. What portion of the plants in the field are attacked? (*Ans.* ·08.)

(13) 1,000 lb. of cane yield as much as 124 lb. of refined sugar. What is the quantity by weight of sugar given by 1 lb. of cane? (*Ans.* ·124 lb.)

(14) 60 lb. of juice give 12 lb. of jaggery. What weight of jaggery is obtained from 1 lb. of juice? (*Ans.* 2 lb.)

TOPIC 16.—DOMESTIC AND CULTIVATION ACCOUNTS.

(**Scope of the treatment**.—*An over-elaborate treatment of the elements of Book-keeping with the single-entry and double-entry systems is not meant to be taken up here. As far as possible, the accounts suggested for maintenance should be practicable for an ordinary cultivator who handles the soil personally and within the power of apprehension of pupils at this stage. Hence, as agreed upon by some of the officers of the Agricultural Department, special emphasis is laid upon the right understanding of the correct maintenance of the Chittah and the Avarza. As an extension of the Avarza, the pupils need be insisted on the maintenance of cultivation accounts, field-war and crop-war, for a few years so that they may pick up good experience about the requisite outlay and approximate out-turn from cultivation of various crops and in various seasons. As a few of the pupils are likely to manage big estates as agents, or to take up large extent of lands for lease, the rudiments of gauging one's true financial position from the accounts kept and the drawing up of the statement of affairs may be incidentally referred to. Too much of worrying with exercises is not suggested, for, it is expected that the pupils will be insisted on maintaining even during the period of their stay in the agricultural school, a chittah regarding their receipts and payments, and the cultivation accounts not only of what they hear and observe in the farm, but also of the lands cultivated by their parents and guardians which they may observe during the holidays and hear of during the working days. In fact, these cultivation accounts should be given as much importance as practical work note-books.*

If there is time available during the course, say, in the second year, the elaborate details of the single and double entries may be taught during the revision, but they are not considered to be within the scope of treatment of this publication.)

Necessity for keeping correct accounts.

Question pupils how much they spent, and on what account, every 'dabbu' from their purse in the course of

the past two months. Impress the fact that they cannot trust their memory as regards *moneys* earned and spent, as they very soon forget what they received and paid. Point out that some people run headlong into debt without any idea of their income and their worth, and without any thought as to whether an expenditure incurred is such as can be met from their income. Point out also that, if correct and systematic accounts are kept, they will be able to find out what they have received in a particular year, how much they have spent, whether their expenditure is within their income or exceeds it, and, if it exceeds, what items could be cut short. In this connexion refer to the fact that by the maintenance of systematic and correct accounts they can not only find out easily the extent of their savings or debts at any time, but also forecast a certain year's expenditure and income from the previous year's figures.

Different kinds of account books.

(a) *The Chittah.*

Bring home to the pupils the need for knowing how much cash has been received and on what dates and how much has been spent and when. Point out that for this purpose a *chittah* may be kept which will show the cash received or spent every day. Impress the fact that they would then be able to see whether the cash on hand tallies with the balance in the cash-book. Show pupils a specimen page of the chittah as maintained by the local merchants with the following headings:—

No. (1) Chittah of M. R. Ry. Chekka Suriyanarayana opened on

Dhurmathinama Samvatsara Chitra Sudda Paddimi Sthiravaram (corresponding to 9-4-21).

Date-war details (Tharikuvakaira).		L.F	Receipts (Tzama).	Payments (Kartsu).
Date.	Particulars.			
1	2	3	4	5
			RS. A. P.	RS. A. P.

Tell pupils that the cash-book starts with the cash on hand, that all amounts received or paid, whether they may be the actual income or expenditure of the man or merely sums borrowed or lent by him, should find a place in the book and that all items of expenditure not actually paid should not be entered in it. Intimate to pupils that it would be highly desirable if every day the amount on hand is checked with the balance got from the cash book. Point out that in making entries in the cash-book, all amounts received are entered on the left hand side or debit side, that is, in the "*tsama*" column and that all payments made are entered on the credit or the right hand side, i.e., in the "*Kartsu*" column; and that this system of entry is followed more or less as a convention in the case of all accounts. Question pupils in which column the following details should be entered :—

April 2 Sold a palmyra log for Rs. 2.

4. Bought a pair of bangles for Rs. 60.
Bought a pair of cloth for Rs. 12.
Sold a basket of Jama Pandlu for Rs. 2.
Paid house-rent for last month Rs. 6.

6. Borrowed Rs. 20 from Golla Appadu for purchasing ragi and rice for household consumption.
Bought ragi for Rs. 30 and rice for Rs. 10.

7. Sold some planks for Rs. 10.
Sold an old house for Rs. 1,000.
Repaid a loan of Rs 800 to Bodazu Narayanamurti Garu.

8. Gave an advance of Rs. 15 to the masons for putting up an extension in the dwelling house in the ensuing week.

9. Gave the carpenter Rs. 5 for repairing furniture.

10. Gave the milkman Rs. 5 as last month's dues.
Bought a busta of rice for Rs. 19-8-0 and a manugu of jaggery for Rs. 3-5-0.

Point out that the balance should necessarily be struck off at the end of each month, compared with the cash on hand, and carried over to the next month.

Next suggest to pupils if it would *not* involve much time, space and worry if petty items like "Bought a post-card for 6 pies," kerosene for 3 pies for cleansing a screw,

gave a beggar 2 pies, etc., were to be entered in the cash-book against a particular date. Tell them that the total of such expenditure may be put in under a single item "Sundries for the day."

Next propose where items like 10th April—Sold a jewel on credit for Rs. 100, "Bought a cart on credit for Rs. 70," etc., are to be noted. Point out that as the cash-book deals with only cash transactions, items like purchases and sales on credit have no place in it, that if the credit items be sufficiently large, a daily credit book may be opened on the same lines as the cash-book but only to enter credit items. As with ordinary people such credit items are generally not fairly large in number, tell pupils that it is enough if a few pages at the end of the cash-book are set apart or even a small "advance" book opened wherein credit transactions with other than regular customers may be promptly noted with particulars, the entries thus serving as memoranda containing details of things lent, borrowed, sold or bought for credit, etc., wages for work done, but not paid at the time. Let pupils note, however, that when actual cash is received, e.g., for a sale on credit or actual cash is paid for a purchase on credit, the necessary entries regarding the receipt or payment of cash should be recorded promptly in the cash-book and the facts jotted also in the memorandum credit book.

(b) *The Avarza or the Ledger.*

Make pupils realize that every man should be able to ascertain without delay and without any omission what amounts he owes to persons like bankers and shopkeepers from whom he usually borrows and what amounts are due to him from others, e.g., from bankers to whom he sends frequent deposits. Tell pupils that for this purpose separate personal accounts are desirable. Show them an actual copy of the account book 'avarza' maintained by merchants. Write on the black-board the opening page of an 'avarza' say, as follows :—

Page in Avarza or ledger.	Name of persons arranged in alphabetical order.
1	A.C.
5	A.V.
7	B.A.
	B.C.

Question pupils why they have such an index and point out how it helps ready reference to the personal accounts of a particular customer. Point out also that even a page or two may be allotted to each person with whom regular transaction is had. Next present before pupils a page in the personal accounts of a customer as found in the 'avarza,' say, with the following headings :—

Name of customer { Karuppan village / Kanitha Audinarayana.

Year and month.	Date.	Particulars.	*Receipts* (Tzama).	*Payments* (Kartsu).
1	2	3	4	5

Tell pupils that there is a peculiar feature to be noted in interpreting the columns (Tzama, column 4) and (Kartsu, column 5) as entered in these accounts :—All the transactions that are entered in the personal accounts opened in the avarza are considered from the point of view of the person who is lending or borrowing the money and not from that of the person who is maintaining the accounts. In other words, the transactions are entered in the same way in which the person who is lending or borrowing the money will do in his cash or credit books; for example, the accounts paid to a particular person, say, Kanitha Audinarayana, are entered on the debit side, that is, in the 'tzama' or receipts' column (see column 4 above) and the amounts due from or paid by him are entered on the credit side, that is, in the Kartsu or payments' column (see column 5 above). Illustrate this further as follows :—

Suppose a pupil, say, Gourisam, borrowed from Kanitha Audinarayana. Rs. 15 on 1st January 1922, Rs. 20 on 2nd January 1922 and Rs. 15 on 11th March 1922, and repaid him Rs. 35 on 13th February 1922, and Rs. 15 on 1st April 1922, then in the Avarza maintained by Gourisam, the entries in the page opened in the name of Kanitha Audinarayana *will be as follows.*

Karuppam village. *Kanitha Audinarayana.*

Year and month.	Date.	Particulars.	Receipts (Tzama).	Expenditure (Kartsu).
1	2	3	4	5
			RS. A. P.	RS. A. P.
	1922.			
1	Jan.	By cash, Loan to Gourisam .	...	15 0 0
2	Do.	Do. ..	...	20 0 0
18	Feb.	To cash, repaid by Gourisam,	35 0 0	...
11	March.	By cash, loan to Gourisam ...	...	15 0 0
1	April.	To cash, repaid by Gourisam.	15 0 0	...

Point out to pupils that, whenever Kanitha Audinarayana lent money, it will be a payment (Kartsu) entry in his cash-book and will appear so in the page of the Avarza opened in his name by Gourisam and also that whenever Kanitha Audinarayana is given back any money, it will be a receipt (tzama) entry in his cash book and will appear so in the page of the Avarza opened in his name by Gourisam. Lead pupils to note that the Kartsu entries in the above Avarza will appear as 'tzama' entries in the cash-book maintained by Gourisam and the 'tzama' entries in the above Avarza will appear as 'Kartsu' entries in the cash book. Tell pupils that in order that ready reference may be possible from the chitta to the Avarza, and vice versa, the column (3) left vacant in a page of the chitta described above may be filled with the page number in the Avarza wherein are entered particular items in the chitta and between the columns (3) and (4) in the avarza may be shown the page number in the chitta relating to the items concerned.

Change the dates, names of persons and amounts in the above illustration and set a series of exercises to pupils about opening personal accounts in the avarza. Direct them also to enter correctly details of cash transactions both in the avarza and in the cash-book, dealings with the grocer, cloth-merchant, banker, etc., being taken for the purpose.

Gauging the actual worth of persons from accounts.

(After the system of keeping the chitta and the avarza as detailed above is fully understood by the pupils, the teacher may take up the question of finding the true financial position of a person at any time from the above accounts, supposing the accounts to have been kept systematically and correctly.)

Now point out that the receipts' (tzama) column does not give the actual total income of the man nor the kartsu column his total expenditure in as much as it contains all receipts including sums borrowed which do not form part of the income and all payments, including sums lent, which do not form part of his expenditure, and also as it excludes those amounts due to him, but not actually collected, e.g., the sale of an article on credit and all amounts due from him, but not actually paid, e.g., the purchase of an article on credit, wages due but not paid, etc. Hence let them note that another account may be necessary showing the actual income and expenditure of the man, that such an account will start with the total value of the man's property on the debit side (tzama) side, that in it items of income whether actually received or merely due will be entered against the respective dates on the debit (tzama) side, that all items of expenditure whether paid in cash or not will be entered on the credit (kartsu) side and that a balance may be struck at the end of the year which will give the value of the man's property then. Impress the fact that all sums borrowed or lent should not be entered in the account as they do not form part of the man's income and expenditure. Tell pupils that if they are cultivators of small holdings and not managers of vast estates, too many accounts would be hard to keep, that then they need not maintain such an income and expenditure account separately. Get them to realize that many of the items, in such an account, would have been already entered in the daily cash-book and it would be hence sufficient if the items of income and expenditure that appear in the cash-book are totalled at the end of every month. Inform them also that to this total all items of actual income and expenditure, not entered in the cash-book, may also be added, such as sale or purchase on credit, etc., and the resulting amount so obtained entered month-war in a separate page at the end of the cash-book under the heading "month-war income and expenditure."

Cultivation accounts.

Talk to the pupils about the receipts and payments of a cultivator solely in regard to the land he cultivates. Point out to them that agriculturists generally do not keep a separate account for noting down the income and expenditure relating to the cultivation of their fields, that they do not know whether their business works at a gain or loss and that often they draw large amounts for their domestic expenditure, and think that they have not realized any profit from their labour. Show from this that there is a clear necessity for the keeping of separate accounts relating to cultivation. Point out also that the right and systematic maintenance of the accounts in regard to the crops grown would give the cultivators a fairly correct idea as to the income and expenditure relating to each crop and enable them to realize which crop in which season and according to what method of cultivation would be most paying, and what lease amount may be offered without loss for leasing a certain garden or field, etc.

Make pupils understand that to enable the cultivator to know whether he has gained or lost on the whole during a *panta* or year, he must strike a balance at the end of the period. Get them to realize that to prepare such a statement of affairs the following details should be taken into consideration, viz., (i) the value of the land, implements, cattle, etc., at the beginning, (ii) their value at the end, (iii) the total expenditure incurred during the period solely in the cultivation, both in cash and on credit, inclusive of the quit-rent, tax, etc., and (iv) the total income from the lands by the sale of the produce, etc., either in cash or credit.

Point out that here again all the accounts which have been kept for noting down the transactions in connexion with the household management may be kept—the chitta wherein all cash transactions—receipts and payments may be promptly entered in chronological order, a small credit book wherein may be promptly recorded all purchases and sales on credit, wages for work done for others or by others not paid at the time, etc., and the avarza or the ledger wherein may be recorded the accounts of the various persons with whom the cultivator has steady and regular transactions.

Tell pupils that the maintenance of such separate accounts purely in connexion with cultivation will be

practicable when there is a separate agent deputed for the work, that such an agent will be employed when there is a vast estate to be looked after and that as the ordinary cultivator does not keep the cash used for domestic purposes separate from that used for cultivation, the same cash-book may be used both for household and cultivation purposes in as much as the object in both cases is only to find out the cash on hand. Point out similarly that the *avarza* or the accounts to be kept in the names of persons with whom the cultivator has regular transactions may be, so far as cultivation is concerned, combined with the personal accounts kept for domestic purposes.

Next tell them that in order that the cultivator may find out whether he is gaining or losing on each kind of crop grown and how much he is gaining or losing it is essential that at least before he picks up sufficient experience an account to estimate average cost should be kept showing field-war and, wherever practicable, for the whole year, or crop-war for the different pantas, the details of expenditure and income relating to the cultivation.

Field No. Area—1 acre.

Name.

Name of crop—Gingelly (*Tolakari*).

Year and month. (1)	Date. (2)	Particulars. (3)	Receipts (Tzama). (4)	Payments (Kartsu). (5)
1922.			RS. A. P.	RS. A. P.
April ...	2	Ploughings—two ..	2 8 0	
May ..	1	Penning sheep, 1,000 ...	6 0 0	
	8	Ploughings—two ...	2 6 0	
	9	Seed—2 seers ...	0 9 0	
	10	Ploughing, sowing—1 .	1 0 0	
	25	Ploughing, harrowing—2.	1 4 0	
June ...	5	Hoeing, 16 women ...	2 0 0	
August ...	5	Harvesting, pulling out and storing—2 men and 4 women.	1 0 0	
	10	Threshing—2 men ...	1 0 0	
		Yield 40 kunchams ...	...	28 0 0
		Total ...	17 11 0	28 0 0

Point out that in making the entries as above, all that the field received is entered on the left hand side, i.e., the receipt or *tzama* side and all that the field yielded is entered on the right hand side, i e., the payment or *kartsu* side. Let pupils work out the totals of the tzama and the kartsu columns and calculate by how much the payments exceed the receipts. Point out that this excess gives the profit of the cultivation Lead pupils to note that for the season (*paira*) from December to April, they will have to enter in addition the expenditure on *pannu* or *sistu* over and above the cost of cultivation and that if under any circumstances the payments equal the receipts there would be neither gain nor loss and if the payments exceed the receipts the difference would indicate the loss incurred in the cultivation, e g , when crops wither or fail. Point out that the cash transactions relating to the crop accounts will also find a place in the cash book with the difference that the tzama entries in the crop or field account will appear as kartsu entries in the cash book, that the kartsu entries in the crop or field account will appear as tzama entries in the cash book (see the reasons explained when dealing with the maintenance of the avarza), that often the entries for more than one field, say, regarding ploughing charges, will appear as one entry in the cash book and that credit transactions, e.g., wages outstanding, loan or borrowal would not find a place therein.

Next suggest to pupils how entries like ploughing charges will appear in the crop-account if the cultivator keeps his own bullocks or buffaloes and ploughs the field. Get them to realize that as they are charges incurred they may be inserted at the current rates in the crop account, that a separate page may be opened for cattle maintained, with the receipts and charges on their behalf being correctly shown and the ploughing charges entered therein as receipts got from the maintenance of the cattle. Similarly, tell them that they may open a separate page in the crop account showing the quantity of grain in stock, the quantity sold and the balance on hand in chronological order; also, that an implements account, cows and milch-buffaloes' account, poultry account, etc., may be maintained on separate pages of the book, the requisite entries being also made in the cash book simultaneously.

Statement of affairs.

Point out that to enable a person to know whether he has gained or lost on the whole in running a business during a particular period, say, a year, or in leasing some lands and undertaking their cultivation during the period, a statement of affairs may be prepared at the end of the year by totalling up the *isama* and *kartsu* columns in the chittah, credit memorandum book, and avarza including the crop, cattle, implements, etc., accounts and entering them as follows :—

Statement of affairs on 31st March 1923 (or the last day of Palgunam of a Telugu year).

Liabilities.	RS.	Assets.	RS.
Value of the land at the beginning of the year ...	2,000	Value of the land at the end of the year	2,500
Value of the implements at the beginning of the year	150	Value of the implements at the end of the year ...	140
Value of the cattle at the beginning of the year ...	350	Value of the cattle at the end of the year	250
Value of the stock of grain on hand at the beginning of the year ...	250	Value of the stock of grain on hand at the end of the year	1,150
Cash on hand at the beginning of the year ...	150	Cash on hand at the end of the year	150
Debts due to others at the beginning of the year ...	400	Money due from others at the end of the year ...	150
Total Rs. ...	3,300		
Profits during the year ...	1,040		
	4 340	Total ...	4,340

Note on depreciation.

In estimating the values of implements, etc., at the beginning and at the end of a period, say, a year, get pupils to note that, owing to general wear and tear, the value of implements, tools, buildings, etc., may tend to diminish in course of time and that, therefore, when a cultivator wants to ascertain how much he was worth at the beginning, and, is worth, at the end of a period, say, a year, he must take into account the probable dimunition in value above referred to. Tell pupils that it is usual to deduct a certain amount from the value of implements, etc., at the beginning of a period and thus obtain the value at the end of the period. Point out to

them that one advantage of such depreciation being noted is that the accounts of the cultivator would continue to show fairly approximately the actual value of his belongings. Intimate to pupils that the depreciation in value above referred to is best effected by deducting from the value at the beginning of a period a certain fraction of it and then writing off that amount; for example, if a cart is valued at Rs. 100 at the beginning of a year and if the depreciation in value is taken as one-tenth, Rs. 10 will be written off as depreciation, being obviously one-tenth of Rs. 100, and the value at the end of the year represented as Rs. (100 — 10) or Rs. 90.

(Incidentally get pupils to infer that if there is enhancement in value, the enhancement should also be similarly noted.)

Propose the following exercise :—

1. Bapannah starts with Rs. 800 cash and lands worth Rs. 3,000, intends to make agriculture his occupation and the following are his transactions from 1st September 1922 to 1st January 1923 :—

1922.		Rs.
1st Sep. ...	He takes from D. Kamayya 15 acres of lands on a rent of Rs. 500 in money.	
5th ,, ...	Bought seed	160
10th ,, ...	Ploughs and cattle	470
15th ,, ...	Ploughing charges paid	40
20th ,, ...	Sowing charges paid	25
25th ,, ...	Bought on credit seeds from Sitarama Patrudu—value Rs. 100.	100
30th ,, ...	Watchman's wages	25
5th Oct. ...	Manure for cash	15
10th ,, ...	Bought manure on credit	20
20th ,, ...	Spreading manure	4
28th Nov.	Coolies' wages	12
15th Dec. ...	Reaping charges	50
26th ,, ...	Sold produce of paddy for cash at a flat contract rate	400
27th ,, ...	Sold produce of paddy for cash at a flat contract rate	800
28th ,, ...	Paid S. Papayya	100
29th ,, ...	Paid S. Venkannah...	20
31st ,, ...	Paid rent	500
1923.		
1st Jan. ...	Value of implements, cattle	380
Do.	Paddy on hand	100

Fill up the entries in the cash book and the credit book in the above transactions, showing the monthwar balance.

2. The following details refer to a farm account for the period from 1st September 1922 to 31st January 1923:—

1922.			RS.
1st Sep.	...	Value of land on 1st September 1922	3,000
30th Sep.	...	Cash expenses from cash book for September 1922	300
Do.	...	Credit expenses	100
31st Oct.	...	Cash expenses from cash book for October 1922	25
Do.	...	Credit expenses	20
30th Nov.	...	Cash expenses from cash book for November 1922	80
Do.	...	Credit expenses	50
31st Dec.	...	Cash expenses	300
Do.	...	Cash sales	1,000
Do.	...	Credit sales	500
1923.			
1st Jan.	...	Stock of paddy	900
Do.	...	Depreciation of implements, cattle, etc., during the period	100

Prepare the gain or loss from the whole transaction.

3. Bapayya is appointed as an agent under Sanyasi Razu on a salary of Rs. 100 per mensem. He joins duty on the 1st September. He had Rs. 10 with him when he began to keep his accounts for the month. The following are his receipts and expenses for the month. Enter them in the proper accounts and find out his financial status at the end of the 1st October:—

1922.			RS.	A.	P.
2nd Sep.	...	Bought cloth on credit from B. Hanumanta Rao	15	0	0
3rd "	...	Bought milk	0	4	0
4th "	..	Bought provisions on credit from Datta Suryanarayana	37	8	0
5th "	...	Advance given as house-rent	2	0	0
10th "	...	Took a loan from D. Acharya	15	0	0
15th "	...	Bought a box for cash	15	0	0
16th "	...	Petty expenditure	0	4	0
1st Oct.	...	Received pay	100	0	0
Do.	...	Paid balance of house-rent	5	0	0
Do.	...	Paid back B. Hanumanta Rao	15	0	0
Do.	...	Cleared debts	40	0	0

Change the numbers in the above exercise and propose a series of exercises.

4. *It is given that to maintain a carting pair bough* for Rs. 360, it costs Rs. 5-10-0 a month for ordinar

food, Rs. 5-10-0 a month for special food, watching and cleaning sheds Re. 0-15-0 a month, the value of the bullocks at the end of the year is Rs. 410, ploughing charges, manure, etc., saved by the maintenance of the pair is Rs. 80 for the year. Distribute the above items monthwar and datewar taking any nominal dates and figures which would make up the given total. Work out the profit or loss in maintaining them for the year under question.

5. Distribute, as in the previous exercise, details for sugarcane cultivation over a field 1 acre in extent one year and work out the net profit of the cultivation:—

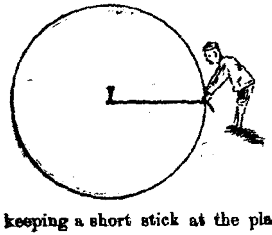
Sets—10 puttis Rs. 35; sheep-penning 2,000, Rs. 10; castor cake 10 bags, Rs. 55; bamboos 4,000, Rs. 20; bamboos 2,000, Rs. 20; irrigation Rs. 5; ploughing charges Rs. 5; ridging Re. 0-10-0; extracting juice and making jaggery 96 men at 6 annas each; mill hire Rs. 30; yield 6,000 lb. of jaggery valued at Rs. 500 in the locality.

TOPIC 17.—THE CIRCLE.

(**Scope of the treatment**.—*Even though any flat surface which appears to be round is in common language said to be circular, it must be pointed out to the pupils that in mathematics the term is applied only to particular shapes which have the following fundamental property :— equidistance of points on the rim from a central point. The methods of describing circles on the floor and on paper by using the compasses or a string should be clearly explained to the pupils. As exercises in the use of the compasses for drawing circles, certain designs may be set up before them, e.g., those underlying Mr. Sampson's economical method of planting trees, and the economy in space effected thereby should be well impressed. The relation between the diameter and the radius may be dealt with and notions about equal and unequal segments imparted. Too much of terminology, e.g., like chord, segment, sector, need not be brought in here as the pupils are likely to get confused. The method of measuring the diameter by means of a string may be explained and the relation between the circumference and diameter roughly derived. Applications of the circumference as a locus, etc., may be relegated for later treatment whenever they are found necessary.*)

Notion of a circle.

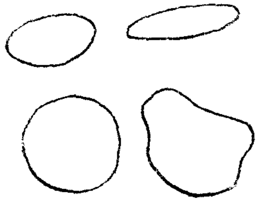
Take the pupils to a plot of level ground near the school-yard. Get a peg driven into the ground somewhere near the middle of the plot. Tie to the peg one end of a piece of string about 11 feet long. Direct a pupil to stretch the string straight and catch hold of it at a distance of 10 feet from the peg. So holding the string and keeping a short stick at the place of grip, let him walk



around the peg and trace a mark with the point of the stick along his path. Point out that a ring-shaped curved line is thus drawn on the ground. Tell pupils that the portion of surface enclosed by the curved line is called "a circle" and the curved line bounding the circle its "circumference." Ask pupils to stand at several places along the ring. Question how far they are then from the peg. Let them measure with a tape, if necessary. Get them to realize that the spots chosen on the rim of the ring are all at equal distances of 10 feet from the peg. Hence, elicit that the place where the peg is driven is "the centre" of the circle. Incidentally, lead pupils to infer that any spot chosen outside the ring should be at a distance greater than 10 feet from the peg and points lying within the circle are at shorter distances from the peg.

Ask pupils to suggest circular rims and surfaces they have seen, e.g., the bottom of a jaggery-pan, the ends of cylindrical logs, the cart wheel, the silver, copper and gold coins, the path of the "ganuga" bullock, etc. Point out that though in common language we speak of all these as circular in form, we would be correct in so styling, only if the property above referred to exists. Impress on them that judged from such a criterion the ends of many logs, etc., would be found not to be actually circular in form even though they appear to be so.

Let pupils compare a circular form with other curvilinear forms, e.g., the oval, etc., drawn on the blackboard as shown below :—



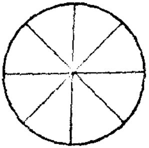
See that they realise that unlike the other curvilinear forms the circle curves in the same manner all through.

Describing circles with compasses.

Propose to pupils to draw a circle on the blackboard or on paper. Let them suggest methods, e.g. by using a string or circular rims, by holding the pencil by the fist and turning it round, etc. Point out how a pair of compasses may be used to obtain a good circular form. Let pupils handle a pair of compasses, note that its moving ends are adjustable to different distances and hence and otherwise decide the superiority in using it for describing circles over the other methods suggested above. Ask pupils to draw a circle on paper and on the black-board by using the compasses. Caution them to mark the centre-point first so that, if while describing the circle the metal point of the compasses should slip, it may be restored to its original position. If while describing the circle the chalk or the pencil point jerks, point out to them that if they hold the legs of the compasses in a slanting position they would be able to draw the line freely.

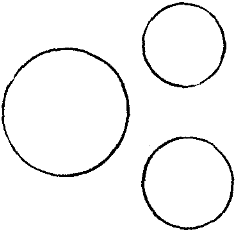
Equality of radii in a circle.

Ask pupils to draw a straight line from the centre of the circle they have drawn to its circumference. Let them draw some more such straight lines emerging from the centre in different directions. Ask them to compare their lengths and lead them to note by measurement or by laying a string that the lines so drawn are all equal. Tell them that each straight line is called a "radius" of the circle. Question how many such straight lines can be drawn in a circle. Actually draw some more of the lines. Let pupils infer that *in one and the same circle many radii can be drawn and that they must all be equal.*



Ideas of equal and unequal circles.

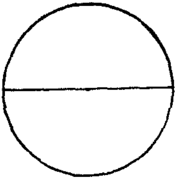
Ask pupils to draw a circle of radius 1 inch in their note-books. Let one of them draw such a circle on tissue or on translucent paper. Pass this sheet round



and let pupils place the circle drawn on this sheet over that drawn in their note-books so that the centres coincide. Hence lead them to infer that *circles drawn with equal radii are equal.* Repeat the above exercises with two or more circles drawn with unequal radii and let pupils infer that circles drawn with unequal radii are unequal. Hence point out that a circle can be determined if the position of its centre is fixed and the length of the radius given.

Relation between the diameter of a circle and its radius.

Ask pupils to draw a circle of any radius on paper and on the blackboard. Let them draw a straight line so as to pass exactly through the centre. Let them next produce the straight line either way until it meets the circumference at two points. Point out to them that a straight line has thus been obtained passing through the centre right across the circle. Tell them that such a straight line is called "a diameter" of the circle. Next ask them to compare the length of the diameter with that of the radius of the circle. Bring out the fact that the diameter is twice the radius. Point out incidentally that a circle can be determined if, instead of the radius, its diameter is given.



Ask pupils to draw some more diameters in the circles. Point out that any number of diameters can be drawn in a circle.

Idea of a semi-circle.—Let pupils draw a circle on tissue paper, putting a cross-mark to denote its centre. (Circles cut-out on paper may be used for this purpose

as an alternative.) Let them next draw a diameter in the circle. Ask them to show the two parts into which the circle is divided by the diameter. Direct them to fold the sheet along the diameter. Let them note that the two parts of the circle on either side of the diameter exactly fit in with each other. Hence let them derive that each part is half a circle. Tell them that each half so obtained is called a "semi-circle." Let pupils repeat the above exercise with some other diameter and get at the same fact. Point out by using paper-circles that it is only when the circles are folded along the diameter they get the circle divided into two halves and that when they are folded straight along any other line they get only two unequal parts.

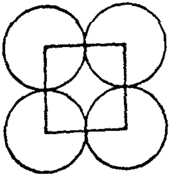
Let pupils next draw a pair of circles in their note-books, show in one circle the division into two equal parts and in the other the division into unequal parts. Let them shade one of the two parts thus obtained in each circle.

Exercises.

Propose :—

1. It is known that a missing field-stone was 120 links from a palmyra tree. Along what line may the field-stone be expected to have remained ?

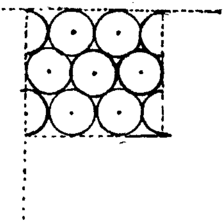
2. The Agricultural Department advises that mango



grafts should not be planted closer than 40 feet apart. Draw a figure to show where, around a particular graft, other grafts may be planted at the distance above stated. Represent 20 feet by 1 inch.

(If the pupils suggest a square form of arrangement point out by drawing the corners of a hexagon as shown in the accompanying figure how such an arrangement is also possible.)

3. Copy the design in the marginal figure, choosing 2 inches as the length of the radius.



4. Mark two points, A and B, 2 inches apart. With A and B as centres describe circles with radii 1 inch.

(Show that they get equal circles touching each other. Place the rim of a quarter anna coin contiguous to that of a *rupee coin* and impart the notion of unequal circles in contact with each other.)

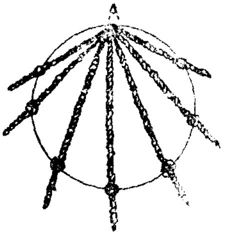
Attach to the blackboard the diagram containing Mr. Sampson's economic method of planting trees at specified distances apart. Ask pupils to copy the diagram (to a convenient scale). Point out the economy in space effected by adopting Mr. Sampson's plan.

Measuring the diameter of a circle.

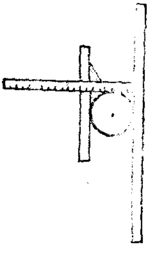
Refer pupils to a cylindrical jaggery pan. Question them what measurement they would take in order to secure another of the same size and shape. Get them to suggest that the height and girth or the height and the diameter may be measured for the purpose. Point out to them that the centre point of the circular face is not clearly marked. Propose to them how they would then measure its diameter. Tell them that the diameter could be measured by means of a piece of string and, preliminary to such measurement, they will have to learn about an important property of the diameter of the circle not dealt with before Dwell on the property as follows:—

Draw a circle on the board with its centre clearly marked. Mark a point anywhere on the circumference.

Ask a pupil to hold a string with one end fixed at A and stretch it straight so that the string assumes the position of a straight line drawn from the point A to any other point on the circumference. (See figure for guidance.) Place the string in several positions and question in which of the positions the point on the circumference marked out by the string is remotest from A. Let pupils actually compare the lengths and note that when the string is laid along the centre the remotest point from A is obtained on the circumference. Point out to them that a rotation of the string in that position round A would convince them that there is no point on the circumference remoter from A.



Get pupils to fix an end of a string against a spot on the rim of the jaggery pan in question and try by a series of adjustments to get at the remotest spot on the rim. Then let them measure the distance between the two spots, keeping the string stretched straight. Lead them to note that the length so obtained would give the diameter of the pan in question.

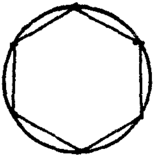


If there be a set of sliding calipers in the school the teacher may make use of the apparatus for showing how the diameter of any cylindrical object may be measured with it. Otherwise, this method alone need be dealt with as it would lead to the measurement being taken with the materials on hand, e.g., a string and a measuring rod. Avail of a visit to the local fibre factory to show how the diameters of cylindrical bundles are measured with a set of sliding calipers.

Deriving the relation between the circumference and diameter of a circle.

(i) Place a cylinder before the pupils with the centre point of a circular end distinctly marked. Get pupils to measure aright the circumference by means of a piece of string and mark out the portion of the thread covering the entire circumference. Let them next place the thread along the diameter and see how often the diameter is contained in the portion so marked out. Thus get them to note that the circumference is slightly greater than 3 times the diameter but less than $3\frac{1}{2}$ times the diameter.

Referring to a diagram like the accompanying one, get pupils to note that the circumference is divided into six parts, each part is greater than the straight line which joins its ends and which is equal to the radius and thus derive that the entire circumference is greater than six times the radius, i.e., three times the diameter.



Refer pupils to the fact that the length of tyre required for a wheel should, therefore, be greater than six times the length of a spoke but less than seven times the length.

TOPIC 18.—VULGAR FRACTIONS.

(Scope of the treatment.—*Clear notions may at first be given of what is meant by fractions of a unit. The pupils may then be introduced to the fractional notation and to the significance of the numerator and the denominator. The principle of equivalent fractions may be explained and applications of it shown in expressing fractions in different denominations so that they may be easily compared, added and subtracted. The notion of improper fractions and of the equivalent mixed numbers may be next given and the rules for converting improper fractions into mixed numbers and the latter into the former derived. The processes of multiplication and division as applied to fractions may next be taken up and their applications to working questions on denominate numbers shown. The tables for the common fractions may be derived and the pupils directed to get them memorized. Some of the intelligent indigenous methods of calculation involving the use of the above tables may be explained and plenty of oral drill may be given in them. Most of the rules may be got derived by reference to the capacity measures and the units of money and wherever diagrams are suggested it may be noted that rough sketches will do as otherwise a good deal of time will have to be spent and thereby rapidity in progress will be difficult of achievement.*)

Idea of a fraction and the fractional notation.

1. Distribute among pupils rectangular sheets of paper (octavo size). Ask them to fold a sheet into two equal parts and then into four equal parts.

2. Place a " nimmakaya " on the table. Ask pupils to divide it into two equal parts.

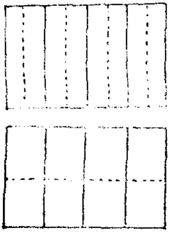
3. Draw a rectangle on the ground so as to represent a field. Ask pupils to divide it into two equal parts so that they may sow " Mahipali " in one part and " Bobbiliganti " in the other part.

Let pupils note that in every article there are two halves and that the fraction "half" may be written in a more

concise form as "$\frac{1}{2}$" just to indicate "one" out of the "two" equal parts into which the whole is divided.

4. Place a "Gummidikaya" or a guava fruit on the table. Ask pupils to divide it equally among four of them. Let them show one-fourth, half and three-quarters of the fruit cut. Lead them to note that there are four-quarters or fourths in each fruit. Question how one-fourth and three-quarters may be written briefly by using figures. Let them represent the fractions as $\frac{1}{4}$ and $\frac{3}{4}$ respectively. Point out that the numbers "1" and "3" respectively indicate one out of the four and three out of the four equal parts into which the whole fruit is divided.

5. Draw a rectangle on the board. Tell pupils that it represents a field. Ask them how they would divide it into eight equal parts. Let them draw lines freely and thus sketch out roughly the eight parts required. Suggest, if necessary, how crosswise lines may also be drawn to divide the rectangle as required. Ask pupils to show 1, 2, 3, 4, 5, 6, 7, of the parts so obtained and state what fraction of the field is thereby got in each case. When they give out that the part shown is one-eighth, three eighths, etc., ask them to write the fraction in figures.



6. Next propose to pupils that a long piece of cloth has to be divided into three equal parts so as to be fit for wearing. Question how they would proceed to divide the cloth as required. Let them actually handle a shorter piece of cloth, fold it into three equal parts and thus see how the division may be done. If any of the pupils suggest that the whole length may be measured, then divided by three and thereby the length of the parts obtained, point out that the division by folding is handier. Without tearing the cloth into three equal pieces, let them show one-third of it and two-thirds of it. Next ask them to represent these fractions in figures.

7. In place of the piece of cloth suggested in the last exercise, propose that a long pole should be shown divided

into three equal parts and then into six equal parts. If necessary, suggest to them the use of a string or of a rope for the purpose. Lead them next to explain how it may be stretched straight along the length of the pole and how the portion so stretched may first be *folded into three* and then *into six equal parts*.

After the portions have been marked out, let pupils show 1, 2, 3, 5, of the parts and state what fraction of the length of the pole is shown in each case. Let them simultaneously write the fraction in figures.

8. Mark out a rectangular plot of ground in the open yard in the neighbourhood. Propose to pupils to set out three-eighths of the plot so that they may raise Cambodia cotton in it as an experimental measure. Let pupils suggest several methods. After they mark out three-eighths of the plot by adopting any of the methods, ask them to indicate the boundaries of the plot drawing a rough path with the foot.

Now impress on pupils the following facts:—

(i) *When a fraction* of anything is talked of it is assumed that the whole thing is divided into a certain number of equal parts and that some of the parts are taken.

(ii) A fraction so chosen may be expressed in words as well as in a shorter form by using figures, e.g., in writing three-eighths as $\frac{3}{8}$.

(iii) In writing a fraction in figures, e.g., three-eighths as $\frac{3}{8}$, two numbers are used, one written above and the other below a line drawn thus "—" or "/". The number written below the line represents the number of equal parts into which the whole thing is divided. It thereby points out to what denomination the fraction belongs, e.g., eighths in the fraction 3/8. This number is hence called the "*Denominator*" of the fraction. The number written above the line represents how many of the parts so obtained should be taken to form the required fraction, e.g., 3 parts in 3/8. It is hence called the "*Numerator*" of the fraction. (*Numerator* means "*Numberer*.") Both the denominator and numerator form the numbers that are required to denote a fraction in figures and they are hence known as the "*Terms*" of the fraction.

Next describe to pupils how the common fractions are expressed in the indigenous system by means of strokes.

Propose:—

1. A putti contains 20 kunchams. How many kunchams are there in half a putti, quarter of a putti and three-fourths of a putti?

(*Ans.* 10 kunchams, 5 kunchams, 15 kunchams.)

2. An Imperial seer (Calcutta seer) weighs 80 tolas. How many tolas would three-quarters of the seer weigh? How many tolas would five-sixteenths of the seer weigh?

(*Ans.* 60 tolas, 25 tolas.)

3. A rupee could be exchanged for 48 dabbus. How many dabbus can be had for half a rupee, quarter of a rupee, and one-eighth of a rupee?

(*Ans.* 24 dabbus, 12 dabbus, 6 dabbus.)

4. A "stothrium" contains 168 acres of land. My family has a twelfth of the stothrium. How many acres of lands has my family? (*Ans.* 14 acres.)

5. A bundle of "aratti akulu" contains 200 leaves. I take one-fourth of it. How many leaves do I take?

(*Ans.* 50 leaves.)

6. A ton contains 40 "*gundulu.*" I buy seven-eighths of a ton of firewood. How many gundulu would I get? (*Ans.* 35 gundulu.)

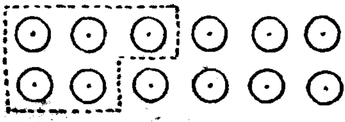
Repeat the above exercises by a change of numerical data.

Expressing one quantity as a fraction of a larger quantity of the same kind.

Propose:—

Sugarcane cultivation in a field fetches 12 puttis of jaggery. If the landlord claims from the cultivating tenant five puttis as his share, what fraction of the yield does he claim?

Draw 12 rings on the blackboard to represent 12 puttis. Let pupils mark out five rings as representing the



quantity claimed by the landlord. Lead them to note that if the whole quantity be divided into 12 equal parts the landlord would claim 5 of the parts. Thus bring out the fact that he claims five-twelfths or $\frac{5}{12}$ of the total yield of jaggery. Point out how the number of puttis in the total yield may be written first, a line next drawn over it and the number of puttis claimed by the landlord written above the line and thus the fraction 5/12 written without delay. Point out also that in writing the fraction 5/12, it is usual to write "5" first, next draw the line and then write "12" below the line.

Propose:—

1. Gourisam owns 50 cents of garden lands. He has raised pasupu on 12 cents and sugarcane on 25 cents. What fraction of his lands is under pasupu and what fraction under sugarcane?

(*Ans.* Pasupu $\frac{12}{50}$, sugarcane $\frac{25}{50}$.)

(Do not of your own accord invite the attention of the pupils at present to cancel common factors from the terms of tho fraction above expressed as answers.

If the pupils, of their own accord, express $\frac{25}{50}$ as $\frac{1}{2}$, let them do so. The teacher should for a short time at present be satisfied with answers like $\frac{4}{48}$, $\frac{12}{16}$, $\frac{40}{120}$, $\frac{25}{40}$, $\frac{15}{40}$ $\frac{12}{18}$, and $\frac{200}{300}$ for the following six exercises.)

2. What fraction of a rupee is 4 dabbus?

3. What fraction of a rupee is 12 annas?

4. The entire cost of a country cart is Rs. 120 at Anakapalle. The tyres cost Rs. 40. What fraction of the entire cost is the cost of the tyres?

5. My grandfather had 40 acres of wet lands. He gave 25 acres to my paternal uncle and 15 acres to my father. What fraction of the lands did he give to each?

6. A field which was usually served with 18 cart-loads of farmyard manure was served with only 12 cart-loads of the manure one year. What fraction of the usual quantity was it served during the year?

7. A garden which was usually yielding 300 kunchams of paddy during Vari Panta yielded only 200 kunchams of paddy one season. What fraction of the usual yield was then obtained?

Expressing fractions in different denominations and the principle underlying the process.

Propose :—

What fraction of rupee is a beda ?

If pupils say that a beda is 1/8 of a rupee ask them how *they got the result. Point out that a rupee would* fetch 8 bedas as change and hence a beda is 1/8 of a rupee. Next get pupils to argue that a beda would fetch 2 annas and a rupee 16 annas and hence express that a beda is $\frac{2}{16}$ of a rupee. Proceeding similarly, let them express that a beda is the equivalent of 6 dabbus, a rupee of 48 dabbus and hence infer that a beda is $\frac{6}{48}$ of a rupee. Let them next express a beda and a rupee in *terms of their equiva*lents in coppers, ardhana and pies respectively and hence derive that a beda is 8/64 of a rupee, or 4/32 of a rupee or 24/192 of a rupee.

As soon as each equivalent is obtained, write on the blackboard.

A beda	$= \frac{1}{8}$	of a rupee.
"	$= \frac{2}{16}$	"
"	$= \frac{6}{48}$	"
"	$= \frac{8}{64}$	"
"	$= \frac{4}{32}$	"
"	$= \frac{24}{192}$	"

Impress the fact that the value of a beda and that of a rupee is unalterable and hence let pupils conclude that the fractions 1/8, 2/16, 6/48, 8/64, 4/32, and 24/192 are equal in value. Tell them that fractions which are equal in value are called *"Equivalent Fractions."* Get them next to examine in what respects difference is noticeable among the fractions above written. Point out that the denominations in terms of which the values of the beda and the rupee have been successively expressed have been different at different times and hence the denominators differ. Illustrate this further by pointing out that when expressing the value of the beda in annas, we get the fraction 2/16 and when in dabbus the fractions 6/48. Next lead them to realize that, as the denominators change, the numerators also change accordingly. Let them examine more closely the changes in the denominators and those in the numerators and get them to infer that if the denominator in one fraction can be obtained by multiplying the

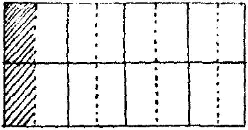
denominator in an equivalent fraction, say, by 3 the numerator in the former fraction can also be obtained by multiplying the numerator in the second fraction by 3, e.g., in the case of 2/16 and 6/48, or if the denominator in one fraction can be obtained by dividing the denominator in an equivalent fraction say, by 12, the numerator in the former can also be obtained by dividing the numerator in the latter fraction by 12, e.g., in the case of 24/192 and 2/16.

Illustrate the principle contained in the above by taking some more examples, e.g.—

1. 10 annas $= \frac{10}{16}$ of a rupee.
$= \frac{5}{8}$ of a rupee.
$= \frac{30}{48}$ of a rupee, etc.

2. 5 Visams $= \frac{5}{8}$ of a maund.
$= \frac{10}{16}$ of a maund.
$= \frac{20}{32}$ of a maund, etc.

In this connection, draw also a rectangle on the blackboard. Mark out a part to represent $\frac{2}{16}$ of it. Erase the line marking out sixteenths keeping those marking the eighths. Thus lead pupils to derive that $\frac{2}{16} = \frac{1}{8}$.



Next divide each part into still smaller subdivisions and show that $\frac{10}{16} = \frac{20}{32} = \frac{40}{64}$, etc.

Let pupils compare the terms of the fractions as before and conclude likewise.

Thus lead them to generalize that *the same fraction can be expressed in different denominations by having both its denominator and numerator multiplied or divided by the same number* and impress on them the fact that the value of a fraction would be unchanged when its terms are multiplied or divided by one and the same number.

Application of the above principle.

Point out to pupils that the answer in exercise (1) above namely the fraction $\frac{25}{50}$ of the lands may be expressed in a shorter and more familiar form as "$\frac{1}{2}$" by dividing the numerator and denominator of the fraction $\frac{25}{50}$ by 25. Similarly, point out that the fraction $\frac{6}{32}$ may be written as $\frac{3}{16}$, 10 annas instead of being expressed as $\frac{10}{16}$ of a rupee

may also be expressed as $\frac{5}{8}$ of a rupee, and 12 annas as $\frac{3}{4}$ of a rupee instead of as $\frac{12}{16}$ of a rupee. Impress on pupils that to express a fraction like $\frac{12}{16}$ in a familiar form known to us as so many eighths or fourths, etc., the numerator and the denominator may be divided by one and the same number whenever possible. Thus, if from a bundle of 1,000 leaves one takes away 625, the fraction taken may be represented as $\frac{625}{1000}$ of the bundle ; this fraction may be expressed in a simpler form as $\frac{125}{200}$ by dividing both the numerator and the denominator by 5; this again as $\frac{25}{40}$ by adopting the same device and this again in a still more simpler form as $\frac{5}{8}$. Point out to pupils that the form 5/8 is quite handy and simple and is more familiar to us than $\frac{625}{1000}$.

Comparison of Fractions.

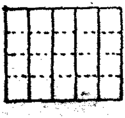
Propose :—

B. Devaiyya sows *horse-gram in one-fourth* of his dry lands and jonna in two-fifths of his dry lands. Which cultivation occupies a greater extent of the lands ?

Let pupils suggest how they would make the comparison in the above case. Suggest to them first that they may consider Devaiyya's lands to be 100 cents, then get them to note that he will have horse-gram in 25 cents and jonna in 40 cents. Thus lead them to find that jonna occupies a greater extent of land and thereby note that the fraction 2/5 is greater than $\frac{1}{4}$.

Tell them that they may adopt the following method too :—The answer to the above question may be easily found by deciding which of the two fractions is greater, $\frac{1}{4}$ or 2/5. Let pupils work as follows :—If one rupee be taken as a unit, Re. $\frac{1}{4}$ will come to four annas, and 2/5 of a rupee will fetch six annas and odd. As the latter amount is the greater, 2/5 is greater than $\frac{1}{4}$.

Next present before them the following illustration also. Let pupils draw on the blackboard a rough sketch to represent Devaiyya's lands choosing a rectangle for the purpose, say 10″ × 8″. Let them next divide it into 5



equal parts by drawing lengthwise lines *roughly*. Let them shade 2/5 of the diagram so divided. Let them next divide the rectangular sketch into four equal parts by drawing breadthwise lines roughly and shade one-fourth of the

sketch. Let them thus see that the whole field is divided into 20 equal parts, jonna occupies $\frac{8}{20}$ of the field and horse-gram $\frac{5}{20}$ of the field, and hence conclude that jonna occupies a greater extent. Let them next set down on the blackboard the comparison above made of the fractions ¼ and 2/5.

2/5 is equal to 8/20; 1/4 is equal to 5/20; hence 2/5 is greater than ¼.

Point out that in this method the number 20 has been chosen as the denominator of the equivalent fraction, i.e., as the number of equal parts into which the whole field may be considered as divided so that when a fifth or a fourth of the field has to be found the division may be exact and easy. Let pupils suggest that the number 20 chosen above as a common denominator is a multiple of 4 as well as of 5 and is thus a common multiple of both the denominators. Point out that instead of the number 20 any other common multiple of 5 and 4, e.g., 40, 60, 100 and 120 may also be chosen. Point out that if they should choose 60 as a common multiple 2/5 may be expressed as 2 × 12/60 or 24/60, as 5 multiplied by 12 will give 60, and ¼ may be expressed as 15/60 as 4 multiplied by 15 will give 60, that of the two fractions 24/60 and 15/60 so obtained *the former is the greater and hence it may be concluded* that the fraction 2/5 is greater than the fraction 1/4. Next propose to pupils which of the above methods may be adopted in order to work out an exercise quickly. Let them choose any of the methods they find convenient.

Improper fractions and mixed numbers.

Question pupils what meaning they assigned to a fraction like 5/8 up till now. Point out that they considered the unit as divided into eight equal parts and 5 of the parts taken to represent the fraction. Hence show that they invariably got the numerator less than the denominator. Next tell them that in some cases they may meet with fractions expressed like 15/8. Point out to them that even though they may consider it impossible to take 15 parts out of the eight parts into which the unit is divided another interpretation may be given to such a fraction by considering it as denoting 15 eighths. Thus point out to them that the denominator may be given the above kind of interpretation. Tell pupils that such fractions are known

as *improper fractions* to distinguish them from fractions in which the numerator does not exceed the denominator. Suppose they take 15 eighths of a rupee. Question how many full rupees they could get out of it and what fraction of a rupee would remain. Point out that eight eighths of a rupee would fetch them a rupee and hence for 15 eighths they may get Re. 1 and 7 eighths of a rupee. Tell them that this quantity, namely 1 rupee and 7 eighths of a rupee used to be written as Re. $1\frac{7}{8}$. (Read as "one and seven eighths" or as "one and seven by eight".) Let pupils similarly express 9 fourths as $2\frac{1}{4}$. Thus point out to them that fractions in which the numerator happens to be greater than the denominator may also be expressed as made up of an integer and a fraction less than unity, i.e., as a *mixed number.*

Propose :—

1. Express 17 sixteenths of a rupee by using a mixed number.

2. Express Re. $1\frac{5}{8}$ by using an improper fraction. Repeat the above exercises by changing the numerical data and let pupils note that to express an improper fraction as a mixed number they have only to divide and to express a mixed number as an improper fraction, they have only to multiply and add. Keep the equivalents obtained in working the above exercises and make them explain which nnmbers have to be divided or which numbers to be multiplied in the above conversion.

Addition and subtraction of fractions.

Propose :—

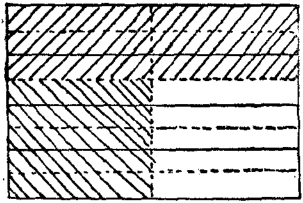
A ryot has onions planted in 3/8 of his lands and sweet potatoes in 5/16 of them. What portion of his lands is thus under the above cultivation?

Let pupils note that here 3/8 and 5/16 have to be added to obtain the portion required. Tell them that there are several methods for obtaining the sum of the fractions 3/8 and 5/16 and that they may adopt any one of the methods stated below :—

Method I.— Suppose it is 3/8 of a rupee and 5/16 of a rupee which have to be added. As 3/8 of a rupee is 3 bedas or six annas, and 5/16 of a rupee is 5 annas, the total comes to 11 annas and this may be expressed in

rupees as 11/16 of a rupee. Lead pupils to note that just as 3/8 of a rupee plus 5/16 of a rupee is 11/16 of a rupee, 3/8 of a plot plus 5/16 of the plot must come to 11/16 of the plot.

Method II.— Let pupils draw a rectangle to represent the whole field and divide it into eight equal parts by means of horizontal lines. Let them shade three of the parts to indicate 3/8 of the field. Let them draw a line vertically and thus get the rectangle divided into 16 parts. Ask them to shade 5 of the parts so obtained to indicate $\frac{5}{16}$ the field. Question into how many equal parts the whole field is then divided and how many of the parts are covered by the portions shaded. Thus get them to note that the portions shaded contain 11 out of the 16 equal parts into which the whole field is shown divided and hence derive that the cultivated portion is $\frac{11}{16}$ of the whole field.



From the methods I and II above, lead pupils to derive the following general method:—$\frac{3}{8}$ is in the denomination eighths and $\frac{5}{16}$ is in the denomination sixteenths. In order to add the two fractions they must be expressed in one and the same denomination, e.g., sixteenths or forty-eighths or sixty-fourths, etc. Let them choose sixteenths as being the simplest. Applying the principle of equivalent fractions get them to derive that $\frac{3}{8}$ is equal to $\frac{6}{16}$ and therefore $\frac{3}{8}$ plus $\frac{5}{16}$ equals $\frac{6}{16}$ plus $\frac{5}{16}$ or $\frac{11}{16}$.

As far as possible, impress on pupils to adopt method I given above as it would lead to results being found orally and quickly. The problems given under addition of fractions are expected to be those mostly occurring in common use, e.g., $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{1}{16}$, etc., and hence the application of method I would be possible in almost all these cases.

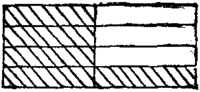
If any pupil should suggest how a question on addition would be solved if instead of $\frac{3}{8}$ and $\frac{5}{16}$ one had $\frac{1}{3}$ and $\frac{5}{16}$, point out that even then $\frac{1}{3}$ may be expressed as $\frac{16}{48}$ of a rupee as a rupee contains 48 dabbas, and $\frac{5}{16}$ as $\frac{15}{48}$ of a

rupee as one anna contains three dabbus and hence the total may be expressed as $\frac{23}{48}$ of a rupee. Thus show that even then the result may be obtained by the money method. Point out further that it is the custom in some parts to reckon 5 kasulu for a pie after an old coin now extinct and thus work out results. Get pupils to note that the nominal use of the term "Kasu" would admit of the above method of addition and subtraction being conveniently applied even with reference to fractions having 5, 10, 15, etc., as denominators.

Next propose :—

A ryot sows green-gram in $\frac{3}{8}$ of his lands and black-gram in $\frac{1}{4}$ of them, and the rest is yet to be cultivated. What portion of the lands is yet to be cultivated ?

Let pupils work this question by any of the methods above adopted. Get them to note that $\frac{3}{8}$ and $\frac{1}{4}$ should be added and the sum deducted from "1". Point out that $\frac{3}{8}$ and $\frac{1}{4}$ of a rupee would give 3 bedas plus 2 bedas or 5 bedas, and hence there remain in the rupee 3 bedas still, i.e., $\frac{3}{8}$ of a rupee. Hence lead them to deduce that $\frac{3}{8}$ of the lands is yet to be cultivated. If pupils attempt the above combination of addition and subtraction by the other method, point out that the whole field may be considered as divided into 8 eighths, that green-gram cultivation occupies 3 eighths, black-gram cultivation 2-eighths, and hence the portion yet to be cultivated is $\frac{3}{8}$ of the field. Illustrate this also by drawing a diagram—



Next propose :—

I took $\frac{3}{4}$ of an acre of land on lease, cultivated pasupu in $\frac{7}{10}$ of an acre and used the rest for growing vegetables. What fraction of an acre would then be under vegetables ?

Get pupils to note in this case that if an acre be considered as divided into 40 equal parts, $\frac{3}{4}$ of it will cover 30 parts of it, $\frac{7}{10}$ will cover 28 of the parts and hence the portion under vegetables would cover two of the parts, i.e., $\frac{2}{40}$ of an acre

or $\frac{1}{20}$ of an acre. Point out to pupils that instead of considering an acre as being divided into 40 equal parts or into 20 or 60 or 80, etc., equal parts a subdivision of an acre has been made in land survey into 100 equal parts called "cents," $\frac{3}{4}$ of an acre will come to 75 cents, $\frac{7}{10}$ of an acre to 70 cents and hence the portion under vegetables is 5 cents, i.e., $\frac{5}{100}$ of an acre and this may also be otherwise expressed as $\frac{1}{20}$ of an acre.

Give plenty of exercises of the following type to be done orally :—

Find the sum of :—

(1) $\frac{3}{4}$, $\frac{1}{4}$; (2) $\frac{1}{4}$, $\frac{1}{4}$; (3) $\frac{1}{2}$, $\frac{1}{8}$; (4) $\frac{3}{4}$, $\frac{1}{8}$; (5) $\frac{1}{8}$, $\frac{1}{8}$; (6) $\frac{1}{8}$, $\frac{1}{16}$; (7) $\frac{1}{16}$, $\frac{1}{16}$; (8) $\frac{1}{4}$, $\frac{1}{16}$; (9) $\frac{1}{2}$, $\frac{1}{16}$; (10) $\frac{1}{4}$, $\frac{1}{16}$; (11) $\frac{3}{8}$, $\frac{1}{16}$; (12) $\frac{9}{16}$, $\frac{1}{16}$; (13) $\frac{3}{4}$, $\frac{1}{16}$; (14) $\frac{3}{4}$, $\frac{3}{16}$; (15) $\frac{7}{8}$, $\frac{1}{16}$.

Find the values of :—

$\frac{1}{2}-\frac{3}{8}$; $\frac{5}{8}-\frac{3}{16}$; $\frac{3}{8}-\frac{1}{12}$; $\frac{1}{6}-\frac{1}{8}$; $\frac{5}{12}-\frac{1}{4}$; $\frac{3}{4}-\frac{5}{12}$; $\frac{7}{8}-\frac{1}{4}$; $\frac{7}{12}-\frac{3}{8}$; $\frac{7}{12}-\frac{5}{16}$; $\frac{4}{5}-\frac{2}{5}$; $\frac{7}{12}-\frac{3}{8}$; $\frac{7}{16}-\frac{3}{8}$; $\frac{1}{4}-\frac{1}{6}$; $\frac{3}{4}-\frac{5}{8}$ and $\frac{5}{8}-\frac{1}{3}$.

Addition and subtraction where mixed numbers are involved.

Propose :—

I had $5\frac{1}{8}$ maunds of jaggery. I sold in retail $3\frac{3}{16}$ maunds. How many maunds would remain if no piece of jaggery was wasted ?

Point out that 3 may be subtracted first from $5\frac{1}{8}$, from the difference so obtained $\frac{3}{16}$ may be subtracted, as $\frac{1}{8}$ is less than $\frac{3}{16}$, $\frac{3}{16}$ may be subtracted from $1\frac{1}{8}$ and the result $\frac{5}{16}$ added to 1 and thus the difference required expressed as $1\frac{15}{16}$. Point out to pupils that this method would lead to the result being orally worked out to be $1\frac{15}{16}$.

Tell them that an alternate method would be to convert both the mixed numbers given into improper fractions as $\frac{41}{8}$ and $\frac{51}{16}$, express these fractions in a common denomination, say sixteenths and then find the difference.

Point out also that special artifices may be used which would suit the particular cases, e.g. $2\frac{1}{8}$ maunds minus $\frac{3}{16}$ maund may be replaced by Rs. 2-2-0 minus 3 annas, and when the result Rs. 1-15-0 is obtained it may be expressed in the alternate form Rs. $1\frac{15}{16}$, and hence the

balance required in the question may be expressed as $1\frac{15}{16}$ maunds. Get them also to suggest similar artifices say, by expressing maunds into visams, etc.

Propose :—

(1) Find the values of the following :—

(Assume that the fractions given below denote so many rupees or maunds of jaggery or puttis of chillies.)

(1) $1\frac{1}{2} + 3\frac{3}{4} + 6\frac{1}{2}$; (2) $5\frac{3}{4} - 1\frac{1}{2}$; (3) $7\frac{1}{2} - 6\frac{3}{4}$; (4) $5\frac{1}{2} - 3\frac{1}{8}$ (5) $4\frac{5}{16} - 3\frac{3}{8}$; (6) $5\frac{1}{8} - 3\frac{3}{16}$; (7) $7\frac{1}{8} + 8\frac{3}{4} - 9\frac{1}{2}$; (8) $4\frac{1}{2} + 3\frac{3}{4} - 7\frac{3}{8}$; (9) $5\frac{1}{16} + 7\frac{1}{2} - 8\frac{1}{4}$; (10) $5\frac{5}{16} + 7\frac{1}{4} - 7\frac{7}{16}$; (11) [illegible] + [illegible] + [illegible] $- \frac{1}{2}$; (12) $\frac{7}{16} + 1\frac{1}{8} - 1\frac{1}{4}$.

(13) $5\frac{1}{8} - \frac{1}{4}$; (14) $8\frac{3}{8} - 5\frac{5}{8}$; (15) $8\frac{1}{2} - 3\frac{1}{6} - 4\frac{1}{12} + 3\frac{1}{6}$.

(2) Beginning with a given fraction or a mixed number, say, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{8}$ $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$, $5\frac{1}{2}$, $6\frac{1}{4}$, $7\frac{1}{8}$, $5\frac{3}{4}$, $\frac{7}{16}$, count forwards by $\frac{1}{2}$'s, $\frac{1}{4}$'s, $1\frac{1}{2}$'s, $2\frac{1}{4}$'s, $\frac{1}{8}$'s, $\frac{5}{16}$'s, $\frac{7}{8}$, $\frac{7}{16}$, etc. Also count backwards by the same fractions, starting from the above fractions, namely $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$, etc.

Multiplication of fractions.

Multiplication of a fraction by an integer.

Propose :—

A vessel measures half a seer. If milk is measured out 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 16, 20, 30, 50, 60, 70, 80, 90 and 100 times with the vessel how many seers of milk will there be in each case?

Answer the above question by taking in turns that the vessel measures $\frac{1}{4}$ seer, $\frac{1}{8}$ seer, $\frac{3}{4}$ seer, $\frac{1}{16}$ seer, etc. Thus get pupils to construct the multiplication table for $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{3}{4}$ and memorize them.

Propose :—

1. A seer of gingelly costs 14 annas. Find at this rate the cost of half a seer, $\frac{1}{4}$ seer, $\frac{1}{8}$ seer and $\frac{1}{16}$ seer.

2. A ponaku used for carrying manure in carts costs Re. $\frac{3}{4}$. Find at this rate the cost of 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 15, 16, 20 ponakulu.

3. A maund of ghee sells at Rs. 12. Find at this rate the cost of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{8}$ and $\frac{1}{16}$ maund of ghee, respectively.

4. A yoke pole (puse) costs Re. $\frac{1}{4}$. Find at this rate the cost of 2, 20, 30, 4, 40, etc., yoke-poles.

Multiplication of one fraction by another.

Propose :—

A seer of milk costs $\frac{1}{4}$ of a rupee. Find the cost of $\frac{3}{4}$ seer.

Here point out that the value of $\frac{3}{4}$ of Re. $\frac{1}{4}$ has to be found. Let pupils work out that for $\frac{3}{4}$ seer they will have to give $\frac{3}{4}$ of 4 annas, i.e., 3 annas or $\frac{3}{16}$ of a rupee. Hence get them to note that $\frac{3}{4}$ of $\frac{1}{4}$ is equal to $\frac{3}{16}$.

Propose :—

A viss of pasupu sells at $\frac{5}{8}$ rupee. Find at this rate the price of $\frac{3}{4}$ viss.

Lead pupils to note that they have to find the value of $\frac{3}{4}$ of $\frac{5}{8}$ of a rupee. Get them to work as follows :—

$\frac{5}{8}$ of a rupee is 10 annas, which is equal to 20 arthannas. Therefore $\frac{3}{4}$ of 20 arthannas equals 15 arthannas, i.e., $\frac{15}{32}$ of a rupee, as 32 arthannas make one rupee.

Propose :—

A kuncham of Krishna Katukallu sells at $\frac{5}{8}$ rupee. Find at this rate the value of $\frac{7}{8}$ kuncham.

Let pupils work as follows :—

10 annas equals 40 coppers.

$\frac{7}{8}$ of $\frac{5}{8}$ of a rupee is equivalent to $\frac{7}{8}$ of 40 coppers, i.e., 35 coppers, i.e., 35/64 of a rupee as 64 coppers make 1 rupee. Let pupils therefore note that 7/8 of 5/8 is equal to 35/64.

Propose :—

A ryot cultivates only $\frac{3}{4}$ of his dry lands near hill slopes. In 7/10 of the portion under cultivation is grown *harika* or kodo-millet. What part of his dry lands is under *harika* cultivation ?

Let pupils make a rough sketch on the floor or on the paper a rectangle to represent the ryot's dry land. Let them divide it into 4 equal parts by drawing 3 breadthwise lines and shade $\frac{3}{4}$ of it to represent the part under cultivation. Let them draw 9 lengthwise lines, dividing the field into 10 equal parts and thus mark off 7/10 of the portion cultivated. Question into how many equal parts the whole lands are divided and how many such parts there are in the portion under *harika* cultivation. Get

pupils to note that the whole lands are considered as divided into 40 equal parts, of these 21 are under *harika* cultivation and therefore the ryot has this cultivation on 21/40 of his lands. Let pupils hence derive that 7/10 of $\frac{3}{4}$ is equal to 21/40. Tabulate the results of multiplication in each of the above cases as follows:—

$\frac{3}{4}$ of $\frac{1}{4}$ = 3/16;
$\frac{3}{4}$ of $\frac{5}{8}$ = 15/32;
$\frac{7}{8}$ of $\frac{5}{8}$ = 35/64;
and 7/10 of $\frac{3}{4}$ = 21/40.

Propose to them whether they could derive the product in each case without arguing out in so many steps. Suggest to them that they may first compare the numerators of the given fractions and those of their products and the denominators of the given fractions and those of their products. Get them to suggest that the numerator of the product could be obtained by multiplying the numerators of the given fractions and the denominator of the product by multiplying the denominators of the given fractions.

Propose:—

1. A cow requires $\frac{1}{4}$ viss or yebalam of gingelly cake every morning. How many maunds of the cake will be required for feeding it for 8 months at the above rate? Find the cost of the quantity required at 2 annas a viss. (Reckon 30 days for a month.) (*Ans.* 7$\frac{1}{2}$ mds.; Rs. 7$\frac{1}{2}$.)

2. Horse-gram sells at 3/8 rupee per kuncham. Find the cost of 7$\frac{1}{2}$ kunchams of horsegram.

Here point out that the multiplication of a proper fraction by a mixed number is involved. Get pupils to note the following methods of working:—

Method I.—The cost of a kuncham is $\frac{3}{8}$ rupee or 6 annas and therefore the cost of 7 kunchams is equal to 42 annas, that of half a kuncham is 3 annas; altogether the cost of 7$\frac{1}{2}$ kunchams is 45 annas, that is Rs. 2-13-0.

Method II.—7$\frac{1}{2}$ is equal to 15/2. Therefore $\frac{3}{8}\times 7\frac{1}{2}$ equal $3/8 \times 15/2$ that is, Rs. 45/16, i.e., Rs. 2-13-0.

Get pupils to note that in the second method a mixed number like 7$\frac{1}{2}$ involved in the multiplication has been converted into the equivalent improper fraction so that the rule above derived for the multiplication of one fraction by

another may be adopted. Repeat the above exercises by a change of numerical data and let not pupils be taught cancelling until they find it out of their own accord or until a thorough grounding is given in the application of the general rule for multiplying one fraction by another.

Propose :—

1. An acre of wet lands sells at Rs. 800. Find at this rate the cost of $\frac{7}{8}$ of an acre, 7/10 of an acre, $2\frac{1}{2}$ acres and $\frac{3}{4}$ acre respectively.

(*Ans.* Rs. 700 ; Rs. 500 ; Rs. 200 and Rs. 600.)

2. A bag of paddy sells at Rs. 12. How much may I have to pay for $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{5}{8}$, 3/16 of a bag respectively?

(*Ans.* Rs. 6, Rs. 9 ; Rs. $1\frac{1}{2}$; Rs. $7\frac{1}{2}$ and Rs. $2\frac{1}{4}$.)

3. A kuncham of selected paddy-seed sells at Re. $\frac{3}{8}$. How much may I have to pay for $2\frac{1}{2}$ kunchams, $3\frac{3}{4}$ kunchams, $4\frac{1}{4}$ kunchams, $5\frac{3}{4}$ kunchams, $7\frac{1}{2}$ kunchams and $8\frac{1}{2}$ kunchams respectively?

(*Ans.* Re. $\frac{15}{16}$: Rs. $1\frac{13}{32}$; Rs. $1\frac{10}{32}$; Rs. $2\frac{5}{32}$ Rs. $2\frac{13}{32}$ and Rs. $3\frac{3}{16}$.)

4. A busta of castor-cake sells at Rs. 8. A man takes $\frac{3}{10}$ of a busta. How much may he have to pay?

(*Ans.* Rs. $1\frac{1}{2}$.)

5. A putti of cane gives $\frac{1}{8}$ putti of jaggery. I have 20 puttis of cane. How many puttis of jaggery may I get? My friend has $7\frac{1}{4}$ puttis of cane. How many puttis of jaggery will he get? (*Ans.* $2\frac{1}{2}$ puttis, $\frac{29}{32}$ putti.)

6. A carpenter earns Re. $\frac{27}{32}$ a day. If he earns like this for only 20 days during the month, what sum may he be earning for a month? (*Ans.* Rs. $16\frac{7}{8}$).

7. In walking round a farm I make 1,000 strides in all. My stride is $1\frac{7}{8}$ ft. What is the distance round the farm? (*Ans.* 1,875 ft.)

8. I walk round a sugarcane plantation and I make 600 steps. If a step is $2\frac{1}{4}$ ft. long, find the length of fence required for the plantation. (*Ans.* 1,350 ft.)

9. A viss of ghee costs Rs. $2\frac{3}{4}$. Find at this rate the cost of $\frac{3}{4}$ viss. (*Ans.* Rs. $2\frac{1}{16}$)

10. A man sells in retail $17\frac{3}{16}$ puttis of *pesalu* at Rs. $17\frac{1}{2}$ a putti. Find how much he will get.

(*Ans.* Rs. 300-12-6.)

Here point out that the product Rs. $17\frac{3}{16} \times 17\frac{1}{2}$ involves the multiplication of one mixed number by another and that the multiplication may be done as follows :—

17 times $17\frac{3}{16}$ is given by 17 times, 17 plus 17 times $\frac{3}{16}$, $\frac{1}{2}$ of $17\frac{3}{16} = \frac{1}{2}$ of 17 plus $\frac{1}{2}$ of $\frac{3}{16}$.

Get them to note that the total of the results obtained would give the product required.

Get pupils to suggest that another method would be to convert the given mixed numbers into their equivalent improper fractions, multiply their numerators as well as their denominators and thus get the product of the mixed numbers. Lead pupils to note that in many cases the application of the first method would help at working out the result orally and quickly.

11. A cart holds $\frac{1}{3}$ ton generally at Anakapalle. How many tons will I have if I buy 6 cart-loads of fire-wood? (*Ans.* 2 tons.)

12. A thousand tiles sell at Rs. $4\frac{1}{2}$. If I buy $25\frac{3}{4}$ thousand tiles, how much will I have to pay ?
(*Ans.* Rs. 115–11–0.)

13. A *mura* of cloth sells at Re. $\frac{7}{8}$. How much will I have to pay for buying $4\frac{1}{2}$ muras? (*Ans.* Rs. 3–15–0.)

14. At Rs. 25 as the monthly rent for your school building, how much will be given as rent for a year ?
(*Ans.* Rs. 300.)

Get pupils to note that Rs. 25 = $\frac{1}{4}$ of Rs. 100 that 12 times $\frac{1}{4}$ = 3 and therefore the rent for a year is 3 hundred rupees.

15. At Rs. 125 as the price of a ton of fish guano, calculate how much the agent to an estate should remit for 84 tons of the manure required for the estate.

(Point out that Rs. 125 is an eighth of Rs. 1,000 and hence 84 times Rs. 125 is given by 84 eighths of Rs. 1,000, i.e., $10\frac{1}{2}$ thousand rupees or Rs. 10,500.)

Find by applying the methods above indicated the values of Rs. 250 × 18, Rs. 375 × 15, Rs. 625 × 54, Rs. 875 × 18, Rs. 50 × 77, Rs. 75 × 19, Rs. 25 × 88, Rs. 125 × 97, Rs. 125 × 87, Rs. 250 × 13.

Next give pupils exercises like the following :—

Find the values of $\frac{17}{32} \times 44$, $\frac{17}{32} \times \frac{18}{15}$. Point out to pupils that instead of multiplying the numerators and denominators in full they may also apply the principle of equivalent fractions and divide both numerators and denominators by one and the same number, e.g. , in expressing $\frac{17 \times 18}{15 \times 34}$ as $\frac{6}{10}$ or as $\frac{3}{5}$.

Division of a fraction by an integer.

Propose :—

A kuncham of paddy seed sells at Re. $\frac{1}{2}$. How much does an adda cost ?

Point out that as 4 addas = 1 kuncham, the price of an adda is given by Re. $\frac{1}{2} \div 4$. Remind pupils that the multiplication table 4 times $\frac{1}{8} = \frac{1}{2}$ gives that Re. $\frac{1}{2} \div 4$ equals Re. $\frac{1}{8}$ and therefore the price of an adda of paddy seed is $\frac{1}{8}$ rupee. Point out also that the division by 4 may be effected by multiplication by $\frac{1}{4}$ as one adda is $\frac{1}{4}$ of a kuncham and therefore the price of an adda is Re. $\frac{1}{8}$.

Get pupils to note that 8 viss make a maund and that the price required is therefore Rs. $3\frac{1}{4} \div 8$. Point out also that a viss is $\frac{1}{8}$ of a maund and that therefore the above result may be expressed as Rs. $3\frac{1}{4} \times \frac{1}{8}$. Get pupils to work out the value to be Re. $\frac{13}{32}$.

Propose :—

1. A viss of *mamidi allum* sells at Re. $\frac{5}{16}$. What is the price of a seer ?

Taking 5 seers for the viss, get pupils to derive that the price required will be given by Re. $\frac{5}{16}$ divided by 5, i.e., by $\frac{1}{5}$ of Re. $\frac{5}{16}$ or Re. $\frac{5}{80}$ or Re. $\frac{1}{16}$.

2. The price of 100 sugarcanes is Rs. $6\frac{1}{4}$. What is the price of a cane ?

3. A kuncham of horse-gram sells at Re. $\frac{5}{8}$. What is the price of a sola?

Get pupils to generalize from the above results that when a fraction is divided by an integer the resulting fraction will have its numerator the same as that of the original fraction and its denominator equal to that of the original fraction multiplied by the integer to be divided. Write on

the blackboard a series of exercises of the following type:—
Re. $\frac{3}{4}$ divided by 8; $\frac{5}{6}$ putti divided by 12; $\frac{7}{10}$ viss divided by 16, etc., and get pupils to work out the values orally and quickly.

Division of a fraction by another.

(a) *Division of a larger fraction by a smaller one.*

Propose :—

1. A bundle of firewood weighs $\frac{1}{8}$ of a gundu. If I want $\frac{3}{4}$ gundu of firewood, how many such bundles should I take?

Get pupils to note that the result required could be found by considering how many $\frac{1}{8}$ gundu's there are in $\frac{3}{4}$ gundu and this may be represented by the division $\frac{3}{4}$ gundu $\div \frac{1}{8}$ gundu. Point out that the division may be effected as follows :—

Method I.—$\frac{3}{4}$ gundu $\div \frac{1}{8}$ gundu.
$= \frac{6}{8}$ gundu $\div \frac{1}{8}$ gundu.
$=$ 6 eighths of a gundu $\div$ 1 eighth of a gundu.
$= 6$.

Method II.—The multiplication table for $\frac{1}{8}$ shows that six times $\frac{1}{8} = \frac{3}{4}$ and therefore $\frac{3}{4} \div \frac{1}{8} = 6$.

Method III.—Consider $\frac{3}{4}$ gundu in terms of its equivalent 42 lbs. and $\frac{1}{8}$ gundu as 7 lbs.

Then $\frac{3}{4}$ gundu $\div \frac{1}{8}$ gundu.
$=$ 42 lbs. $\div$ 7 lbs.
$= 6$.

Method IV.—Instead of $\frac{3}{4}$ gundu and $\frac{1}{8}$ gundu, consider $\frac{3}{4}$ rupee and $\frac{1}{8}$ rupee.

As $\frac{3}{4}$ rupee $=$ 6 bedas and $\frac{1}{8}$ rupee $=$ 1 beda
$\therefore$ $\frac{3}{4}$ rupee $\div \frac{1}{8}$ rupee $=$ 6 bedas $\div$ 1 beda
$= 6$.
$\therefore \frac{3}{4} \div \frac{1}{8} = 6$.

Propose :—

2. A person has $\frac{3}{4}$ ton of fish guano. At 3/20 ton for an acre, how many acres could be applied with the manure?

Get pupils to note that the result required would be given by the division.

$$\tfrac{7}{8} \div 3/20.$$

Let pupils work that $\tfrac{7}{8} = 35$ fortieths and $3/20 = 6$ fortieths.

and therefore $\tfrac{7}{8} \div 3/20 = 35$ fortieths $\div$ 6 fortieths $= 35/6$ or $5\tfrac{5}{6}$.

Point out that instead of fortieths they may express $\tfrac{7}{8}$ ton and 3/20 ton in terms of their equivalents in lbs., or quarters, e.g., as 70 quarters and 12 quarters and thus get by division 70/12 or 35/6 as the number of acres required.

Propose :—

3. A family consumes $\tfrac{3}{4}$ kuncham of rice every day. How long would $7\tfrac{1}{2}$ kunchams last the family ? (*Ans.* 10 days.)

4. A cart holds $\tfrac{1}{2}$ a garisa of limestone. How many carts should be engaged to bring $2\tfrac{1}{2}$ garisas ? (*Ans.* 5.)

Change the numerical data in the above and propose a series of exercises on such division.

(b) *Division of a smaller fraction by a larger one.*

Propose :—

1. A cultivator finds that a plot yields $\tfrac{7}{8}$ putti of paddy. The landlord demands 9/16 putti of paddy being given to him. What part of the total produce does the landlord demand ?

Get pupils to note that the fraction required is given by the division 9/16 putti $\div$ $\tfrac{7}{8}$ putti. Point out to pupils that both the fractions may be expressed in a common denomination and the result expressed as follows :—

$$\tfrac{7}{8} = 14/16.$$
$$\therefore\ 9/16 \div \tfrac{7}{8} = 9/16 \div 14/16.$$
$$= 9 \text{ sixteenths} \div 14 \text{ sixteenths}$$
$$= 9/14.$$

Propose :—

2. A ryot has 9/10 putti of jaggery. He then sold 3/8 putti of jaggery. What part of the jaggery he had did he sell ? (*Ans.* 3/72 or 5/12.)

3. A landlord has $4\frac{3}{4}$ acres of lands. He makes over $2\frac{1}{2}$ acres to his son. What part of the entire lands does he make over to his son? (*Ans.* 10/19.)

(Here point out that the mixed numbers $4\frac{3}{4}$ and $2\frac{1}{2}$ may be expressed in terms of their equivalent improper fractions and the division proceeded with as before.)

4. There are $7\frac{1}{2}$ garisas of paddy in a granary. The owner sells $2\frac{3}{4}$ garisas. What part of the quantity does he sell? (*Ans.* 11/30.)

5. A landlord has Rs. $25\frac{1}{2}$ with him. He buys a bag of castor-cake for Rs. $8\frac{1}{2}$. What fraction of his money does he give away in buying the castor cake? (*Ans.* $\frac{1}{3}$.)

6. A person got $7\frac{1}{2}$ cartloads of straw from his lands. He stored $3\frac{1}{4}$ cartloads in his yard for the use of his cattle and sold the rest. What part of the total quantity of straw did he store for the purpose stated above? (*Ans.* 13/30.)

Change the numerical data in the above and propose a series of exercises on such division.

After a good number of exercises on the division of one fraction by another of both the types indicated in (*a*) and (*b*) above have been worked, get pupils to tabulate the results in a few cases, as follows:—

$$\frac{7}{8} \div \frac{3}{5} = 35/24.$$
$$\frac{4}{5} \div \frac{3}{8} = 32/15.$$
$$\frac{1}{6} \div \frac{1}{5} = \frac{5}{6}.$$

etc.

Question pupils how the division may be interpreted when the dividend is greater than the divisor and when it is less than the divisor. Point out that in the former case the number of times the dividend contains the divisor is found and in the latter case what part the divisor is of the dividend is found. Next lead them to examine if the numerator and denominator of the resulting fraction could be derived from those of the given fractions. Get them to derive that to divide one fraction by another the divisor may be inverted and multiplied. Let pupils apply the rule and see if it holds good in the case of each question on division hitherto done by them.

Propose :—

1. There are 2½ puttis of jaggery obtained from sugarcane cultivation in a plot. The landlord takes away 1 putti and the cultivator takes away the remaining quantity. What fraction of total quantity does each of them get ?

(*Ans.* Landlord $\frac{2}{5}$; Cultivator $\frac{3}{5}$.)

2. A tenant estimates that there are $3\frac{1}{8}$ hundred kunchams on the threshing floor and thinks of making over to the landlord 1¼ hundred kunchams. What part of the estimated yield does he contemplate to make over to the landlord ? (*Ans.* $\frac{2}{5}$.)

3. A cwt. of fish guano sells at Rs. 5½. How many cwt. could I buy for Rs. 19¼ ? (*Ans.* 3½ cwt.)

4. Jaggery is kept ready in the market-yard for sale in the shape of blocks. If a block of average size weighs $1\frac{3}{16}$ manugus, how many such blocks should you pitch upon for weighing to the extent of 38 manugus ?

(*Ans.* 32 blocks.)

5. A tenant undertakes to supply his landlord with 1¼ puttis of chillies for every acre of land allowed for cultivation. According to this contract he supplied his landlord with 2¾ puttis of chillies. How many acres of lands was he allowed to cultivate ? (*Ans.* $2\frac{1}{5}$ acres)

6. A railway maund is $82\frac{2}{7}$ lbs. Express in railway maunds the weight of a ton of fish guano.

(*Ans.* $27\frac{2}{9}$ maunds.)

7. A piece of mull 20 yards long costs Rs. 18. How much does this work out for a yard ?

Familiarize pupils with the following indigenous methods :—

20 times Re. ½ = Rs. 10 ; Rs. 8 remains.
20 „ „ ¼ = „ 5 ; „ 3 „
20 „ „ ⅛ = „ 2½ ; „ ½ or 8 annas remains.
20 times ¼ anna or a copper = 5 annas ; 3 annas remains.

20 times half a copper = 2½ annas ; ½ anna remains.
½ anna ÷ 20 gives a sum less than half a pie.
∴ the price of a yard = Re. ½ + ¼ + ⅛
plus a copper and a half plus something less than half a pie.
= 14 annas 5 pies to the nearest pie

Here again remind pupils of the use of the equivalent '1 pie = 5 kasulu' to working out the last step above as follows :—

$\frac{1}{2}$ anna = 30 kasulu.
20 times $1\frac{1}{2}$ kasulu gives 30 kasulu
∴ the price required = Re. $\frac{1}{2}$ plus Re. $\frac{1}{4}$ plus Re. $\frac{1}{8}$ plus a copper plus half a copper plus $1\frac{1}{2}$ kasulu.
= Re. 0-14-5 to the nearest pie.

Applying the above method let pupils work the following exercises.

8. A busta holding 25 kunchams of pesalu sells at Rs. 20. How much is this for a kuncham ?

(Ans. 12 annas 10 pies to the nearest pie.)

9. Find, to the nearest pie, the price of a kuncham of each of the articles below, given the price of a busta :—

(*a*) A busta of 26 kunchams of Akullu sells at Rs. 19.

(*b*) A busta of $22\frac{1}{2}$ kunchams of Doosi Sannellu sells at Rs. 20.

(*c*) A busta of $26\frac{1}{2}$ kunchams of Krishna Kattukallu sells at Rs. 17.

(*d*) A busta of 25 kunchams of horse-gram (ulavalu) sells at Rs. 14.

(*e*) A busta of 20 kunchams of green-gram sells at Rs. 9–12–0.

(*f*) A busta of 20 kunchams of red-gram sells at Rs. 21–8–0.

(*g*) A busta of 20 kunchams of Bengal-gram sells at Rs. 18.

(*Ans.* (*a*) Re. 0–11–8, (*b*) Re. 0–14–3, (*c*) Re. 0–10–3, (*d*) Re. 0–9–0, (*e*) Re. 0–7–10, (*f*) Re. 1–1–2, (*g*) Re. 0–14–5.)

10. A busta of $7\frac{1}{2}$ maunds of Samalkota sugar sells at Rs. 31–4–0. What is the price of a maund and that of a viss ?

(*Ans.* Rs. 4–2–8 a maund, Re. 0–8–4 a viss.)

11. A cloth 15 muras long sells at Rs. 3–14–0. How much does this come to for a yard ?

(*Ans.* Re. 0–4–2 to the nearest pie.)

12. Find the cost of 1,680 coconuts at Rs. $8\frac{3}{4}$ for a hundred.

Let pupils work as follows :—

100 nuts cost Rs. $8\frac{3}{4}$.

$\therefore$ 1,000 nuts cost 10 times Rs. $8\frac{3}{4}$ or Rs. $87\frac{1}{2}$.

600 nuts cost 6 times Rs. $8\frac{3}{4}$ or Rs. $52\frac{1}{2}$.

50 nuts cost $\frac{1}{2}$ of Rs. $8\frac{3}{4}$ or Rs. 4–6–0.

25 nuts cost $\frac{1}{2}$ of Rs. 4–6–0 or Rs. 2–3–0.

(25 nuts being $\frac{1}{2}$ of 50 nuts)

5 nuts cost $\frac{1}{5}$ of Rs. 2–3–0 or Rs. 0–7–0.

5 nuts being $\frac{1}{5}$ of 25 nuts.

$\therefore$ the cost of 1,680 nuts = Rs. $87\frac{1}{2}$ plus Rs. $52\frac{1}{2}$ plus Rs. 4–6–0 plus Rs. 2–3–0 plus Rs. 0–7–0 = Rs. 147.

Get pupils to suggest other short cuts also for the above question, e.g., finding the cost of 17 hundred nuts and then deducting from it the cost of 20 nuts.

13. Find the cost of 7 maunds 5 visams of jaggery at Rs. 3–4–0 a maund.

Let pupils work as follows :—

The cost of 7 maunds is 7 times Rs. $3\frac{1}{4}$ or Rs. 21 plus $1\frac{3}{4}$ or Rs. 22–12–0.

The cost of 4 visams = that of $\frac{1}{2}$ of a maund
= $\frac{1}{2}$ of Rs. 3–4–0 or
Rs. 1–10–0.

The cost of 1 visam = that of $\frac{1}{4}$ of 4 visams
= $\frac{1}{4}$ of Rs. 1–10–0
= Rs. 0–6–6.

$\therefore$ The cost of 7 maunds 5 visams of jaggery.
= Rs. 22–12–0 plus Rs. 1–10–0 plus
Rs. 0–6–6.

Point out to pupils that it is for obtaining convenient aliquot parts the quantity 7 maunds 5 visams is split into 7 maunds plus 4 visams plus 1 visam.

14. Find the total daily wages disbursed to the coolies engaged in a farm during the five consecutive days for which particulars are given below :—

(1) 24 at $6\frac{1}{4}$ annas each, 4 at $4\frac{1}{4}$ annas each, 2 at $3\frac{1}{2}$ annas each and $5\frac{1}{2}$ at $2\frac{1}{2}$ annas each.

(*Ans.* Rs. 11–11–9..

(2) 14 at 5¼ annas each, 6½ at 5 annas each, 17 at 4 annas each, 21 at 2½ annas each and 8½ at 6 annas each.
(*Ans*. Rs. 17-5-6.)

(3) 6 at 4 annas each, 6 at 3 annas each, 1 at 2¾ annas, 7 at 2 annas each, 1 at 4¼ annas.
(*Ans*. Rs. 3-15-0.)

(4) 7 @ 6¼ as. each, 11 @ 5¼ as. each, 6 @ 2¼ as each, 7 @ 5 as. each, 16 @ 4 as. each, 21 @ 2½ as. each.
(*Ans*. Rs. 16-10-6.)

(5) 7 @ 4 as. each, 5 @ 2¾ as. each, 2 @ 2½ as. each, 2 @ 2 as. each. (*Ans*. Rs. 3-2-9.)

15. Bangaru China Somanna sends us 15 bags of cotton seed and charges at the rate of Rs. 5-14-3 a bag. What will we have to pay? (*Ans*. Rs. 88-5-9.)

16. Find the total of the following purchases made by a landlord in the course of a day :—

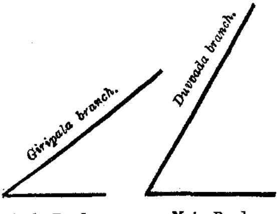
8 plough ropes @ Re. 0-1-8 each.
⅛ viss of margossa oil @ Re. 1 a viss.
17½ seers of green-gram seed @ 3½ seers a rupee.
8 wooden troughs @ Rs. 3-8-0 each.
12 screws @ 3 pies each.
7½ kunchams of quicklime @ 5½ as. a kuncham.
300 palmyrah leaves @ Rs. 7½ a thousand.
(*Ans*. Rs. 38-12-8.)

TOPIC 19—ANGLES (INTRODUCTORY).

(**Scope of the treatment**.—*A correct conception of angles may be given as denoting amount of turning. A few exercises may be set on the comparison of angles by the amount of turning required and by tracing and by superposition. The method of naming angles may be shown with the necessity for the same. Notions of an angle of complete revolution and of a right angle as a quarter of it may be imparted. A few important properties of the right angle may be dealt with, e.g , of all right angles being equal, of the outer arms of two adjacent right angles being in a straight line, of four right angles set out around a point on a plane surface covering up the space around the point in full, etc. A few exercises may be set to enable pupils to recognize at sight whether an angle is equal to, greater than, or less than a right angle. Items like the independence of the size of an angle to its arms, the reflex angle, etc., need be dealt with only if the pupils betray confused notions which necessitate the teacher's interference for the purpose of elucidation.*)

Notions of Angles.

Draw the following diagrams on the blackboard. Tell



pupils that these diagrams represent how certain roads leading to Giripala and Duvvada branch out from the

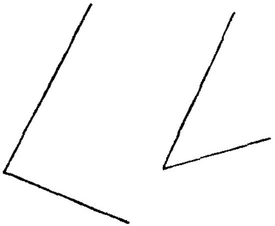
high road near the farm. Question them if they notice any difference in the way in which the roads branch out. Lead them to suggest that the Duvvada branch road deviates to a greater extent from the main road than the Giripala branch road as judged from the above diagrams. In addition to such an inference being drawn by pupils, use the following device to give a clear idea of the above :—

(A pair of narrow strips of cardboard screwed likewise may also be used for the purpose.) Ask them to close the arms of the dividers, place the screw over the point in the diagram representing the branching corner. Keeping one of the arms so as to lie along the line representing the main road, let one of the pupils turn the other arm so that it may lie along the line representing the branch road. Let another pupil do likewise with reference to the other figure. Thus get pupils to note that the rotating arm of the dividers has to be turned to a greater extent along the surface of the board to point out how the Duvvada road branches out from the main road than in pointing out how the Giripala road branches out from it.

Next tell pupils that when from one and the same spot two roads emerge they are said to branch out in common language and that in mathematics one road is said to make *an angle* with the other. Point out to them that it has been found that a greater amount of turning is required to place the arm of the dividers from the direction of the main road to point towards the Duvvada road than towards the Giripala road and hence it may be said that the Duvvada road makes a greater angle with the main road than the Giripala road instead of saying in common language that the former deviates to a greater extent from the main road than the latter.

(In rotating the cardboard round the screw it may be turned clock-wise as well as counter clock-wise from the position of pointing out the main road to that of pointing out the branch road. If a doubt should arise in pupils as to which method of turning should be taken into account, clock-wise or counter clock-wise, direct them to choose that which requires less amount of turning. The teacher need touch upon this question only when the pupils press him for an answer. **Otherwise, he need not deal with the reflex angle at all.)**

Draw diagrams on the board as shown below and tell pupils that the diagrams represent how two bunds emerge from a corner in certain fields with straight boundaries. Let pupils examine how the bunds diverge and note that they diverge differently in different cases. Get them to use the dividers as indicated above and find out in which case the divergence is greater and in which case less. Tell pupils once again, that instead of saying in common language that there is greater divergence between the two bunds in one case than in the other case, it may be said that the bunds make an angle with each other and that the angle between the bunds is greater in the former than in the latter case.



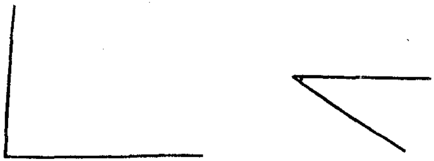
Next refer pupils how the twigs branch out from the stem of a tree and examine the difference between the way in which they branch out, how the leaves branch out from the stem of a plantain or coconut, how the rafters emerge from the ridge-piece, etc. Tell them that in all these cases instead of saying that they emerge in different ways, it may be stated that the angles which they make with each other are different.

Representation of an angle on the blackboard or on paper.

Question pupils how pairs of bunds, roads, twigs, leaves, etc., which emerge from a common spot may be represented on the board. Point out that to represent them straight lines may be drawn so as to proceed from one and the same point and that this point would represent the common branching corner. Tell pupils that straight lines so drawn to form angles will hereafter be referred to as *the arms* of the angles and the point from which they emerge as *the angular corner* or the *vertex* of the angles.

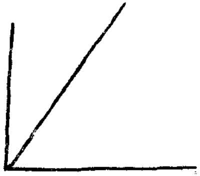
Comparison of Angles.

Draw *two angles* on the black-board. Tell pupils that they represent how adjacent bunds emerge from a common corner in the fields. Propose to them to find in which case the angle between the bunds is greater. Let pupils



use the dividers or a pair of narrow strips of card-board screwed at an end as directed already and find which of the two angles given is greater. Then point out to them that the arms of the dividers are thicker than the lines over which they have to be placed in so comparing and in order that the difference between the angles may be more accurately perceived tell them that the following method may be adopted :—

Let them trace the lines forming one of the angles given.



Let them next place the traced angle over the second angle so that the corners coincide and one arm of the traced angle falls along one arm of the other angle and see if the other arm of the traced angle would coincide with the other arm of the second angle. If the arms coincide, lead them to note that the angles are equal. If they do not coincide, let them infer that the angle which has its second arm lying outside is the greater.

Draw a series of equal and unequal angles on the blackboard and get pupils to draw such angles on paper. Get them to use the dividers as well as the tracing paper to find which angles are equal, which angle is the greatest and which angle is the least.

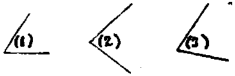
(If while comparing the angles in the manner indicated above, a pupil comes to conclude without tracing or with-

out using the dividers that a particular angle is greater than another from the fact of the arms of the former appearing longer than those of the latter, direct him to compare correctly by using tracing paper or the dividers as shown before. Then point out to pupils that a long pole or a set of short dividers may be used for indicating the direction of the lines and that they must not take into consideration the length of the pole or the arms of the dividers used to represent the direction but judge which angle is greater from the amount of turning required. The teacher will do well to note that this question of the independence of the size of an angle with reference to its arms should not be given out by him on his own initiative unless the pupils are found to commit mistakes of the type above indicated which require his intrusion to set them right. From the way in which the notion of angles is presented the confusion is not likely to arise and hence he should attend to it only when it arises).

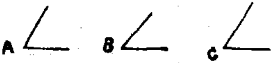
Naming of Angles.

Point out the necessity for naming angles when there are several of them drawn either on paper or on the black-board just as they name individuals. Lead pupils to suggest several methods, like the following :—

1. Write the numbers, (1), (2), (3), etc., within brackets against the angles and refer to the angles by means of the numbers so written, e.g., as angle (1), angle (2), etc., just as police constables are referred to by means of the numbers appearing in their turbans.

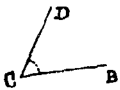


2. By using the letters of the alphabet, name the corners at which the turning indicated by the angles takes place and name the angles by the letters of the alphabet as angle A, angle B, etc.



3. Draw several lines from one and the same corner. Point out that the second system will not hold good in naming angles when there are many angles having one and the same corner.

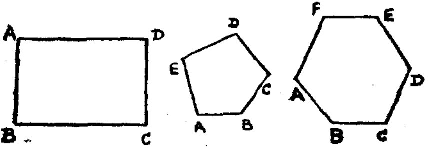
Lead them to refer to an angle, specifying the lines forming the angle, e.g., as the angle between CB and CD, etc. Tell them that the angle between two lines CB and CD is usually known in a briefer form as the angle BCD and is indicated by a small curve proceeding from a point in CB to a point CD as shown in the figure. Point out to pupils that, in so naming, the name of the angular point is placed in the middle. Tell them that the angle may also be named as the angle DCB.



If any of the pupils point out the reflex angle and ask how it may be named, tell them that the word "reflex" should be added to the name of the angle "BCD or DCB." If they do not refer to the reflex angle, let them alone and do not refer to the angle at all, as the pupils would not mean the reflex angle when naming the angle BCD or DCB.

Draw a series of angles on the blackboard and get pupils to name them.

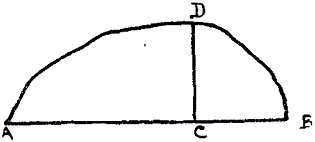
Draw on the blackboard polygons like those drawn below, give out the names of some angles in it and propose to pupils to point out the angles.

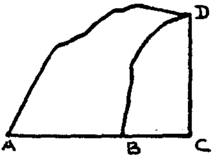


The Right Angle.

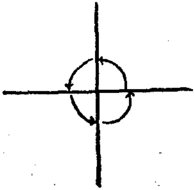
Taking a sheet of paper let pupils fold it so as to get a straight crease-line. Let them refold the sheet so that a part of the crease-line lies along the remaining part. Let

them next unfold the sheet and note that four angles with a common vertex are marked out by the crease-lines. Get them next to actually trace one of the angles and compare it with each of the remaining angles by superposition. Let them note that the four angles are equal.





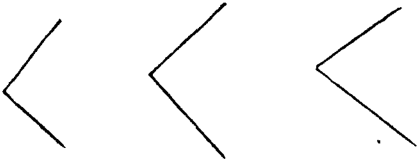
Next place a pencil so that one of its ends is at the common corner of these angles and the pencil itself lies along an arm of one of the angles. Turn the pencil once completely around the point on the surface of the paper. Point out to pupils that an angle of revolution is then made and that each of the four angles turned through in succession as marked out by the crease-lines requires a quarter of a revolution. Tell them that each of the four angle is called a right angle.



Next refer them to the edges of the set squares, the try-square, the note-book, the desk, the table, the bench, the blackboard, the door-way, etc., which emerge at right angles.

Mark with the chalk-piece lines on the graph-board forming right angles. By tracing and superposition get them to recognize that the angles are all right angles and as each of them is a quarter of an angle of revolution the angles must all be equal. Point out that a right angle is the simplest angle to make and that they may therefore use it as a *standard* for comparing angles.

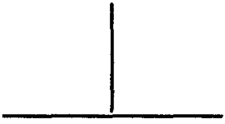
Draw a series of angles on the blackboard which are equal to, greater than and less than a right angle.

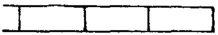


Set pupils to sort them into angles which are greater than, *equal to* and *less than right* angles. If pupils should mistake an angle to be a right angle from the point of view of mere appearance question them how they are to judge whether their conclusion is correct or not. Remind them of the method of setting out a right angle by paper-folding. Thus get them to have a sheet of paper folded with its rims forming a right angle and by placing it over the angles drawn above decide whether those angles which look like right angles are actually right angles or not. (The teacher will do well to instruct his pupils to set out a right angle on stiff paper as described above, cut off the sheet and keep it ready in their instrument box so that they may use it for similar tests later on.) Point out to them also that they may assume the right angle between the two edges of a set square as being correctly laid out and hereafter test right angles with the set square. Point out also that in the case of right angles with long arms set out on the floor the correctness of the angles may be tested by superposing the right angle in the try-square in place of that in the set square.

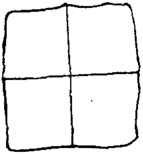
Propose to pupils to cut out two sheets of paper with two edges in each meeting at right angles. Let them

place the sheets side by side on the board or on the desk so that the vertex of one right angle and one of its arms fall along the vertex and an arm of the other respectively as in the accompanying figure. Get them to note that the outer arms fall along a straight line. Let them repeat the exercise with any two angles other than a right angle and note that the above property does not hold good and that it is characteristic of the right angle only. Point out also that it is because of this property a series of bricks or slabs having edges meeting at right angles could be laid along a straight row.





Let pupils cut out four right angles on paper and place them successively around a point on the surface of the board or the desk. Get them to note that they cover in full the space around them. Repeat this exercise with four equal angles other than a right angle. Point out that the above property is characteristic only of right angles.



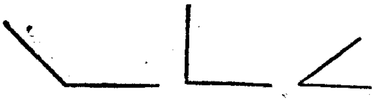
TOPIC 20.—PERPENDICULARS AND THEIR CONSTRUCTION.

(**Scope of the treatment.**—*At first the notion of one line being perpendicular to another may be given and a few exercises set on pupils' recognizing perpendicular edges in familiar objects. The perpendicularity between vertical and horizontal lines may be shown and the pupils made to realize that the words "perpendicular" and "vertical" are not synonymous. How perpendiculars may be drawn by using the try-square and the set square may next be explained and a few properties of perpendiculars derived, e.g., of the possibility of only a single perpendicular to a straight line through a given point, of the perpendicular being the shortest line that could be drawn from a point to a given straight line, etc. How the cross-staff may be used for setting off lines on the ground perpendicular to a given line and for marking out the perpendicular to a line from an external point may be dealt with and some out-door work given in that connexion. The errors arising while using instruments for drawing perpendiculars may be briefly touched upon and the pupils advised to take the neccssary precautions to be free from such errors as far as possible.*)

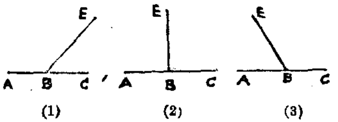
Notion of perpendiculars.

Draw on the blackboard pairs of straight lines at right angles to each other. Get pupils to note that in each pair one straight line is at right angles to the other. Tell them that when two straight lines meet or cut at right angles any one of them is said to be *a perpendicular* to the other.

1. Get a straight stalk of some plant. Have it partly broken. Then present the parts in various positions as shown in the accompanying figures. Get pupils to recognize when the parts appear to be perpendicular to each other.



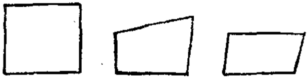
2. Hold one pencil over another successively in the positions (1), (2) and (3) as shown in the accompanying figures.



Point out that in position (1) the pencil BE appears to be slanting towards the part BC, in position (3) towards the part BA, and in the position (2) it is upright, neither slanting towards the part BC nor towards BA. Get pupils to note that in such a position the pencil held in the position BE is perpendicular to the pencil held in the position AC.

3. Set pupils to show edges on the floor and on objects in the class-room which are perpendicular to each other, e.g., the adjacent edges of a blackboard, of the table, of the desk, etc.

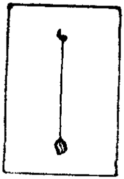
4. Draw figures of the following type and tell pupils that they represent the bunds of certain fields. Set them to recognize which of the bunds may be said to be perpendicular to each other.



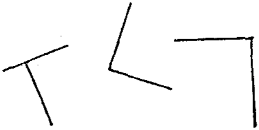
5. Ask pupils to place a staff on the ground so as to be perpendicular to the ground surface. Question them to hold the staff also in positions not perpendicular to the ground surface.

6. Set pupils to fold a sheet of paper or strawboard along a straight line. Let them next unfold the sheet keeping the two pieces in position perpendicular to each other. Let them keep the pieces also in positions not at right angles to each other. Thus give them an idea of one surface being perpendicular to another. Illustrate further by getting them to turn the doors so as to be perpendicular to the wall surface, by showing the perpendicular faces of a cube, cuboid, brick, plank, desk, etc.

Notions of vertical and horizontal lines: Distinction between the use of the terms vertical and perpendicular.

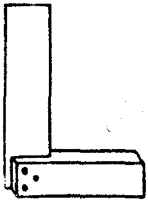


Attach a plummet to a string and get it suspended freely. When it is at rest, direct pupils to hold straight edges perpendicular to the string. Tell them that the string is said to be in a vertical position and the edges held perpendicular to it are all said to be in a horizontal position. The teacher must see that the pupils do not take perpendicular lines to indicate only vertical lines. He should draw slanting lines perpendicular to each other and show that the lines need not be necessarily vertical and horizontal respectively in order that they may be perpendicular to each other.

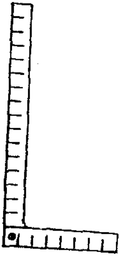


The use of the try-square in setting out perpendiculars on floors, planks, etc.

Ask pupils how the carpenter arranges to get perpendicular edges for the planks, door-ways, etc., which he



gets ready and how in house-building the mason lays out



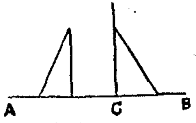
the edges of floors perpendicular to each other. If the pupils have not already observed this, a visit to the carpenter's and to a house under construction may be availed of for the purpose. Show pupils the try-square used by the carpenter and the mason. Let them examine and find the perpendicular edges on its surfaces. Place a try-square on the floor and draw lines along its outer and inner edges. Thus explain to them how, by using the instrument, lines may be drawn perpendicular to each other. Let pupils next suggest how the try-square may be used to mark perpendicular edges on planks, beams, etc. Get them to note that it is by a similar method perpendicular edges are cut out along Cuddapah slabs used for flooring purposes.

Drawing perpendiculars with set-squares.

Question pupils if they would find it handy to draw perpendicular straight lines in a page of their note-book with the carpenter's try-square. Intimate to them that for setting out such lines on paper they would find it handier to use the set squares they have in the mathematical instrument box than the carpenter's try-square.

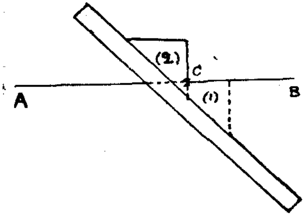
Propose to pupils to draw perpendiculars to lines by using the set squares. Let them suggest how they would proceed. Get them to adopt one of the methods detailed below :—

Draw any straight line AB on the blackboard. Mark a point C on it. Ask pupils to show roughly where the perpendicular to it through C is likely to fall. Let them next attempt to draw a perpendicular to AB through C using the set square. If they place an edge of the set square against AB with its right angular corner against C and then draw a line along the perpendicular edge, point out to them that the line they would be drawing will not be passing exactly through C and that besides it would be making



a curve where it meets AB, prominently noticeable especially when they draw the lines quickly. To be free from such errors, direct them to use the set square as follows :—

Choose one of the two perpendicular edges of the set square. Place the set square so that a straight line drawn along that edge would fall along A B, e.g., in the position (1) show in the accompanying figure. Let pupils note which edge of the set square is perpendicular to AB. Get the set square fixed in that position by placing a straight edge of the set square along the edge opposite to the right angle as shown in the accompanying figure. Then slide the set square up along the straight edge until the edge perpendicular to AB occupies the position in which a line drawn along it would pass through C. Ask pupils to draw such a line and produce it if necessary. Point out to them that it passes through C and is perpendicular to AB.



Propose :—

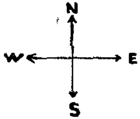
1. Get a sheet of paper folded along a straight crease-line. Fold the sheet again so that a part of the crease-line falls along the other part. Fold likewise repeatedly through various points in the crease-line first obtained. How many crease-lines will be obtained perpendicular to the first crease line ? How many perpendicular crease-lines could be set out to the original crease-line through a particular point in it ?

2. Draw a straight line on paper placing a straight edge in various positions. Draw perpendiculars to it by using the set square as above suggested.

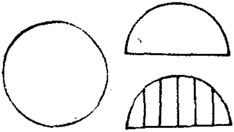
3. Draw a straight line XY in various positions horizontal, vertical and slanting. Mark points P, Q, R, S in it. Draw perpendiculars through these points, using the set square as above directed.

(While the pupils draw the perpendiculars in the above exercises point out that, through a point P in a particular straight line XY, only a single perpendicular could be drawn to a straight line.)

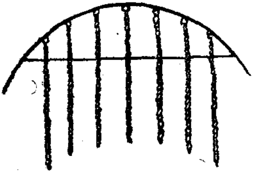
4. Using the magnetic needle, set out on the floor a north-to-south line. Through a point in the line set out a perpendicular straight line, using the try-square. Thus set out the four cardinal points.



5. Cut out a paper circle. Fold it in two so as to get a straight crease-line not necessarily passing through the centre of the circle. Cut out along the crease-line and fold one of the pieces repeatedly so that the crease-lines newly formed are perpendicular to the straight edge of the piece. Compare the lengths of the crease-lines so obtained. Which of the lines is the longest?

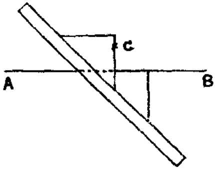


(Get pupils to note how, by a similar method of comparing perpendicular distances, e g., along plumb-lines hung up from various points in an arch up to to the span-line, the middle point of an arch can be fixed by deciding where the perpendicular line is the longest).



(*b*) Next propose to pupils to draw a perpendicular to a straight line

AB through a point C outside it. Direct them to use the set square as indicated already. Show them repeatedly the necessity for placing one of the perpendicular edges in a position in which the straight line AB could be drawn along it and then sliding the set square up or down along a straight edge until the position is secured when a line drawn along it passes through C.

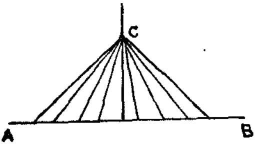


(The teacher will do well to note that the pupils generally get satisfied when they see that the edge perpendicular to the given straight line passes through the point specified. He must point out to them that such a position will not suffice and that when they draw a straight line along the edge it is likely to fall outside the point marked. He must hence caution them to so adjust the edge that a line drawn along it would pass through the point in question.)

The perpendicular as denoting the shortest distance from a point to a straight line.

Propose:—

Draw a straight line AB Mark a point C outside it. Draw lines from C to A B as shown in the figure, choosing a perpendicular from C to A B also as one of the lines. Compare the lengths of the straight lines so drawn, using a pair of dividers or a piece of string for the purpose.



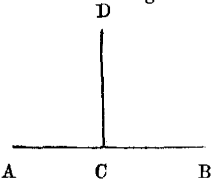
By repeating the above exercise with some more straight lines and points marked outside them, get pupils to note that the shortest distance from a point to a straight line is along the perpendicular drawn from the point to the straight line. With reference to the above figure, point out to pupils that if C be a well and A B a road or a bund and if the distance from the well to the bund or the road

were required the perpendicular distance should be taken as meant and hence that distance alone should be measured.

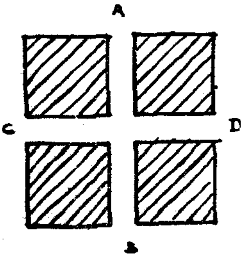
The use of the cross-staff in setting out and measuring perpendiculars on the ground surface.

Propose :—

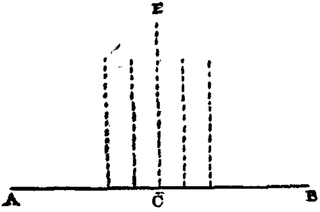
AB is a straight bund. C is a place in the bund 50 links from A. D is a missing field stone and is known to be 80 links from C as measured along the perpendicular to AB. How would you determine its position ?



Point out to pupils that with the perpendicular edges of the set square or try-square they could draw only small lines and hence that the use of the above instrument for drawing a line perpendicular to AB through C would be inconvenient. Get them to note that they may not only find it difficult to produce the perpendicular drawn with such instruments but also go wrong while producing them. Tell them that another instrument is therefore used for setting out perpendiculars on the ground. Show the pupils the top surface at the *head* of the cross-staff. Let them note that there are two grooves in it running perpendicular to each other. Let a boy plant the cross-staff at the place C. Let him turn to the head of the cross-staff so that one groove lies in the direction leading from A to B. Point out that this may be done by adjusting the staff so that looking through the groove the pupil is able to sight A or B. Get pupils then to note that the other groove points to a perpendicular direction. Ask another boy to look through the other groove. Point out that the line of sight of the second boy is perpendicular to AB. Call



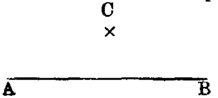
upon a third boy to stand on a line that appears to be perpendicular to AB as drawn through C and let the second boy caution the third to move to the right or to the left until he sees him in the line of his sight. If E represents the position then assumed by the third boy, get pupils to note that CE should be perpendicular to AB. Direct them to measure a distance of 80 links along CE and thus fix the position of the missing field stone D.



Give plenty of exercises of the above type involving the setting of perpendiculars to a given line along the ground by using a cross-staff.

Propose :—

A B in the accompanying figure represents a road and C is a well. It is required to find out how far the well is from the road. How would you lay out the perpendicular from C to AB and measure along it the distance from C to AB ?



Get pupils to suggest how the cross-staff should be used for the purpose. Let them note that if one of the grooves at the head of the cross-staff be placed in the direction of A B the other groove would help them look along a straight line perpendicular to A B. Point out to them that they should try the cross-staff at several places along A B until the spot is reached at which one is able to sight the position of C, looking along a groove adjusted perpendicular to AB.

Give plenty of exercises of the above type.

Errors in setting out perpendiculars.

Point out to pupils that while they set out perpendiculars with the try-square, the set-square or the cross-staff

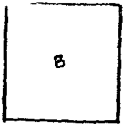
the edges in the first two instruments which appear to be perpendicular may be meeting at angles slightly differing from a right angle because of the wearing out of the edges from frequent use or that the grooves at the head of the cross-staff may not be exactly perpendicular to each other. Suggest to pupils that whenever a doubt arises about the right angular form in the instrument being spoiled as pointed above they should get ready a right angle set out by paper-folding as described in page (201) and test if an exact right angle is maintained or not. Point out also that while holding the staff errors in measurement would arise because of incorrect positions assumed by the staff. Hence impress on them the necessity for holding the staff upright while using the cross-staff. Lastly, point out to pupils that perpendiculars set out with the cross-staff etc., could be relied on only for short distances.

TOPIC 21.—RECTANGULAR AND SQUARE FORMS.

(**Scope of the treatment.**—*At first, notions of a square and a rectangle may be given by getting pupils observe the faces of a cube and a cuboid. The pupils may next be introduced to the side and angle properties of the above figures. The method of cutting them out by paper-folding and of constructing them by means of instruments may be explained. The diagonal properties of the figures may next be dealt with. Practical applications of the properties by the carpenter and the mason in laying out rectangular and square forms may be shown. How the surveyor uses the side properties of the figures in measuring out distances along obstructions may be explained and some out-door work conducted in such measurements being taken.*)

Notions of a square and a rectangle.

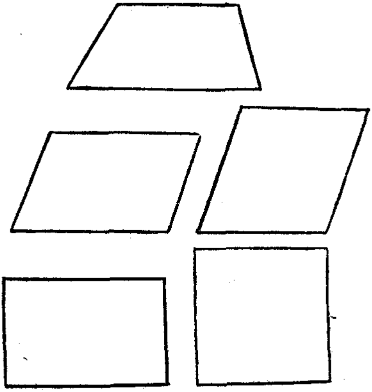
Place before pupils a cube and a cuboid. Show them a face of each and ask them to observe the faces carefully and answer the following questions :—" How many corners has each face ? What kind of angles do they see at the corners of each face ?" Thus lead them to note that the angle at the corners of each face appear to be right angles. Get them to set out a right angle by paper-folding and verify by superposition if the above angles are all right angles. (If the right angular sheet already cut out on stiff paper while doing the topic "angles" is preserved, direct them to use it for the purpose.)



Next question how many edges surround the face of a cube or a cuboid. Using a string or a set of dividers, let them compare the edges of a face of a cube. Thus lead them to infer that they are all equal. Next let them compare the edges of a face of a cuboid which bears an oblong shape. Lead them to note that in this case the opposite

edges alone are equal. Tell them that a portion of surface like the face of the cube they have examined is called "*a square*" and that one like the face of the cuboid they have examined is called "*a rectangle*," or "*an oblong*." Let them next point out square and rectangular forms in familiar objects. Get them to realize that the floors of rooms and verandahs, the surface of the walls, benches, desks, tables, blackboards, etc., are rectangular in form and that in the case of some bricks, tiles and planks, a face is square in form. Set them to draw roughly on the blackboard a square and a rectangle. Impress on them that in so drawing they must arrange to have right angles at the corners so as to get a rectangular or a square shape and that, besides, in setting out a square they must examine if the bounding lines are all equal while in the case of a rectangle it is enough if they see that the opposite boundary lines alone are equal.

Place before pupils pieces of cardboard or paper cut out in the shape of different kinds of quadrilaterals, e.g., a rhombus, a parallelogram, a rectangle, a square, a trapezium, etc., and get pupils to recognize correctly which of them are rectangular and which of them are square in shape.

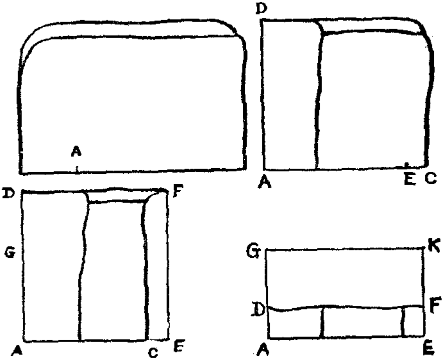


Construction of rectangles and squares on paper.

(a) *By paper-cutting.*

Propose :—

1. Fold a sheet of paper along a straight line and then refold it so that a part of the crease-line already obtained falls along the remaining part. Thus cut out a piece having two perpendicular edges, e.g., AC, AD, in the figure. Fold this piece through a point E in AC so



that EC lies along EA. Cut out the piece along the crease-line through E and thus get a piece having right angles at two corners. Again fold the sheet through a point G in AD so that GD lies along GA. Mark the point K where the new crease-line meets EF. Examine if the angle at K is a right angle.

Thus get pupils to cut out rectangular sheets of paper by paper-folding.

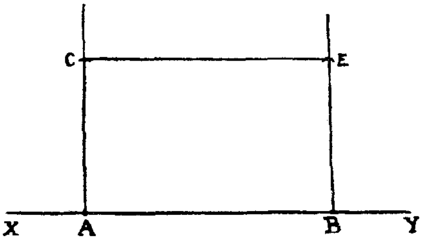
Next question them what adjustments they would make in order to get a square sheet. Make them realize by actual construction that they may adjust AE and AG equal in order to get a square form.

If there be small pieces of cloth available, get pupils to adopt the above method and cut out rectangular and square pieces of cloth.

(b) *By using the set square and straight edge.*

Propose :—

2. Draw any straight line XY. Along it step off a



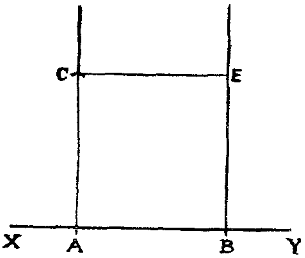
length AB equal to 4 inches. Through A and B draw perpendiculars to XY so as to lie on one and the same side of it. Using the compasses and the foot-rule, step off along the perpendicular through A a length AC equal to 3 inches. Through C draw a perpendicular to AC. Name the point E where this perpendicular meets the perpendicular to XY through B. Examine by placing the set square whether BE is perpendicular to CE. Measure AC, CE, EB and BA.

(Tell pupils that the boundary lines of a figure will hereafter be known as its '*sides.*' Point out that the portion of the surface enclosed by the four lines AC, CE, EB and BA is a rectangle, its opposite sides are equal and its adjacent sides are perpendicular to each other. Tell pupils that it is usual to name the portion of the surface so obtained by mentioning the names of the corners in the order in which one walking along the boundary lines passes them, e.g., naming the above figure as $ABCE$, $ACEB$, $CABE$, etc. The teacher may note that while writing down the name of the figure the pupils are likely to commit the mistake of putting a dot after each letter as in writing $A.\ B.\ E.\ C.$ and he should see that they write down the names of the letters with no dots between them.)

Repeat the above exercise with a change of numerical data and let pupils realize that a rectangle could be determined by two measurements, viz., the lengths of any two adjacent sides and that these measurements are generally known as its **Length and Breadth.**

Propose :—

3. Draw a straight line XY. In it step off a segment

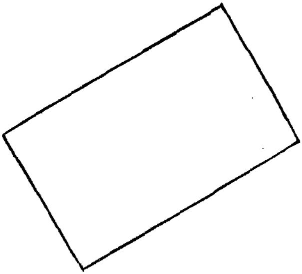


AB equal to 4 inches. Draw AC and BE perpendicular to AB so as to lie on one and the same side of it. Along the perpendicular through A step off a length AC equal to 4 inches. Through C draw a perpendicular to AC meeting the perpendicular BE at E. Examine if the lines meeting at E are perpendicular to each other. Using the dividers, compare the lengths of AC, CE, EB and BA.

Let pupils note that the figure obtained is a square and may be named $ABEC$ as in the case of the rectangle. Ask them to construct some more squares, choosing 5 inches 2 inches, $1\frac{1}{2}$ inches, etc., as the lengths of their sides. Point out that for determining a square it is enough if one measurement alone is given, viz., *the length of any one of the four sides.*

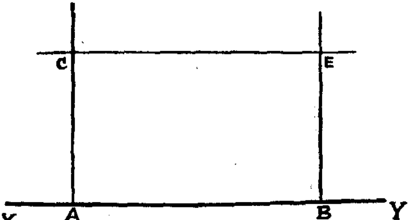
Alternate method of construction as used by the masons, carpenters, etc.

Propose to pupils to draw a big square or a rectangle on the blackboard with its sides exceeding 2 feet. Point out



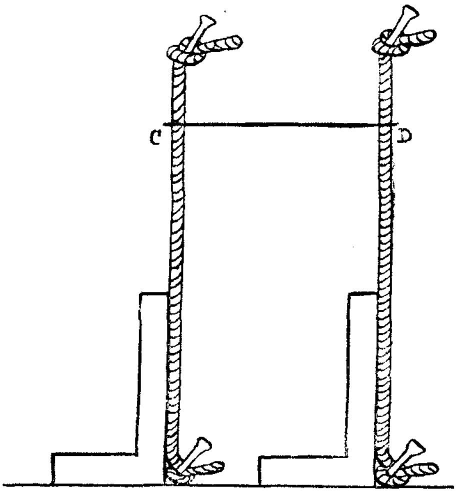
to them that with the perpendicular edges of the try-square or set-square they could draw only small lines and that it is by producing the lines they get the figures required. Get them to note that errors are likely to arise while producing the lines and hence the figures obtained by so producing are not likely to be correct. Point out to them also how they feel irksome to draw perpendiculars to lines appearing slanting. Tell them that instead of setting out perpendiculars at each of the three corners for constructing a square or a rectangle the following alternate method has been devised from a knowledge of some of the side properties already observed.

Suppose a square or a rectangle *ACEB* has to be constructed. After the length *AB* is set off along a straight line



and the perpendiculars *AC* and *BE* are drawn, suggest to them to cut off equal lengths *AC* and *BE* along these perpendiculars in accordance with the measurements given in the particular case and then to join *CE*. Point out to

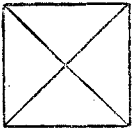
them that this method would facilitate the construction of the square or a rectangle a good deal and that they would find it far more convenient than the method hitherto adopted. Tell them also that it is by this method the

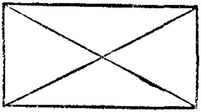


mason sets out rectangular verandahs and floors. Take pupils to a place where oblong-shaped floors or verandahs are to be raised in house-construction. Show how the mason sets off lines, perpendicular to chosen lines, e.g., along the base-line of a wall, measures equal distances along the perpendiculars and lays out oblong-shaped surfaces for verandahs and floors.

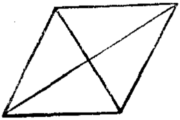
Tell them also that it is by this method the carpenter sets out rectangular and square forms while preparing doorways, desks, blackboards, etc. Point out how he uses the try-squares for setting out perpendiculars and how by using the side properties of the figures tests whether the planks, doors, etc., have assumed the correct shape required.

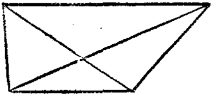
The diagonal properties of the rectangle and the square.

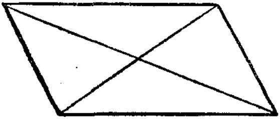




In the squares and oblougs alreadydrawn direct the pupils to join two opposite corners by a straight line. Draw figures on the blackboard in illustration of what they are to do. Point out that each line so drawn crosses the figure straight from one corner to the opposite corner and tell them that it is called *'a diagonal'* of the figure. Let them next join the other two opposite corners and compare the lengths of the two diagonals so obtained by means of dividers. Lead them to note that the two diagonals are equal. Next draw on the blackboard figures like the rhombus and a parallelogram. Ask them to join the diagonals by means of a blackboard ruler. Set







them to compare the lengths of the sides first and then those of the diagonals in each figure. Hence get them to realize that the property regarding the equality of diagonals is a special feature of squares and rectangles and not of any other type of four-sided figures which have their opposite sides equal.

Application of the above property in testing the correctness of rectangular and square forms set out on planks, floors, etc.

Describe to pupils how the carpenter after setting out rectangnlar and square forms on planks, etc., also tests if the diagouals of the door, planks or door-ways are equal

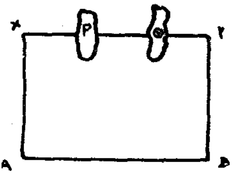
in length. Get pupils to note that a similar test may also be applied to ascertain the correctness of the square and rectangular surfaces set out on the floor.

Propose some exercises to pupils in setting out square and rectangular plots of ground by using the try-square and the cross-staff and testing the correctness of the figures obtained by ascertaining if the diagonal properties exist.

The use of the side properties of rectangles and squares in finding the distance between two places having an obstruction between them.

Propose that the distance between the two corners of a garden has to be measured. *P* and *Q* are two big pools of water or places containing prickly-pear through which one cannot walk when measuring. How is the length between the two corners to be then measured ?

Represent on the blackboard the two corners of the garden by marking *X* and *Y*. Draw also some irregular figures as denoted by *P* and *Q* in the accompanying figure to denote the pools or places offering obstruction to measurement. Suggest to pupils that one way of measuring the distance between *X* and *Y* is by laying out a rectangle *XABY* in which the side *AB* opposite to *XY* would admit of being easily measured. Let pupils propose how they would set out the rectangle. Get them to note that the cross-staff may be used to set out lines perpendicular to *XY* on one and the same side of it, that equal distances may be set off along these perpendiculars and that thereby the points may be marked on the ground corresponding to the corners *A* and *B* of the rectangle.



Give plenty of exercises to pupils in measuring the distances between places having obstructions between them, e.g., blocks of buildings, trees, walls, prickly-pear, etc., chosen within the school premises and see that the pupils become familiar with the above method.

TOPIC 22.—RECTANGULAR AREAS.

(**Scope of the treatment**.—*Elementary space-notions may at first be given of area by referring to the several plots in the adjoining lands and a few exercises set on the comparison of areas. The necessity for units of area and the method of deriving the units may next be dealt with. How rectangles and figures of various shapes may be formed out of chosen units of area may be shown. The pupils may be led to derive the formula for the area of a rectangle in terms of its dimensions. A few applications of the formula may be given and then the table of square measure may be directed to be built up. Sufficient out-door work should be conducted to familiarize pupils with the extent of space covered by the larger units, viz., the square chain, acre and cent used in surveying. Reference may also be made to the indigenous units in vogue and the convenience in using the units in the British system pointed out. Problems may be set on the calculation of the cost of seed or seedling, manure, etc., required for crops raised in fields of varying dimensions, of the yield that may be expected, of the assessment that has to be remitted, etc. A few problems may also be given on paving floors, roofing, plastering, etc., in their practical bearings on house-construction.*)

Comparison of areas and the necessity for a unit of area.

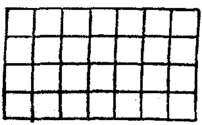
Take pupils to the lands adjoining the school. Referring to two unequal plots, say, one under sugar-cane and another under turmeric, question pupils which crop covers a greater extent of the lands. Next refer to two other plots yet to be cultivated and tell pupils that gingelly is proposed to be sown in them. Question them which of the plots should be sown a greater quantity of the seed, the same method of sowing being adopted. Next point out two fields under ragi or paddy and tell them that they are classed as the same kind of land. Question them which field requires a greater amount of 'Pannu' or 'Sistu' being paid. Thus give a series of exercises to pupils to find out which of the two plots is bigger. Get them to estimate by the eye as far as possible and thereby decide in each case which plot is bigger.

Next propose to pupils to find out in each case above how much one plot is bigger than another. Get them to note that the comparison would be easily made if the extent of space covered by each plot is measured out and expressed in terms of a unit known to them. Also point out that it is by using such a unit they express the extent of lands, owned by several persons, declare how much more of 'Mettu' or 'Pallamu' one landlord has than another, what extent of the lands a ryot has set apart for different crops, say, ganti, sweet potatoes, ragi, etc., in the course of a 'Panta', how much of his lands a landlord has let out for cultivation by a tenant and how which is cultivated under his own personal management, etc.

Notions of units in area measurement.

Take pupils to two plots of ground divided into a number of rectangular or square beds of almost equal size to serve as ragi or onion nurseries. Let them compare the plots and find out which plot is bigger and by how much. Get them to count the number of beds in each plot, and then express which plot is bigger and how many beds it contains more than the other. Next propose for such comparison plots in the neighbourhood which are not so subdivided. Make them realize that if they should subdivide these plots into a number of small rectangular or square beds they could express the extent of each plot as so many beds and then compare the number of beds in the plots. Point out to them that, for purposes of comparison, units may be adopted similar to a bed-space above referred to and that as squares and rectangles admit of being easily set out the amount of space covered by small squares and rectangles may be chosen for purposes of measuring out extent of surface. Tell them that hereafter the term 'area' will be used to denote *extent* of surface.

Set out a rectangular plot, say, about 20′ by 10′ in the school yard. Ask pupils to draw a square, say, of any dimension up to a foot or two in length and ask them to get the whole plot divided into such squares. Get them to start from a corner of the rectangular plot and direct them to divide

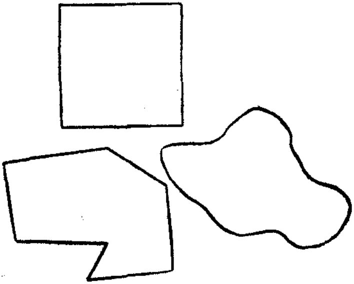


it roughly with a stick into a series of squares of the above

type, arranged row after row (as shown in the figure above.)

Get them to express how many of these squares make up the rectangular plot. Point out to them that if the extent of space covered by the square above be known to everybody it would serve as a very convenient unit for the measurement of area. Point out also that any rectangle may be arbitrarily chosen to serve the same purpose but that it should be one that may be known to many in the locality. Here allude to what they have learnt while constructing squares and rectangles to the effect that in the case of a square a single dimension need alone be remembered, namely the length of a side, as against two dimensions required to fix rectangles. Get them to note that therefore the extent of a square may better be chosen as the unit instead of a rectangle. Make them realize that the extent of a square so chosen for area measurement should be one familiar to people or of easy remembrance by them and that therefore they may choose those squares which have the units of lengths as their sides, namely an inch, a foot, a yard, a chain, etc.

If pupils question as to whether the areas of irregular and curvilineal figures of the following types could be got



by arranging unit-squares as above indicated point out to them that a square of side one foot, say, as cut out on paper may be torn out into a large number of pieces covering all sorts of irregular and curvilineal figures of the shape shown below and hence the figures they referred to could be got out of some unit squares and of their parts. Tell them that at present the areas of rectangles and squares are alone taken up as they are the simplest figures to start

with and that the types of figures they refer to will be taken up later on for treatment.

Expressing small rectangular areas in square inches.

Draw on the graph-board or on plain board a square of side one inch. Tell pupils that the extent of space covered by it is called '*a square inch*'. Get ready beforehand a set of inch-squares of the above type cut out on paper and place them at the disposal of the pupils. Draw a rectangle on the board or on the desk, say, about 4″ by 2″, and ask pupils to find out how many inch-squares will cover that space. Let pupils actually arrange the inch-squares one after another, to cover the rectangles and find out that 8 such squares cover the rectangle. Get them to note that instead of counting they may view the total number of squares as made up of two rows of 4 squares each and hence deduce that 8 such squares cover the whole rectangle. Tell them that the area of the rectangle is therefore expressed as 8 square inches. Direct them to build up rectangles of different dimensions, say, 5″ long and 3″ wide, 4″ long and 4″ wide, 6″ long and 4″ wide, etc., by placing inch-squares side by side as shown above. Get them hence to find out the area of each rectangle as so many square inches. Next let them exhibit on the board in tabular form the facts they have above observed in regard to dimensions and areas of rectangles.

Length in inches.	Breadth in inches.	Area in square inches.
4	2	8
5	3	15
4	4	16
6	4	24

Let them compare the numbers written in the three columns above and find out if there is a short cut for giving out the area in square inches when the length and breadth are given. Get them to note that the number of squares covering the rectangle may in the above cases be easily found out by multiplying the numbers in the first two columns. Let them then record that the number of square inches in the space covered by a rectangle is given by the product of the number of inches in its length and of that

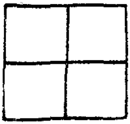
in its breadth. Tell pupils that it is usual to remember this rule in a short form as follows:—

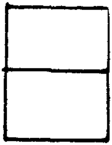
The area of a rectangle is equal to its length × its breadth.

Mark a series of rectangles and squares of different dimensions on the graph-board and let pupils verify whether the above rule would hold good in each case above.

Next draw on the blackboard some rectangles roughly and write out dimensions against the adjacent sides. Question pupils about the length and the breadth of each rectangle. Next propose to them to find out orally the area of the rectangle without setting it out correctly and in full and arranging inch-squares side by side. Get them thus to note that even *without arranging inch-squares* side by side as shown above they could give out the extent of each rectangle by adopting the above rule.

Direct pupils to cut out on paper a square of side one





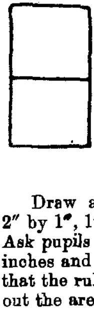
inch and fold the square obtained so that the crease-line passes through the middle point of two opposite sides. Let them cut out along the crease-line, examine the shape assumed by each piece, and observe its *length*, breadth and area. Lead them to note that the area of the piece is half that of the full square and is hence half a square inch, that its length is 1″ and its breadth is ½ inch. Let them next examine if the rule already obtained for finding out the areas of rectangles would hold good even in the present case and note that it holds good.

Next direct them to place the two pieces above

obtained side by side and obtain a rectangle 2 inches long and ½ inch wide. Get them to note that the area is 1 square-inch as it was from an inch square the pieces were cut out. Let them also verify as follows whether the above rule will hold good :—

The number of inches in the length = 2
and " " " breadth = ½
Therefore the number of square inches in
the area = 2 × ½ or 1.

Taking a part forming half a square inch as cut out above, let them fold it again through the middle points of its lengthwise sides and thus get two parts each of which is a square of side ½ inch. Question pupils how much of a square inch each part comes to. Get them to note that each part is ¼ of a square inch and this area could also be obtained by multiplying the number of inches in the length, viz., ½ by the number of inches in the breadth viz., ½.



Draw a rectangle on the graph-board say 1″ by ½″, 2″ by 1″, 1″ by ¼″, ½″ by 2″, 10″ by 5½″, 8½″ by 4½″, etc. Ask pupils to exhibit in a tabular form the dimensions in inches and the area in square inches. Point out to them that the rule already derived may safely be applied to find out the area. Tell pupils that a rectangle 4″ long and 1½″ wide will hereafter be referred to as one 4″ by 1½″.

Next draw squares on the graph-board of length 2″, 3″, 2½″, 8½″, 5″, etc. Let pupils find the area of each square. Get them to note that to find the area of a square they have only to multiply the number of inches in the length by itself.

Ask pupils to roughly sketch out rectangles on paper or on the blackboard. Let them write against the adjacent sides any lengths in inches and fractions of an inch they arbitrarily choose. Get them to find out the area. While

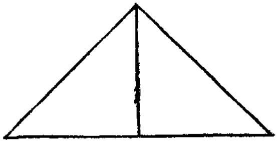
they write out the dimensions, caution them to choose the length of the rectangle so as to be greater than its breadth if they do not do so.

(As multiplication of one decimal by another is postponed to a later stage, the teacher may direct them to avoid at present dimensions of the following type 8·8″ by 4·9″, etc.)

While the above exercises are being done, question them how they would find the length of a rectangle when its area and breadth are given or how they would find its breadth when its area and length are given. Point out that as the number of inches in the length × the number of inches in the breadth = the number of square inches in the area, the number of inches in the length may be found by dividing the number of square inches in the area by the number of inches in the breadth and similarly for finding the number of inches in the breadth they may divide the number of square inches in the area by the number of inches in the length.

Ideas of space-forms which may cover a square inch.

Place at the disposal of the pupils some inch-squares. Get them to divide an inch-square into two parts by cutting along a diagonal. Let them place the pieces together as shown in the figure below. Question them how much of space is covered by the pieces. Point out that even though the pieces do not then appear to be square in form, their area is 1 square inch.



Next let them take another inch square and cut it out

into rectangular pieces 1″ long and ¼″ wide. Let them next arrange the pieces into a rectangle 4″ by ¼″. Question

what the area then is. Point out that though a square shape is not then assumed the area is one square inch as before.

Propose to pupils to suggest various irregular and curvilineal forms for the pieces into which the inch squares may be cut out and re-arranged and thus impress on them the fact that an extent of one square inch may be of different space-forms and need not necessarily be square in form.

Necessity for bigger units in expressing areas.

(a) *The Square-Foot.*

Propose to pupils that the area of a rectangular floor in the school or that of a rectangular field hard by has to be found out. Question them whether they would be satisfied with expressing the area in square inches. Point out that the adoption of larger units would be convenient in expressing the areas. Draw on the floor or on the blackboard a square 1 foot by 1 foot. Let pupils note the dimensions. Tell them that the area covered by it is called *one square foot.* Point out to them that the areas of school floors, or of the surface of walls and other like surfaces may be expressed in terms of this unit. Draw on the blackboard a rectangle 3 feet by 2 feet and if the space of the blackboard is not sufficient draw it on the wall or on the floor-space. Get them to note that the rule already applied to find the areas of rectangles will also hold good here.

Let pupils next measure the length and breadth of *Gathis, Ankanams, School-rooms,* rectangular verandhas, rectangular planks, etc., in feet and inches with a measuring tape and find the areas by the above method. Introduce them to the method of expressing square inches and square feet shortly as 'sq. ins.' and 'sq. ft.' respectively. If in expressing the area, the pupils get fractions of a square foot, point out that they may express the fraction of a square foot in square inches provided they remember how many square inches make one square foot. Referring to a foot square drawn on the graph-board or plain board, question how many inch-squares they will have to arrange to cover the area. Let them note that 12 rows of 12-inch squares are required and thus record *that 144 square inches make one square foot.* (As shown in the case of the square inch the teacher will do well to point out here too that a square

foot of space may be of any shape and need not necessarily be a square.)

(b) *The square yard.*

Next draw on the floor or on the board a square of side 1 yard. Tell them that the area it covers is called *a square yard* and that occasionally the areas of figures are expressed in square yards also. Let them divide the yard square into square feet and thereby derive that *1 square yard contains 9 square feet.* Point out to them that in cases where they get a fraction of a square yard in the result they may express it in square feet, taking that 1 square yard contains 9 square feet.

(Here again the teacher may incidentally impress the fact that a square yard of space may be of any shape and need not necessarily be sqnare in form.)

(c) *The square chain and the square link.*

Take pupils to the open ground, direct them to set out a square of side one chain and stand at its four corners. Question them what name they would assign if the extent of space covered by that square be chosen as the standard for expressing area. Get them to note that they may call it a *square chain* after the manner in which the names "*square inch*" '*square foot*", etc., have been assigned.

Let them next express the length of each side in yards and then calculate the area in sqnare yards. Let them record that *1 square chain contains 484 square yards.* Next introduce them to the term '*square link*' as denoting the extent of space covered by a square of side one link. Ask them to express in links the dimensions of a square of side 1 chain. Let them next calculate the area in square links and record that *one square chain contains* 100 × 100 or 10,000 *square links.* (In the case of the square chain and square link too the teacher may draw the attention of the pupils to the fact that a square chain or a square link of space may be of any shape whatever and need not necessarily be square in form.)

(d) ***An acre and a cent.***

Arrange the pupils so that they stand at the corners of a rectangle ten square chains in area. Let pupils themselves choose the dimensions, say, ten chains for the length of the

rectangle, and one chain for the breadth, or five chains for the length and two chains for the breadth, or four chains for the length and two-and-half chains for the breadth, etc. Tell them that the extent of space covered by the rectangle is called *an acre* and that it is the unit of area used in surveying, e.g., in expressing the areas of fields, gardens, etc. Tell them that to express areas smaller than an acre they have the acre divided into 100 equal parts called '*cents.*' Let pupils stand at the corner of a rectangle 50 links by 20 links. Let them work out the area of that rectangle to be 1/10 of a square chain and hence as 1/100 of an acre. Thus give them an idea of *a cent.* Get them to work out the area of the above rectangle in square links and thus derive that 1,000 square links make 1 cent. Tell them that generally the dimensions of a rectangular field are measured in links and then the area is easily calculated and expressed in acres and cents by remembering the equivalents 1,000 *square links* = 1 *cent.*

100 *cents or* 100,000 *square links* = 1 *acre.*

Propose :—

The length of a rectangular field is 550 links. Its breadth is 480 links. Find its area.

Let pupils work out the area to be 550 × 480 square links or 264,000 square links. Question them how many square links make one cent. Then get them to express the area in cents by dividing the number of square links by 1,000, the number of square links in 1 cent. Thus let them find out the area to be 264 cents. Let them next express the result as two acres and 64 cents by using the relationship *that* 100 *cents make one acre.* Point out to them that if they should meet with any area expressed as 2·85 acres they may express it as two acres and 85 cents or 285 *cents* and that an area like 285 cents may also be expressed as 2·85 acres or 2 acres 85 cents. Get them to work out if necessary that one acre contains 100 cents, ·1 or one-tenth of an acre contains 10 cents and ·01 or one-hundredth of an acre contains one cent and therefore 2 acres contains 200 cents, 8 tenths of an acre 80 cents, 5 hundredths of an acre contains 5 cents and 2·85 acres equals 2 acres 85 cents.

The square mile.

Tell pupils that when the areas of taluks, districts and countries have to be expressed a bigger unit is employed

viz., "*The square mile.*" Get them to note that it is the extent of space covered by a square of side 1 mile. Get them to work out that as a mile is 8 furlongs, i.e., 80 chains, a square mile should cover 80 × 80 or 6,400 square chains and, as 10 square chains make 1 acre, *a square mile covers an area of 640 acres, i.e., about 16 times the area of the present Government Farm at Anakaplle.* (The teacher may change the figures if the Government Farm is extended.)

The units adopted in the indigenous system.

Refer pupils to the custom once prevalent in the locality but now almost extinct of expressing the extent of lands that a person possesses from the amount of seed required for sowing or from the amount of yield expected, thus, '*Garisalunara Vithalumu*' meant a field producing 1½ garisas of paddy, etc. Now that the Agricultural Department has demonstrated that an equal outturn could be got by sowing smaller quantities of seed and that the extent of lands may be different for equal yield, e.g., the same outturn of a crop may be had from 2 acres of wet lands or from 4 acres of dry lands, most of the indigenous terms are disappearing and the terms acre and cent coming into popular usage. Still, remind pupils of terms like *Manudu* which means the quantity of land requiring a *manika* of seed (about 2½ cents), *addudu* (5 cents), *Kunchedu* (10 cents) *edumunarisa* (an acre), *Edumu* (2 acres), *Pandumu* (4 acres), *Putti* (8 acres), *Aru* which means the amount of dry lands which a pair of cattle could plough in one day (½ an acre), a *garisa* of wet lands (2 acres), etc. Refer also to terms like *chalaka*, *manchas* and *kati pudus* prevalent in the Agency, a *mancha* meaning the extent of land which can be commanded by a watcher sitting on a raised bamboo platform in the middle of a field to scare away birds and animals and covering roughly about 2 acres, *Katipudu*—the land that can be cleared in one day by one *Kati* or bill-hook covering about an acre.

Next tell them that in some parts they have got the term '*visam*' to denote about 2 acres, the term '*Pathika*' to denote 4 visalamulu, a cawnie to denote a quarter visamu a *Kathu* to denote 24 visamulu, and that in Parvatipur side an extent of 2½ acres is expressed as 25 puttis or 500 kunchams, etc. Tell them also that the extent of land

covered by houses is expressed as so many *Ankanams Gatis*, etc., which mean only smaller rooms in houses and *that the dimensions of these vary according to locality.* Point out that for want of fixity in extent, the indigenous terms are mostly abandoned and the system of measuring in square feet, acres, and cents has come into vogue.

Dimensions of rectangular and square plots covering an acre or cent in extent.

It may be desirable for pupils to remember the dimensions in feet of squares or rectangles of area 1 acre and 1 cent. Let pupils work as follows :—

1 acre equals 10 square chains, i.e., 10 × 22 × 22 square yards, i.e., 4,840 square yards. Point out that 70 × 70 is 4,900. Hence get pupils to remember that a length of a square set out with a length of 69 to 70 yards for its side would cover an area of one acre. Next let them work out that 1 acre is 100 cents and that hence a cent is equal to 48·4 square yards. Get them hence to note that a square of side about 7 yards would help them to set out a square of area 1 cent. Point out that these equivalents should specially be remembered by them in order that they may know how to calculate quickly the number of plants in an acre when planted at different distances as specified, how to set out quickly 10-cent plots, 5-cent plots, 2-cent plots, etc.

Estimation of areas at sight and by stepping along boundaries.

Give plenty of practice to the pupils in estimating by the eye the number of units of length in the dimensions of rectangular plots of ground, rooms, walls, floors, etc., and thus guess at sight the probable areas. Similarly give them exercises in guessing in cents and acres the areas of plots in the fields all around. Get pupils also to obtain notions of the space occupied by 50-cent and 10 cent-plots in the farm by frequently coming into contact with them, then set out these areas along other plots and thus obtain the power of estimating roughly the areas of fields in acres and cents to a fair degree of accuracy.

Let pupils walk with uniform pace along the length and breadth of rectangular plots. By giving plenty of

practice of the above type, get them to note the dimensions of the rectangles in terms of their strides. Then let them find out how many squares of side equal in length to their stride would roughly cover up an acre or a cent. Thus familiarize the extent of an acre and cent by pupils' stepping out uniformly along the boundaries of rectangular plots. Next give them plenty of practice in walking with uniform pace along the boundaries of rectangular fields, in counting the number of paces in the length and in the breadth and then in calculating the area in acres and cents.

Exercises.

Propose :—

1. A house plot is 20 feet wide and 150 feet long. Estimate the value of the plot at Rs. 1–8–0 a square yard. (*Ans.* Rs. 500.)

2. Pasupu is grown in a plot, 100 links by 60 links. How many cents cover the plot? (*Ans.* 6 cents.)

3. A plot of ground where sugar-cane is cultivated is 500 links long and 300 links wide. Find the value of the plot at Rs. 1,000 an acre. (*Ans.* Rs. 1,500.)

4. An acre of wet lands produces 300 kunchams of paddy. At this rate find how much I may expect from fields of the following dimensions :—

(i) 200 links by 150 links, (ii) 500 links by 400 links, (iii) 600 links by 450 links, and (iv) 300 links by 100 links. (*Ans.* 90 kunchams, 600 kunchams, 810 kunchams, 90 kunchams.)

5. Find what assessment has to be paid for the fields noted below :—

Dimensions of the fields.					Money rates per acre. RS.	A.	P.
(*a*) 500	links by	300	links	...	3	0	0
(*b*) 600	„	500	„	...	5	8	0
(*c*) 700	„	600	„	...	1	4	0
(*d*) 425	„	250	„	...	8	0	0
(*e*) 875	„	400	„	...	5	8	0
(*f*) 200	„	150	„	...	7	8	0

(*Ans.* (*a*) Rs. 4–8–0, (*b*) Rs. 16–8–0,
(*c*) „ 5–4–0, (*d*) „ 8–5–0,
(*e*) „ 19–4–0, (*f*) „ 2–4–0.)

Dimensions of the fields.				Money rates per acre.		
				RS.	A.	P.
(g)	70 links by	250 links	...	12	8	0
(h)	550 „	450 „	...	10	0	0

((g) Rs. 24-12-0, (h) Rs. 0-7-0).

6. Find how many cart-loads of farm-yard manure will be required for cultivating turmeric in the following fields at 18 cart-loads for an acre :—

5 chains, by 4 chains ; 800 links by 500 links ; 400 links by 375 links. (*Ans.* 36, 72, 27 cart-loads)

7. Assuming that an acre of gingelly crop yields 40 kunchams calculate the quantity of gingelly that would be got from the fields whose dimensions are given below :—

280 links by 250 links, 540 links by 510 links. 700 links by 200 links, 500 links by 180 links.

(*Ans.* 28 kunchams, 108 kunchams, 56 kunchams, and 36 kunchams of gingelly.)

8. The dimensions of some rectangular fields and the names of crops raised in them are given below :—

Length.	*Breadth.*	*Crop.*
15 chains	8 chains	Ragi
6 chains	6 chains	Mocca jonna
150 links	80 links	Bobbili Ganti
25 chains	24 chains	Gingelly.

Find the cost of cultivation of each crop in the respective fields at the following rates per acre. Ragi Rs. 45, Mocca jonna Rs. 20, Bobbili gauti Rs. 30 and Gingelly.)

(*Ans.* Rs. 540, Rs. 96, Rs. 36, Rs. 1,344.)

9. Mr. Patrudu has two fields, one 500 links by 400 links and another 450 links by 420 links. Which is the bigger field and by how many cents ?

(*Ans.* The former by 110 cents.)

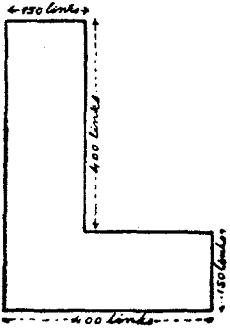
10. A plot is 250 links and 150 links wide. Some years ago the ryots were sowing at the rate of 20 kunchams of paddy seed an acre. How much of the seed would have then been used for the above plot ? If it is found that 5 kunchams will do for an acre, what will be the saving under seed so far as that plot is concerned ?

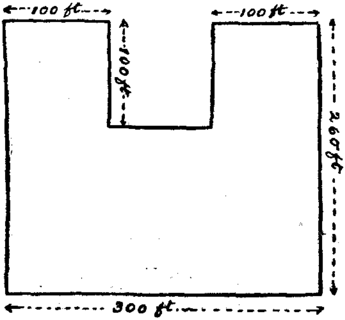
(*Ans.* 7½ kunchams, 5⅝ kunchams.)

11. A cattle shed should be 9 ft. by 8 ft. for a pair exclusive of a space 9 ft. by $1\frac{1}{2}$ ft. for a manger. If you have four pairs of cattle in your house what area of ground should be allotted for erecting the shed according to the above arrangement. Calculate the cost of erecting the shed at Re. 1-8-0 per square foot.

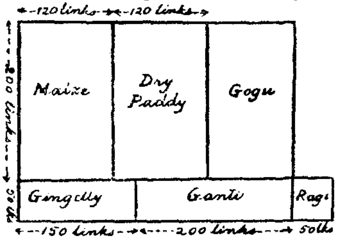
(*Ans.* 342 square feet Rs. 513.)

12. Find the areas of the plots roughly sketched below, making use of the measurements noted against each side.

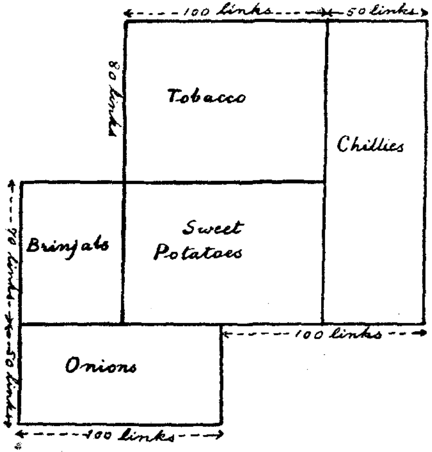




13. The diagram in the margin represents a man's fields wherein he has sown in one season gingelly, ganti, ragi, maize, dry paddy and gogu. Find what area is under each cultivation.



14. Answer the above question with reference to the figure below representing a field in which you have



chillies, tobacco, sweet potatoes, onions and brinjals cultivated.

15. A plot is 20 yards by 10 yards. You suggest to the contractor that a house of a particular design should

be built on it. If he charges at the rate of Rs. 4 for a square foot of the ground-floor, find how much the total cost will come to. (*Ans.* Rs. 7,200.)

16. You have a garden, 20 chains by 10 chains. If you adopt the indigenous system of planting coconuts you could arrange to have only 48 trees per acre. If you follow the plan suggested by the Agricultural Department you will be able to plant 66 trees per acre. How many trees will you be gaining if you should be adopting the latter method? (*Ans.* 360 trees.)

17. Find the outer surface area of a wicker basket open at the top which is 4 ft. 6 in. long, 2 ft. high and 2 ft. wide. (*Ans.* 35 square feet.)

18. In Pārvatīpuram side the area of a house is expressed in gathis, e.g. as 4 gathulu. If a gathi is $10\frac{1}{2}$ feet long and $7\frac{1}{2}$ ft. wide, and costs Rs. 300, find how much this comes to for a square foot. (*Ans.* Rs. 3-12-11.)

19. There are some tiles at your disposal. You contemplate to put up two roofs, rectangular in shape, each 25 feet long and 16 feet wide. If 3,000 tiles are required for covering a space 10 ft. by 10 ft. find the number of tiles required. (*Ans.* 24,000 tiles.)

20. If for covering a space, 10 feet by 10 feet, two masons will have to be engaged for a day at 12 annas each, find how much you have to spend for paying the masons in putting up the roofing referred to in question (19).
(*Ans.* Rs. 12.)

21. You propose to put up a terraced roof for a room 20 feet by 14 feet. If for covering a space 10 feet by 10 feet you require 2,000 flat tiles, find the cost of the flat tiles required at Rs. 2-8-0 a 1,000. (*Ans.* Rs. 14.)

22. Find the area of each wall surface in a rectangular room, 18 feet long, 14 feet wide and 10 feet high assuming that the windows, doors, etc., occupy 1/8 of each wall surface. If you want to replaster the walls the mason charges at the rate of 6 annas a square foot. Find how much the total cost would come to.

(*Ans.* $159\frac{1}{2}$ sq. ft. for each of two opposite walls, $122\frac{1}{2}$ sq. ft. for each of the other two walls, Rs. 210.)

23. A thick rectangular plank of wood 18 feet long, 1 foot wide and $4\frac{1}{2}$ inches thick has been cut into 3 thinner planks 18 feet long 1 foot wide and $1\frac{1}{2}$ inches thick by

applying the saw lengthwise. How many times has the saw been applied? Calculate the wages for sawing at half anna per square foot, *one face out of the two formed* by sawing being alone taken into account. (*Ans.* Rs. 1-2-0.)

24. There is a rectangular wooden box in your house which is 2 feet 6 inches long, 1 foot 6 inches wide and 1 foot 6 inches high. Find what area will have to be varnished. Find the cost of the varnish required at 2 annas for a square foot. (*Ans.* Rs. 2-7-0.)

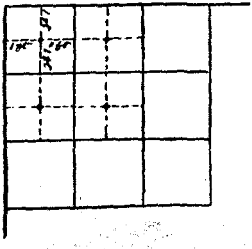
25. In estimating the quantity of straw required for thatching roofs of houses in Pārvatīpuram side it is usual to reckon 15 puttis of straw corresponding to a floor space 7 cubits by 5 cubits, i.e., of a gathi. Estimate at this rate the quantity of straw required for thatching the roof of a house, 14 cubits by 7½ cubits.

(Here get pupils to express the areas of houses in *gathilu* by using a diagram if necessary for the purpose of illustration.) (*Ans.* 45, puttis of straw.)

26. An 'Ankanam' is 12 feet by 6 feet. Its floor has to be paved with Cuddapah slabs 3 ft. by 2 ft. How many slabs will be required? (Ans. 128 slabs.)

27. For thatching a unit room or gathi 5 cubits by 5 cubits, 500 palmyra leaves are required. Estimate at this rate the number of palmyra leaves required for thatching a house 15 cubits by 7½ cubits. (*Ans.* 2,250 leaves.)

28. A plot of ground is 500 links by 200 links. How many tobacco seedlings 2 feet. apart each way could be planted in it?



(Draw a diagram to represent the plot, note against two adjacent sides the length and breadth of the rectangle in feet, 330 feet and 132 feet in this example. Tell pupils that, in actual planting, a distance of half of 2 feet—the spacing given—is left from each boundary line so that for a square 2 feet by 2 feet in the above case there would be a single seedling planted. Illustrate by means of a diagram as in the margin

Thus lead them to note that the number of plants is given by $\frac{330 \times 132}{2 \times 2}$, i.e., the area of the plot in square feet ÷ the area in square feet of a square of side when length is given by the spacing specified. In the diagram extend the division further and illustrate.

29. An acre of land contains 4,840 square yards. Show that the number of plants which could be planted in it 3 feet each way is given by $\frac{4{,}840 \times 9}{3 \times 3}$.

30. Change the number of feet in the spacing in Ex. (29) and propose a series of exercises.

TOPIC 23.—DIRECT PROPORTION.

(Scope of the treatment.—*Notions of quantities varying directly with each other may be given. The unitary method as well as the method of multiplying ratio to working questions on direct proportion may be explained and the convenience of adopting either method according to the occasion may be shown. Cases which do not admit of the application of the method may next be pointed out. The process of simplifying the result required may be directed to be attempted either at the final step or in the intermediate step so as to suit the occasion. A few realistic problems may then be proposed for solution.*)

Ideas of quantities varying directly with each other.

Propose :—A country plough costs Rs. 3. How much should be given for 2, 3, 4, 5, 10, etc., ploughs, respectively, of the same sort and at the same rate ?

Lead pupils to note that the number of ploughs bought is different and hence, the prices to be paid would be different. Question how much the price to be paid would increase for each plough bought extra. Point out that if the number of ploughs becomes double, thrice, four times, etc., as large, the price to be paid would also become double, thrice, four times, etc., as large, that is to say, for 2 ploughs Rs. 3 × 2 or Rs. 6 will have to be paid, for 3 ploughs Rs. 3 × 3 or Rs. 9 will have to be paid, for four ploughs Rs. 3 × 4 or Rs. 12, etc., and so on.

Propose :—An acre of wet land yields 300 kunchams of paddy in one season. Find at this rate the yield from half an acre, one-fourth of an acre, one-eighth of an acre of the same kind of land, etc.

Get pupils to work out that from half an acre half of 300 kunchams or 150 kunchams may be expected ; from one-fourth of an acre, $\frac{1}{4}$ of 300 kunchams or 75 kunchams may be got ; from 1/8 of an acre, 1/8 of 300 kunchams or $37\frac{1}{2}$ kunchams may be got, etc. Point out to them that under similar conditions as the extent of land diminishes, the quantity of yield that may be expected also diminishes in the same proportion, and that if they want to find the yield

from 7/10 of an acre, they have only to take 7/10 of 300 kunchams or 210 kunchams.

From the above two instances, point out to pupils how two quantities may vary simultaneously. Get them to note that corresponding to a change in the number of ploughs bought, there is also a change in the price to be paid and corresponding to a change in the extent of land there is also a change in the quantity of yield. Lead them to realize that the changes in the values of one of the two quantities have been observed to be proportionate to the changes in the values of the other, that is to say, to what extent the number of ploughs increases, to that extent the price to be paid also increases, and to what extent the extent of land decreases, to that extent the yield also decreases. Thus give them an idea of direct variation. Get them to suggest some more instances of quantities varying directly with each other, e.g., the value of land depending on its extent, the quantity of manure required for cultivation depending on the extent of land, the price to be paid for manure depending on the quantity and quality bought; the value of a brick kiln depending on the number of bricks contained in it, etc.

The methods of working questions on direct proportion.

Propose :—The price of 10 cwt. of fish-guano is Rs. 60 at Anakāpalle. Find at this rate the value of 20 cwt., 30 cwt., 40 cwt., and 50 cwt. of fish-guano.

Let the pupils work the above question as follows :—

10 cwt. costs Rs. 60.
∴ 1 cwt. costs Rs. 6.
∴ 20 cwt. costs Rs. 6 × 20 or Rs. 120.

Similarly,

30 cwt. costs Rs. 6 × 30 or Rs. 180.
40 „ „ Rs. 6 × 40 or Rs. 240.
50 „ „ Rs. 6 × 50 or Rs. 300.

If, instead of adopting the above method, they reckon the price of 20 cwt. to be twice that of 10 cwt. and hence find the cost to be Rs. 120, point out to them that it is a quicker method.

Propose :—A ton of bone-meal costs Rs. 80. Find at this rate the price of 7 cwt. of bone-meal.

Let pupils attempt to work the above question by any of the methods shown to them. Point out to them that either the one or the other of the two methods given below may be adopted, whichever they may find easier.

Method I.—Question how many cwt. make one ton and hence what part of a ton is 1 cwt. Thus get them to work that as the price of 1 ton or 20 cwt. is Rs. 80, that of 1 cwt. comes to 1/20 of Rs. 80, i.e., Rs. 4 and hence the price of 7 cwt. is Rs. 4 × 7 or Rs. 28.

Method II.—Question what fraction of 1 ton would give 7 cwt. and point out that 7 cwt. is 7/20 of 1 ton and therefore the price of 7 cwt. of bone-meal should be 7/20 of the price of 1 ton, i.e., 7/20 of Rs. 80 or Rs. 28.

The pupils would generally find the former method to be more convenient. If any of them should find the second method handy, let them adopt it. If not, let them adopt the first method. Point out that the steps in method I should be clearly arranged as follows:—

The price of 20 cwt is Rs. 80.
∴ the price of 1 cwt. is Rs. 80/20 or 4.
∴ the price of 7 cwt. is Rs. 4 × 7 or Rs. 28.

Let pupils examine the steps written in working according to the above method, both in this and in the previous question. Get them to note that there are three main steps and that the second step gives the price of the unit quantity chosen. Tell them that this method is therefore called "the Unitary Method." Point out to them also that in the second step in the above question, the value of Rs. 80/20, admits of being expressed in the simple form "Rs. 4" and that, in cases where such simplification is not easy to effect, the quantity Rs. 80/20 may be carried on to the third step and the result, viz., 80/20 × 7 may be arrived at either from 560/20 or from 4 × 7.

Propose:—A bag of castor-cake weighing 164 lb. costs Rs. 9. Find at this rate the price of 10 cwt. of the cake.

Let pupils work by the above method as follows:—

10 cwt. is equal to 1,120 lb.
164 lb. costs Rs. 9.
∴ 1 lb. costs Rs. 9/164.
∴ 1,120 lb. costs Rs. 9/164 × 1,120.

Point out that the simplification has been reserved to the end, and the value worked out to be Rs. 61–7–5 *to the nearest pie.* Impress on pupils the necessity for expressing values approximately when quantities like fractions of a pie, etc., are involved in the last stage.

Cases in which the application of direct proportion would not hold good.

In working the above exercises, point out to the pupils that when the price of large quantities are given, that of a small fraction of it is calculated by proportionate valuation or when that of a small quantity is given that of a comparatively very big quantity is also calculated likewise. Get pupils to note that in practical shopping a higher rate will be charged when only a small quantity is bought and a cheaper rate allowed or demanded when very large quantities are hought. Thus lead them to note that the propriety of the above method is open to question but this objection could be set right by the use of the expression *at this rate* in each exercise above which requires a supposition being granted.

Let pupils suggest cases when such suppositions will be found quite inadmissible. Refer them to examples like *the following* :—

If the height of a guava tree three months old be 9", what will be the height of the tree in 30 years? Point out the obvious impossibility of the tree growing up to 9 × 12 × 10 or 90 feet even though it is kept on for 30 years. Suggest also other questions of the same type, e.g., the growth of a child, the height of a stone thrown up into the air, etc. Get pupils to discuss the propriety of the application of proportion whenever a question is given.

Exercises.

Propose :—

1. 8 kunchams of dry paddy are required to sow one acre broadcast. How many acres could be so sown with 6, 40 and 68 kunchams? (*Ans.* $\frac{3}{4}$ acre, 5 acres, $8\frac{1}{2}$ acres.)

2. The cost of manufacturing a maund of 25 lb. of jaggery in a farm is 4 annas. Find at that rate the cost of manufacturing $8\frac{3}{4}$ such maunds. (*Ans.* Rs. 2–3–0.)

3. A person has 45 acres of wet lands. In cultivating 2 acres of the lands he got 700 kunchams of paddy. Will he be getting 700/2 × 45 kunchams of paddy from all his wet lands? State the cases when he may get the above quantity and when he may not get it.

4. Selected paddy seed is sold in the Anakapalle farm at Rs. 10 for 24 kunchams. How much should be paid for 32 kunchams? (*Ans.* Rs. 16.)

5. Jaggery costs Rs. 49 for a barva weight. What rate does this give for a maund and for a viss, respectively?
(*Ans.* Rs. 2–7–2 for a maund and Rs. 0–4–11 for a viss.)

6. Turmeric sells at Rs. 30 for a putti weight. How much is this for a viss? (*Ans.* 3 annas.)

7. Dry chillies sell at Rs. 1–4–0 for a maund of 25 lb. At this rate, estimate the value of a ton of dry chillies exported from Golaprole. (*Ans.* Rs. 112.)

8. Onions sell at 8 annas a maund of 25 lb. How much is this for a putti of 500 lb.? (*Ans.* Rs. 10.)

9 Sugarcane sells at Rs. 4–11–0 per 100. How much would I get by selling 2,850 canes from a field?
(*Ans.* Rs. 133–9–6.)

10. A person planting six plantain trees expects to earn Rs. 4–8–0 from them in a year. What sum of money may a gardener expect to earn in a year if he plants 400 plantain trees in his gardens? (*Ans.* Rs. 300.)

How many plaintain trees should he plant so that he may expect to earn from them Rs. 180 in a year?

(In working the second step let pupils argue as follows:—

To earn Rs. 4–8–0, 6 plantain trees should be planted.

∴ to earn Rs. 9, 12 plantain trees should be planted.

∴ to earn Rs. 180, 12/9 × 180 or 240 plantain trees should be planted.

Point out that the result in the third step may be expressed to be Rs. 180/9 × 12 and thereby worked out orally to be Rs. 20 × 12 or Rs. 240.

Get pupils to note that the step "to earn one rupee 4/9 plaintain tree should be planted" would appear ridiculous

when presented separately, hence that step is taken in *mentally* and the third step deduced from it.)

11. From 10 maunds of green-chillies 2½ maunds of dried marketable chilles are obtained. How many maunds of green chillies should be bought to get 50 puttis of dry marketable chillies ? (*Ans.* 200 puttis.)

12. A cow in the farm gives 21 seers of milk in the course of a week. Find the value of milk given by it during a period of 288 days she is in milk. Assume the above rate of yield and that a seer of milk would fetch 4 annas. (*Ans.* Rs. 216.)

(Point out that in working this question, the total quantity of milk may have to be first found, then the value of it and that thereby this question involves the application of the proportion method twice.)

13. Onion seedlings sell at 2 annas for 5 bundles. I want 65 bundles for the purpose of planting in my garden. How much may I have to pay ? (*Ans.* Rs. 1-12-0).

14. Red plantain suckers sell at Rs. 6-4-0 per 100. What is the price of 450 such suckers ?

(*Ans.* Rs. 28-2-0.)

15. The cost of 100 plantain leaves is 4 annas. What is the cost at this rate of 450 leaves ?

(*Ans.* Rs. 1-2-0.)

16. Guavas sell at 6 annas a *pathika* (= 25). How much of money would 150 guavas fetch at the above rate ?

(*Ans.* Rs. 2-4-0.)

17. Fish-guano sells at Rs. 6 a bag of 112 lb. What is the price per ton at this rate ? (*Ans.* Rs. 120.)

18. A busta of Bengal gram weighs 164 lb. and contains 20 kunchams. Express the weight of a kuncham of the grain in lb. (*Ans.* 8 1/5 visam.)

19. A busta of rice containing 26 kunchams sells at the rate of Rs. 18-12-0 (without bag). How many seers does this come to per rupee ? (Take 4 seers for a kuncham.) (*Ans.* 5½ seers and about a gidda extra.)

20. In pounding rice out of paddy the cooly will give 9 kunchams of rice for every 20 kunchams of paddy supplied. For a marriage, I require 5 bustas of rice, each

of 22½ kunchams. How many kunchams of paddy should be given to the cooly for pounding ? (*Ans.* 250 kunchams.)

21. If 1,500 sets of sugarcane weigh a putti and sell at Rs. 3, find the weight of 30,000 such sets as well as their cost. (*Ans.* 20 puttis and Rs. 60.)

22. Thousand palmyrah leaves cost Rs. 10. Find at this rate how many leaves could be bought for Rs. 35. Also find the cost of 6,400 leaves.

(*Ans.* 3,500 and Rs. 64.)

23. To get sand from the Sarada River or black soil for house-building, 3 cart-loads are charged a rupee. I want 10 cart-loads of sand and 10 cart-loads of black soil. How much should I pay ? (*Ans.* Rs. 6–10–6.)

24. A garisa of *sunnamu* (quick-lime) sells at Rs. 20. How much should a man pay for buying 6 puttis of sunnamu ? (Take 30 puttis for a garisa). (*Ans.* Rs. 4.)

25. Amidamupindi (black castor-cake) sells at Rs. 25 a putti at Vizagapatam. Half a putti is applied as manure for an acre of paddy. Find the cost of the manure required for 5½ acres at the above rates ?

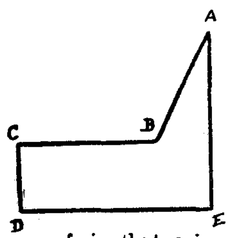
(*Ans.* Rs. 68–12–0.)

TOPIC 24.—DUPLICATES AND SIMILARS.

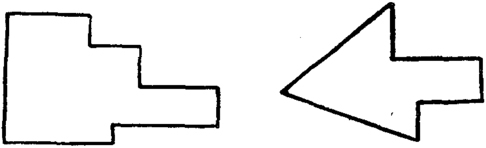
First Notions.

(**Scope of the treatment.**—*The teacher need not be scared away by the high sounding names "duplicates and similars" found in the title of this topic. All that is intended here is to convey to the pupils the fundamental space-notions which would help him to study aright a plan and its original. At first, notions may be given of equality in size and sameness of shape in figures. Next, the pupils may be introduced to ideas of similarity in shape with a change in size. It may be well impressed that in setting out similarity in shape there is proportionate representation of size. It may also be brought home to the pupils that there is constancy of direction, besides, among the corresponding lines in a duplicate and in a similar as also in a plan and its original. A few exercises may be set on the study of the farm-plan, the ground-plan of a school or of any other building, etc., some practical work may be given in drawing figures to scale. Rectangles, squares and figures formed by joining successively the corners of any squares arbitrarily chosen on the graph-board, etc., may be taken up for the purpose*)

Have the following got ready before hand.—1. Fold a large sheet of paper into two. Starting from a point in the crease-line and keeping the paper folded, cut out with a pair of scissors along the lines AB, BC, CD, DE, EA, as shown in the marginal figure. Choose for the figure very long and very short sides and also arrange to obtain very sharp as well as blunt corners. Attach to the blackboard by means of pins the two irregular places so cut out. See that the pieces are at least as large as a quarter sheet of paper.



2. Draw on graph paper a figure like the following :—



Get it traced on translucent paper, and obtain two copies using carbon sheets for the purpose. The farm-plan may be chosen for drawing the outline indicated above. Get also two or more figures of the above shape drawn to scales, say, developed in the ratio 2 : 1, 4 : 1, 5 : 1, etc.

3. Arrange to get photos of one and the same person in different sizes. If there be the King-Emperor's bust supplied to the school, use it for the purpose. Get ready also a few postal stamps containing the bust, as well as coins containing the same.

4. If there be maps supplied to the school keep ready a map of India as shown separately in the wall map of Asia, in the world map, and as shown in the maps and in the atlases, provided the same type of projection is adopted in all of them.

Ideas of equality in size and sameness in shape.

Refer pupils to the pieces got ready in (1) above. Let them examine the pieces closely and compare them. Get them to note that one piece looks exactly like the other. Direct them to remove the pins attached to one of the pieces, and arrange to see if the piece could be placed so as to exactly coincide with the other. Lead them to note that one piece could be made to exactly coincide with the other. Hence get them to realize that the pieces not only look alike but are also equal in size. Next question them which corners in one piece corresponds to which corners in the other, which edge in one piece corresponds to which edge in the other and which angle in one piece corresponds to which angle in the other.

Let pupils then compare by superposition, the lengths of the edges as well as the sizes of the angles which so correspond. Thus get them to note that the sides which

correspond are equal as well as the angles which correspond. Tell them that when two pieces have the above properties, viz., of equality in size and sameness in shape one of them is said to be '*a duplicate*' of the other. Get a few sheets of carbon paper (going in for a loan from the Farm Office, if necessary), and placing these sheets between ordinary sheets of plain paper arrange to draw in the presence of pupils on the uppermost sheet any irregular figure. Send round the class the traces obtained. Set them to compare the size and shape of the irregular figures so drawn. Get them to note that the figures are equal in size and are also of the same shape and that the figure found in one sheet may be said to be a "duplicate" of that found in another. Let pupils trace one of the figures with the help of translucent paper and see that the figure so traced could be fitted in with that in any other sheet. Thus give pupils a clear idea of equality in size and sameness in shape between a duplicate and the original.

Ideas of similarity in shape with a change in size.

Show pupils the figures of the Farm plan prepared to different scales as indicated in (2) above. Without telling them that they represent the outline of the Farm plan, set them to compare one figure with each of the others in succession. Question in what respects the figures appear alike and in what respects they differ. Lead them to recognise by intuition that there is a resemblance in form and diminution in size. Question them if they are able to recognise which corner in one figure corresponds to which corner in the other figure and which side in one figure corresponds to which in the other figure. Let them next compare the lengths of the corresponding sides that so correspond and get them to note that the sides are not equal. Next let them show which angles in one figure correspond to which angles in the other figure. Set them to compare by tracing and superposition the size of the angles which correspond. Get them to note that the angles are equal. Question if one of the figures could be placed over the other so as to exactly coincide with it and let pupils infer the obviously impossibility of it. Sum up the details above dealt with and point out to pupils that the figures may differ in size and still be similar in shape, that corresponding sides, corners and angles may be easily recognised

in such figures and that the corresponding angles will be found to be equal but not the corresponding sides.

Show the photos got ready in (3) above or the busts of the King-Emperor as found in photos, coins, one-rupee notes and postal stamps and get pupils to note that all these differ in size and are yet similar in shape.

Next refer pupils to one and the same map, say of *India*, as appearing in the wall-maps of the world and of the continent of Asia, and in the atlases. (Here the teacher should take care to choose as far as possible maps of the same kind of projection.) Point out to them that, though one and the same country is represented, there is difference in size with similarity in shape. Tell them that hereafter when there is a similarity in shape with a change in size, one of them will be referred to as a similar" to the other and that, the term " duplicate " will be applied only where there is equality in size in addition to sameness in shape.

Similarity in shape and proportionate size.

With reference to the figures got ready by the teacher according to preliminary instruction (2) above, let pupils measure the lengths of sides which have been above observed to correspond, just wipe off two lines which meet and substitute lines far longer. Question if the sameness in shape is preserved. Point out that the resemblance in shape is disturbed with reference to the figures in (2) above. To enable pupils to infer about the proportionality in length without much ado it is desirable that the *teacher has so drawn the copies that the corresponding sides* are in the ratios 2 : 1 or 4 : 1, or 5 : 2, etc. Let them tabulate the results as follows :—

The length of a side in one figure.	The length of the corresponding sides in the other figure.
10″	5″
12″	6″
6″	3″
16″	8″
20″	5″
24″	6″
12″	3″
32″	8″
	etc.

From exercises similar to the above, get them to note that a particular proportion is maintained between the lengths of the sides in the case of figures of *one* and the same shape. Point out that to maintain the proportion a length of one inch may be repesented by 2″ or by 4″ or by ½″ or by 1″ or by ·2″, etc. Let the pupils see if a similar proportion is noticeable between the lengths of lines joining corresponding points, say, in photos and in different sizes of the same bust, e.g., the length of the profile or the distance between the ears, the width of chest in a human body, the height of trees, buildings, etc. reproduced photographically, etc. Thus lead them to infer that in order *to get a good copy of a figure to a smaller* or to a larger size, a uniform proportion ought to be maintained between corresponding sides.

Comparing a duplicate and a similar.

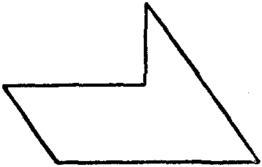
Place on the table figures and sheets containing duplicates as well as *similars* of the *original. Direct* pupils to arrange duplicates along with the original and sort out the similars separately. Set them to describe in what ways the similar and the duplicate agree and where they differ. Remind them that the duplicate is of the same size and shape as the original, that the similar is not of the same size and that it represents the original proportionately in size. Impress the fact that the proportionality has been deduced by comparing the lengths of the corresponding sides. Let them point out in the original different lines not necessarily its sides, i.e., any diagonals, the join of the middle points of the sides, etc. Ask pupils to show the corresponding lines in the duplicate and in the similar and compare their lengths, the sizes of the angles between them, etc. (Let them actually trace and superpose or measure the lines and angles.) Thus get them to conclude that, in 'a similar' and the original', *corresponding lines must be equally inclined, that such lines are equal in length in the duplicate and in the original and are of proportionate lengths in the similar and in the original.*

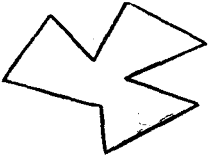
Hence impress on them that whenever they say that a picture or a plan is a representation of the original in regard to sameness of shape but drawn to a larger or smaller size they must be careful not only to test if the corresponding lines are of proportionate length but also

see that they are equally inclined to each other. Point out to them that what they speak *of as the scale of a map* only represents what proportionate lengths should be chosen for drawing the corresponding sides of an original, thus a scale of 1 inch to 100 feet simply means that a length of 100 feet should for want of space be represented by 1 inch. Get them to note that this property of sameness in shape and proportionate representation of size could be noted only between the original and a correct plan of it, and not between the original and a rough sketch of it.

Give plenty of exercises to pupils in showing corresponding lines and corners in the Farm and in the Farm plan, in the school-building and in the ground-plan of the school, etc. Let them note in each case that the plan shows the exact shape of the original but to a changed scale, and that the directions represented by the corresponding lines are the same, because of the equality of angles between them as found both in the plan and in the original.

Draw on the graphboard figures of the following type one after another and ask pupils to produce a duplicate. Get them also to draw the figures to a smaller or a larger size representing a length of 4 inches by 2 inches or ½ inch. Let them examine if the properties above ennunciated are present.

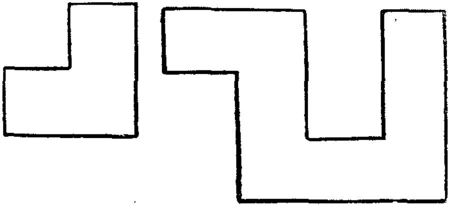




Next take pupils to rectangular plots of ground. Set them to measure the boundary lines and draw a plan of each plot to scale. Ask them to note the lie of the bunds, say north-to-south or east-to-west and get them to indicate the directions correspondingly in the plan. Question them by what length the distance of a chain measured along the bunds is represented on the plan. Thus get them to state the scale in the plan. Point out that every distance

measured along the plan is a constant fraction of the corresponding distance in the original plot.

Repeat the above exercise with plots of ground of the type :—



TOPIC 25—LINE-CHARTS.

(**Scope of the treatment.**—*It is not here intended that the theory of plotting and that of drawing statistical graphs should be taught in full. It is enough if the pupils are made to understand that quantities of the same kind can be represented in their relative sizes by means of lines drawn to scale so that clear ideas about their comparative bigness and smallness may be had at a glance from such diagrammatic representation. A few exercises may be set on the representation to scale of quantities of one and the same denomination. The pupils may next be introduced to a series of specially prepared charts showing the simultaneous variation of two connected quantities. How these charts should be read may be well explained to them and the pupils made to realize how they are prepared. For purposes of the above representation the fluctuations in prices, the growth of plants, the yield of crops under different treatment, the devastation of crops by insects, etc., may be taken up and a few exercises may be set on them. The teacher will do well to remember that the full mathematical significance of a graph is not expected to be dealt with here and hence the title of the topic is put in merely as* '**line-charts.**')

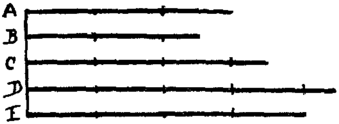
Representation of quantities by means of lines drawn to scale.

Tell pupils that the lengths of some irrigation channels are given to be as follows (See note at the foot of the next page for variants) :—

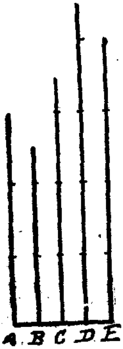
Channel	A	3	miles.
„	B	2½	„
„	C	3½	„
„	D	4½	„
„	E	4	„

Ask them to compare the lengths of the channels. When they compare the numbers 3, 2½, 3½, 4½, 4 point out to them that an easy way of making the comparison and also of remembering it would be by drawing straight lines to

represent the above lengths to scale. Actually draw on the blackboard a diagram like the following :—



Draw their attention to the fact that one small division, say, half-an-inch, has been taken to represent one mile and that the lengths of the irrigation channels have been marked out to scale by means of horizontal lines proceeding from points in the same vertical line. Get them to note that such a method facilitates quick comparison as the other ends of these lines have alone to be looked into. Make them realize that the diagrammatic representation above shown helps them to infer at a glance that D is the longest of the channels, B is the smallest and so on. Tell them that instead of representing the lengths proportionately by means of horizontal lines as above drawn they may also represent them by means of vertical lines of proportionate length emanating from the same horizontal line as follows :—



Also point out that, to lay out proportionate lengths, squared paper may be found to be more convenient than plain paper. (If the teacher should find that channels of the lengths above given are very rare, he may read furlongs for miles, doubling or trebling the number as may be found necessary. Instead of irrigation channels, roads to neighbouring villages, field bunds of so many links may be substituted.)

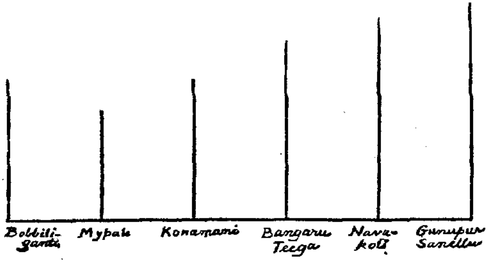
(*b*) Propose :—

The number of grains in an ounce weight of different varieties of paddy has been found to be as follows :—

Bobbiliganti	1,252
Mypali	1,040
Konamani	1,271
Bangaru Theega	1,342
Navakoti	1,512
Gunupur Sannellu	1,531

Compare these varieties and find out in which of them there will be a larger number of grains for the same quantity by weight.

Point out to pupils that here too the comparison may be made in the same way as in the last exercise and, only, instead of representing a larger length by a smaller one to scale they have to represent the number of grains by means of lines drawn to scale. Get them to suggest a scale like 1 inch to 1,000 grains. Question them by what lengths they would then have to represent the number of grains in each variety above mentioned. Point out that lengths like 12·52″, 10·4″, 12·71″, 13·42″, 15·12″ and 15·31″ may have to be represented and tell them that they may better have these lengths laid out approximately to the nearest tenth of an inch on the graph-board.



Let pupils next draw a diagram like the above on graph paper, say, to a scale of 1″ to 1,000 grains or to 2,000 grains. Make them realize how, from the jutting nature of the tips of some of the lines or from their lagging behind another, it may be easily perceived

which variety contains more grains and which variety less in the same quantity by weight. Point out that a comparison from the above diagram would very vividly show them how the different varieties arrange themselves in regard to the number of grains in one and the same quantity by weight, Konamani containing more grains than Mypali, Bangaru Theega more than Konamani, Navakoti more than Bangaru Theega and Gunupur Sannellu more than Navakoti. Refer to the figure and illustrate this fact. Point out also that, once the diagram has been drawn, one has not got to worry himself with the numbers, he has *only to compare* the heights of the lines just like the heights of pupils standing on level ground and thus make the inference. Tell pupils also that a clear understanding *of the above comparison would help them in practical* agriculture to suggest off-hand which of these varieties will produce more seedlings and which less, the germination capacity of the seed being the same, etc.

Propose to pupils to make linear representations of the type above made in regard to the following and get them to compare the quantities in each case as already described :—

1. The number of days in milk for certain cows reared in a farm is found to be as follows :—

Name of the cow.	Number of days in milk.
Lakshmi	249
Gouri	361
Tara	247
Padma	213
Sumitra	316
Indira	246
Ganga	289
Seetha	283

2. The yield of jaggery per acre obtained in a farm from different varieties of sugar-cane grown in plots of equal extent was as noted below :—

Variety.		Jaggery per acre.
B.	1,529	8,570 lb.
B.	208	10,000 „
B.	147	7,450 „
J.	36	9,225 „
B.	6,450	7,800 „
J.	247	9,750 „

3. The varieties of sugar-cane raised in a farm in one year are found to have the following average weights :—

Variety of sugar-cane.	Average weight of a cane.
B. ... 3,412	4·0 lb.
B. ... 147	5·1 „
B. ... 6,450	4·6 „
B. ... 376	4·6 „
J. ... 247	3·1 „
B. ... 1,529	3·5 „
J. ... 36	2·5 „
B. ... 208	3·8 „
Red Mauritius.	3·9 „

4. The money spent on different items of expenditure in raising Rasangi in a plot of 5 acres is noted hereunder:—

Preparatory cultivation	Rs. 30
Manuring	„ 50
Seeds and sowing	„ 20
After cultivation	„ 10
Harvesting, threshing, cleaning and storing	„ 49

5. The value of the milk realized from some milking animals during lactation period is given below :—

Name of the milking animal.	Value of milk.
A.	Rs. 156
B.	„ 204
C.	„ 167
D.	„ 147
E.	„ 112
F.	„ 174
G.	„ 134
H.	„ 284

6. Particulars are given below relating to experiments conducted on sugar-cane crop with the following manures :—

Name of manure and quantity applied per acre.	*Weight of raw-sugar per acre.*
Cattle Manure, 30 tons ...	9,355 lb.
Safflower cake, 4,009 lb. ...	12,671 „
Gingelly cake, 1,964 „ ...	14,966 „
Pungam cake, 3,229 „ ...	14,387 „
Groundnut cake, 1,640 „ ...	13,599 „
Castor cake, 1,992 „ ...	13,166 „

7. The yield per acre in lb. of different varieties of paddy tried in one season is as follows :—

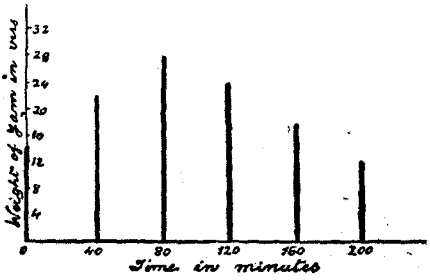
	Grain.	Straw.
Bangaru Theega ...	2,750 lb.	5,335 lb.
Konamani	2,950 „	4,595 „
Maharajabhogam ...	2,479 „	4,375 „
Mypali	2,358 „	4,483 „
Vazzanam	2,395 „	4,460 „

8. Manurial experiments with paddy give the following yield :—

	Yield of grain per acre.	Yield of straw per acre.
Green leaf manure...	3,628 lb.	5,916 lb.
Castor 5 bags per acre	3,304 „	4,640 „
Castor 5 bags and potash 1 cwt. per acre	3,060 „	3,748 „
Castor 5 bags and super-phosphate 2 cwt. per acre ...	3,832 „	5,848 „

Diagrammatic representation of the varying values of two quantities at a time.

Draw a diagram like the following on the graph board and tell pupils that the vertical lines represent the quantity in viss of yam available for sale in the vegetable market, Anakapalle, at different times one morning.



Ask pupils to suggest what quantity (by weight) of yam each vertical line may be taken to represent. Get them to grant a scale, say, of 1″ to 2 viss and let them give out the quantities of yam represented by the vertical lines. Next question what meaning they would assign to the inter-spaces. Point out that they denote intervals of time at which the gauging of the quantities was attempted.

Let them next assign some scale for reading out the intervals of time say of 1″ to 80 minutes. Get them to show the lines representing the following :—

At the time the observer first stepped into the market to ascertain the qnantity, it was so much, forty minutes hence it was so muoh, eighty minutes hence it was so much, hundred and twenty minutes hence it was so much, and so on. Next let them compare how the quantity of yam available changed from time to time. Get them to note the gradually increasing lengths of the first three vertical lines and the gradually decreasing lengths of the next three and thus infer that as time passed on there was a gradual increase in the quantity up to a particular moment and then there was a gradual decrease in the quantity. Impress on them the fact that quantities of varying values have been represented to scale in the diagram, horizontal distances representing periods of time starting from a particular moment and vertical distances representing the quantity of yam (in viss) available in the market.

Get pupils to suggest examples of similar varying quantities, e.g., the number of guavas picked from the neighbouring gardens during different weeks, the expenditure incurred in a house during different months, the quantity of paddy, ragi, or ganti which a ryot got from his field in different years, the price of jaggery during different weeks, the quantity of castor-cake which could be got for different sums of money, the number of plants or trees that could be raised in a plot at different distances apart, etc. Tell pupils that in all these cases a diagrammatic representation could be made of the values of the quantities as done above.

Get pupils to suggest, for example, the price of a maund of jaggery during different weeks in the Anakapalle market. Tell them that the price of a maund during the sixteen weeks beginning from the 8th December one year has been respectively as follows.

Week.		Price.			Week.		Price.		
		RS.	A.	P.			RS.	A.	P.
I	...	3	12	0	IX	...	3	4	0
II	...	3	10	0	X	...	3	0	0
III	...	3	8	0	XI	...	2	12	0
IV	...	4	0	0	XII	...	2	10	0
V	...	3	12	0	XIII	...	2	14	0
VI	...	3	14	0	XIV	...	3	4	0
VII	...	3	10	0	XV	...	3	8	0
VIII	...	3	8	0	XVI	...	3	12	0

Question pupils how they would proceed to represent the above details in a diagram. Lead them to suggest that different weeks may be represented along a horizontal line leaving a space of half-an-inch or so, for a week, and vertical lines may be drawn to represent the prices to scale, say, of 1 inch to Re. I. Point out that fractions of a rupee need only be approximately represented. Have a neat diagram prepared beforehand with the scales neatly written and the axes neatly named as "weeks" and "price of a maund in rupees" respectively and place it before pupils. Let them examine the diagram and note along which line they have to refer to the number of weeks and along which line for the price. Let them have a look at the diagram and then find out what they could read out from the diagram. Lead them to note the rise and fall of the price from week to week, when the price was lowest and when it was highest, when nothing definite could be said as to whether the price would rise or fall and so on. Tell them if necessary that if such a diagram is prepared with a price list for the sixteen weeks not only for the current year but also for the preceding years they are likely to find out that the price is lowest during a particular period of each year. Hence question pupils whether they could forecast for the forthcoming year at about what time the price is likely to fall so that they may purchase at the cheapest price then and also gauge at what time the price is likely to go high so that they may then sell the produce with profit. Pointing out the period when the price was lowest, get them to note that it corresponds to the period of *Makara Sankaranthi* when either out of keen need for money to spend for the festivity or out of hard pressure exerted by the sowcar to have loans repaid or because of the large quantities prepared for sale by the mill-owners during sugar-cane crushing season the jaggery owners

sell their stock cheap and hence they find the price then falls.

Point out that in the above diagram two quantities of different kinds have been represented, namely, the period of the year in weeks and the price that fluctuates and that these have been clearly written by the side of the horizontal and vertical lines along which they have been respectively represented. Point out also that along each line the scales to which the quantities have been represented are also clearly specified. Get them to note that the tips need not be shown as isolated points but may be joined by lines to indicate the rise and fall also, that the lines representing the prices need not then be drawn conspicuously thick and their tips alone need be shown by a cross-mark (×) or a circle.

Show pupils three diagrams of the following types :—

1. A diagram drawn so as to show the variation in the growth of a plant or of its root system in different soils in one season.

2. One drawn to represent the multiplication of moths or of stemborers in a field (A loan of these diagrams from the Assistant Entomologist at Samalkota would be helpful).

3. A chart representing the number of trees per acre corresponding to different spacings. Get pupils to read out these charts with reference to the scale used. If there is a maximum and minimum thermometer in the School, a shadow pole, a rain-gange, etc., let pupils gather particulars regarding the daily mean temperature, the rainfall during the several months and the length of the shadow at different times during the day and represent them in charts according to the above method.

Propose to pupils to chart the following data :—

1. The price per 100 of guavas has been as follows during the successive week-end market days in a village :— 9 as., 10 as., 8 as., 7½ as., 8½ as., 6½ as., 9½ as.

2. The expenditure in Gourisam's house came to Rs. 25, Rs. 28, Rs. 20, Rs. 45, Rs. 50, Rs. 40, Rs. 39, Rs. 52, Rs. 49, Rs. 37, Rs. 55, Rs. 50, respectively, during the twelve months of one year.

3. The price of a maund of ghee has been as follows on 8 successive Sundays at Vizianagram :—Rs. 12, Rs. 13 Rs. 12½, Rs. 14, Rs. 11, Rs. 11½, Rs. 12¾, Rs. 12¼.

4. The rain-fall in inches at Anakapalle has been as follows during 1916–17 :—

April	...	...	1·3″
May	...	...	2·11″
June	...	...	4·44″
July	...	...	8·49″
August	...	...	6·58″
September	...	...	6·58″
October	...	...	5·18″
November	...	...	·44″
December	...	...	·00″
January	...	...	·00″
February	...	...	1·55″
March	...	...	·00″

5. The number of trees that could be planted per acre at different spacings are as follows :—

			Number of trees per acre.
5 feet each away	...	...	1,742
6 do.	...	...	1,210
8 do.	...	...	680
10 do.	...	...	435
12 do.	...	...	302
15 do.	...	...	200
18 do.	...	...	135
20 do.	...	...	110
25 do.	...	...	70
30 do.	...	...	50
35 do.	...	...	35

6. The annual rain-fall at Anakapalle has been as follows during 8 successive years :—

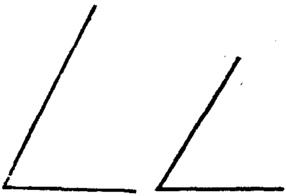
Year.			Rainfall in inches.
1913–14	...	...	36·62
1914–15	...	...	45·68
1915–16	...	...	50·23
1916–17	...	...	42·36
1917–18	...	...	41·84
1918–19	...	...	29·69
1919–20	...	...	44·21
1920–21	...	...	26·63

TOPIC 26.—ANGLES—(*contd.*)

(**Scope of the treatment.**—*The conception of angles as denoting the amount of turning and the comparison of angles by superposition have already been dealt with in a previous topic. The necessity for angle-measurement may be here explained and the use of "the right angle" and "the degree" as units for the purpose may be pointed out. How a protractor may be used for measuring angles and for setting them out may next be dealt with. In this connexion the use of angles to indicate direction and thereby to fix the position may slightly be touched upon. The angle-sum property of a triangle, the sum of the angles on one side of a straight line, the equality of vertically opposite angles, etc., may be incidentally dealt with in the course of exercises.*)

The adoption of the right angle and the degree as units in angle measurement.

Draw two unequal angles on the blackboard. Question pupils by how much one angle is greater than the other. Point out that in the case of two lines the comparison could easily be made by expressing the length of each line in terms of a chosen unit and by then stating that one line is greater than the other by so much. Thus get them to realize that in the case of two angles a similar comparison would be easy to make if they should express each angle in terms of a definite unit. Remind them of the right angle which has been referred as a quarter of the complete revolution and tell them that it is used as a standard for the purpose of comparison. Point out to them that the right angle is the simplest to make and the easiest to recognize and is hence chosen as the standard for comparison.



Propose to pupils whether by adopting the right angle alone as a unit they would be able to express exactly the size of different angles as marked below. Point out to them that they would then be able to only express that a particular angle is greater than, equal to, or less than a right angle but not to state definitely the size of each angle, or by

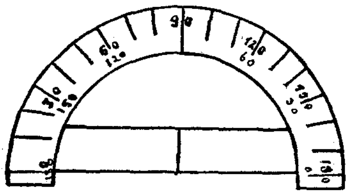
how much it is greater or less than another. Hence get them to note that the adoption of a unit smaller than a right angle would be convenient. Referring pupils to a clock face, point out to them how the minute-hand of the clock could be turned through various fractions of a right angle, e.g., half-a-right angle, $\frac{1}{3}$ of a right angle, 1/15 of a right angle, etc. (Do this by turning the minute-hand from the position of pointing towards XII to that of pointing towards III.) Tell pupils that just as a right angle has been obtained by taking a quarter of a complete revolution a smaller unit for angle measurement has been devised by setting out an angle indicating 1/360 of a complete revolution *and that this smaller unit is called* "a degree". If pupils question how such a unit has been obtained, point out to them how by dividing the rim of the circular clock-dial into twelve equal parts they could arrange to turn the minute or the hour hand through 1/12 of a complete revolution. Let them examine into how many equal parts the circular rim of the clock-dial is divided. Direct them to turn the minute-hand through 1/60 of a complete revolution. Thus get them to note that a similar division of the rim into 360 equal parts would enable them to turn the moving end of the minute hand through 1/360 of a complete revolution. If the pupils question how the division into 360 equal parts may be made, point out that they may measure the length of the rim with a string, then find the length of a part equal to 1/360 of the circumference by

a process of division and then set out such lengths successively along the rim. Point out to pupils that the division of an angle forming a complete revolution into 360 parts is tantamount to dividing each of the four right angles in it into 90 equal parts and hence a degree may be considered also as 1/90 of a right angle.

The protractor or the angle-gauge.

Next show pupils the figure of a circle formed by fitting together two semi-circular protractors. Ask them to show its centre. Direct them to examine the rim. Get them to note that the rim is then divided into 360 equal parts. Choose a *long thin needle* and *place it along* a radius at the base of one protractor with the blunt end coinciding with the centre. Rotate the needle. Let pupils read in each case through how many degrees the needle has been rotated. Set them next to rotate the needle through 1 degree, 5 degrees, 20 degrees, 30 degrees, 40 degrees, 80 degrees, 100 degrees, 140 degrees, 150 degrees, 170 degrees, 190 degrees, and 210 degrees, etc. Point out to them that they have thereby got a handy instrument by means *of* which the number *of degrees the needle* is turned through could easily be measured.

Next show one half of the above circle would easily come off by removing one of the two pieces fitted *together*. Let them note that it is semicircular in form and that its rim shows graduations into 180 degrees.



Point out to them that the angles they have to measure *are generally* less than half a *complete revolution* and hence such an instrument will do for them to measure angles. Tell them that the instrument is known as "a protractor" or "an angle-guage."

Let them next examine which line drawn from the centre of the protractor would make an angle of 1 degree,

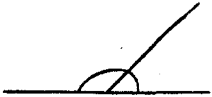
5 degrees, 15 degrees, 35 degrees, 80 degrees, 90 degrees, 120 degrees, 130 degrees, 170 degrees, with a radius at the base of the protractor. Impress on them that there are two readings given by the protractor and that they may be guided in particular cases by the angle made with the radius at the base which has to be measured. Get them to note from an examination of the graduations that the greatest angle that they could measure with a protractor is eqnal to one amounting to half a complete revolution, i.e., half of 360 degrees or 180 degrees and that in measuring a right angle they will have to read 90 degrees. Point out to them to remember that whenever an angle of 90 degrees is referred to they must take it as a right angle.

Exercises in angle-measurement.

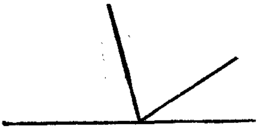
1. Ask pupils to draw several angles of various sizes and with the arms branching out in different directions both on the blackboard and on paper. Direct them to measure the angles drawn on the board with the help of a black-board protractor and those on paper with the protractor they find in the mathematical instrument box. As the semicircular protractor is hollow at the middle and creates some confusion amongst the pupils when giving out the readings the teacher will do well to see that celluloid protractors are supplied to the pupils. If any of the pupils should betray ignorance of the cipher graduation in the protractor, point out to him either end of the base as denoting the graduation. If the arms of the angles drawn on paper are found to be shorter than the radius of the protractor the pupils may complain that they are not able to read out the size of the angle while they used the blackboard protractor. In such cases suggest to them to produce the arms of the angles so as to project beyond the readings.) Point out that there are two numbers written against each graduation in the protractor and that they may use the readings clock-wise and counter-clock-wise according to the way in which the protractor is placed to measure the angles. Caution them to give out the readings correctly from the protractor and not to commit mistakes of the type of giving out the size of an angle of 60 degrees to be 120 degrees or vice versa. Should they commit mistakes of that type, lead them to compare the angle in question with a right angle and hence to decide the

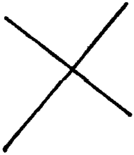
size to be 120 degrees if the angle be greater than a right angle or 60 degrees if less than a right angle.

2. Draw lines so as to form two angles on one side of a straight line. Let pupils measure the angles in succession and find their sum. Let them repeat the exercise with some more such lines. Get them to note that the two angles so formed are together equal to 180 degrees.

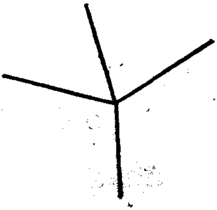


3. Draw lines so as to form three angles on one side of a straight line. Let pupils measure the angles in succession and find their sum. Let them similarly draw lines so as to form four or five angles on one side of a straight line. Let them measure the angles in succession and find their sum. Get pupils thus to note that *the sum of the angles so measured on one side of a straight line is very nearly equal to* 180 *degrees.*



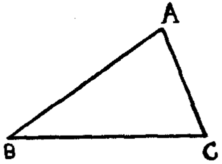


4. Propose to pupils to draw two straight lines to intersect at a point. Question how many angles are formed. Let them measure each angle and note which of the angles are equal.



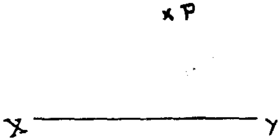
5. Let pupils next draw four or five straight lines emerging from a point. Get them to measure the angles between two lines taken consecutively. Let them find their sum and note that it is very nearly equal to 360 degrees.

6. Draw a triangle on the board. Shade it. Get pupils to note that its three bounding lines meet at three corners and thus three angles are obtained. Tell them that such a figure is called a triangle and is named by specifying the corners in an order, e.g., ABC, CBA, etc., in the accompanying figure. Let the pupils next measure these angles and find their sum. Direct them to repeat the above exercises with some more triangles. Lead them to conclude that the sum of the angles of a triangle is very nearly equal to 180 degrees, i.e., two right angles or half a complete revolution.



Drawing angles of given size.

Draw a line XY on the board. Next mark a point outside it. Propose to pupils that you would wipe out the point marked and that they may take the necessary measurements beforehand to mark the point again at the same spot after it is wiped out. Point out to pupils that if they should join P to X or Y and note down the size of the angle then formed at X or Y as well as the length of the line PX or PY they may be in a position to mark out the point P after it is wiped out. Get them to grant that the point P was above the line XY, the angle which the line XP made with the line XY was 60 degrees and the length of XP to 5 inches. Question them how they would then mark the position of P. Get them to feel the necessity for drawing a line making an angle of 60 degrees with arm XY at the point X. Let pupils suggest a method for drawing the line. If they are unable to suggest the correct method, point out how the semi-circular protractor may be placed so that the middle point of its base may fall on X and its base along the straight line XY,



Let them next decide which graduation in the protractor would indicate the line branching out from the centre and making an angle of 60 degrees with the base line. Let them mark a point against the graduation and join the point so marked to X. Question why the angle then formed may be taken to be exactly 60 degrees.

Propose some more exercises of the above type, e.g., setting out angles of 30, 40, 50, 135, and 150 degrees at points marked in given straight lines.

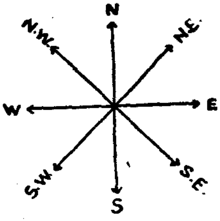
The alternate method of adjusting the base of the protractor so that a line drawn along it would give the line making an angle of the required size with a given straight line and at a given point in it need not be dealt with here.

It is enough if pupils have *notions* as to how angles of a given size may be set out.

The use of angles in setting out the points of the mariner's compass.

Show pupils a *magnetic* needle. Set it, if necessary to show the north-to-south line. Tell pupils that it is with the help of this line they could mark north-to-south lines anywhere on the ground. Next propose to them to draw from a point in the north-to-south line the lines which would indicate east and west. Question them what angle the latter lines should make with the former. Point out to them that the line to be drawn should make a right angle with the north-to-south line. Remind them about the use of a protractor to set out an angle of 90 degrees for the purpose. If pupils suggest that the protractor could not be used for setting out accurately angles of 90 degrees on the floor, suggest to them the use of the cross-staff to set out lines making right angles with a given line. Thus remind them of the method of marking out the four cardinal points of the compass. Next question how from the point of intersection of the above lines they would set out lines representing the directions north-west, south-east, north-east, south-west, etc. Lead them to note that they will have then to set out angles of 45 degrees, successively around a

common point already chosen. Question how this may be done. Get them to point out roughly the line which would make half a right angle with the north-to-south line at the point above referred to. Let them realize that if this angle could be exactly set out they would be able to mark correctly the eight points of the compass. Show them a mariner's compass.



Let them examine how the eight points have been correctly marked out there and let them also mark a diagram of its face by drawing the requisite angles on paper with a protractor. Show them also how a piece of paper could be folded so as to represent the four right angles formed around a point and how by folding the paper again they could divide the right angle into two equal parts and thus get the lines required.

TOPIC 27.—INVERSE PROPORTION.

(**Scope of the treatment.**—*At first some notions may be given of two quantities varying inversely with each other. It may be brought home to the pupils how as one of the quantities increases or decreases the other quantity decreases or increases. The method of working questions on inverse proportion may be explained. The method of unitary analysis alone need be adhered to, that by multiplying ratio being hard of comprehension by the pupils. Cases of joint variation, one direct and one inverse, may now and then be given so that the pupils may adopt the right method of reasoning and decide whether direct or inverse variation is involved in each case.*)

Ideas of quantities varying inversely with each other.

Propose to pupils that there is a certain quantity of manure in the manure-pit and that it is meant for distribution over 5 acres of wet lands or over 10 acres of wet lands. Question in which case an acre of land would be applied with more of manure. Get pupils to note that when there is only a fixed quantity of manure available the quantity applied for an acre would get less as the number of acres to be served increases.

Propose that there is some quantity of ragi in the store-house to be distributed equally among them. Question when they would be getting a larger quantity as their share, if it be distributed amongst 10 of them, or amongst 20 of them, or amongst 5 of them. Point out that as the number of boys among whom the quantity is to be distributed gets less, the share of each would increase and as the number increases the share of each would decrease.

Propose :—Once in four years a ryot will have 192 cart-loads of farm-yard manure at his disposal for distribution amongst his lands. If the manure be distributed equally over four acres of paddy lands, how many cart-loads would be spread over an acre? Answer this question taking that the manure is spread equally over 8 acres, 16 acres, and 24 acres, respectively.

Point out that as the number of acres becomes 2, 3, 4, n times as large the quantity of manure that could be spread per acre is reduced to half, one-third, one-fourth, 1/n respectively of the former quantity.

Next propose :—A ryot buys 20 bags of fish-guano. If he distributes the manure equally over ten acres of wet lands, how many bags of fish-guano would be used for an acre? Answer this question taking 5 acres, 2½ acres, respectively, instead of 10 acres.

Point out that as the number of acres becomes half, one-fourth, . . . 1/n of its former value, the supply of manure becomes twice, four times, . . . n times as large.

From the above questions get pupils to note how as the number of acres increases the quantity served over an acre diminishes and how as the number of acres decreases the quantity served over an acre increases.

Application of the method of inverse proportion to solving problems.

Propose :—A ryot has sufficient manure for 15 acres of ragi crop at the rate of 16 cart-loads for an acre. In how many acres can the same quantity be applied at the rate of 10 cart-loads per acre?

Question pupils whether the number of acres that could be served with the manure would increase or decrease when each acre is served with one cart-load instead of with 16 cart-loads and if so to find how many acres could be served. Get pupils to note that the number of acres that could be served would become 16 times as large, and therefore at one cart-load for an acre, 16 × 15 or 240 acres may be served. Next question whether the number of acres would increase or decrease when each acre is served with 10 cart-loads instead of with a single cart-load. Get pupils to derive that the number would become 1/10th small and therefore the number of acres which could be applied with manure at 10 cart-loads per acre is 240/10 or 24.

Tell pupils that while they argue as above to find the result required they must exhibit the method of working in a neat form by adopting the three steps written below

similar to what they have done in showing the solution of problems on direct proportion by the unitary method :—

At 16 cart-loads per acre, 15 acres could be served.
∴ At 1 cart-load „ 15 × 16 „
∴ At 10 „ „ 15 × 16/10 or 24 acres could be served.

Point out that in the above method the intermediate step has been written which gives the number of acres that could be served with the unit quantity of manure distributed per acre. Get pupils to note that a method of distribution as indicated in the second step is not adopted in the locality and may seem absolutely useless and even absurd and that it is only a supposition which is granted just to derive the third step. The teacher will do well to note that in working out questions on inverse proportion the method of unitary analysis need alone be adopted. The method of multiplying ratio, e.g., in the above question multiplying 15 by 16/10 need *not* be taken up for special treatment as it is found in practice to create some confusion among the pupils. If the pupils, instead of adopting the above method, follow another method, say, of calculating that the stock of farm-yard manure at the disposal of the ryot comes to 16 × 15 or 240 cart-loads and, then by dividing this by 10, get 24 acres as the result, let them adopt that method of working.

If the method of unitary analysis is adopted, caution them to consider with reference to each question whether the two quantities involved vary inversely or directly with each other and then adopt multiplication or division in the second step so as to suit the needs of the case. This caution is specially given here as pupils are likely to adopt division just as in the case of working questions on direct proportion and work, for example, in the above question that at 1 cart-load for an acre 15/16 acre alone could be served with the manure. It is only by repeatedly reminding about mistakes of this type that the pupils could be made to recognize whether a particular question involves direct or inverse proportion and then adopt the appropriate method accordingly.

Propose :—

1. The corn from a threshing ground could be removed to a cultivator's house in 12 cart-loads of 200

kunchams of paddy each. If the cart-man is prepared to carry only 150 kunchams in his cart, how many such cart-loads will have to be carried? (*Ans.*—16 cart-loads.)

2. A ryot has stored 120 bundles of fire-wood each weighing ½ cwt. How many bundles, each weighing ¼ cwt. could be made out of the quantity of fire-wood? (*Ans.* 240 bundles.)

3. If mango grafts are planted in a row, 12 rows would be planted out of a supply of some grafts. If 20 grafts be planted in a row, how many rows would be planted in a garden out of the same number of grafts? (*Ans.* 9 rows.)

4. Walking the distance from here to a cattle market at 3 miles an hour I could reach it in two hours. In what time could I reach the market from here if I get into a cart which moves at 4½ miles an hour. (*Ans.* 1 hour and 20 minutes.)

5. Four men and 8 women would pull out gogu plants from 2 acres of lands in a day. If the gogu crop from 20 acres of lands should be pulled out in 2 days, how many men and women should be employed for each day? (*Ans.* 20 men and 40 women.)

6. If I pay at 3 annas a day, the money at my disposal would allow to engage only 10 men-coolies for a day. If a woman cooly demands only Re. 0–1–6 a day, how many women coolies may I engage for a day with the same sum of money? (*Ans.* 20 women coolies.)

7. A cultivator finds that a cooly could thresh out 9 kunchams of paddy a day and so he thinks it would be enough if he engages 20 coolies for a day to thresh out the paddy he has. If the coolies are prepared to thresh out only 6 kunchams each a day, how many coolies should the cultivator engage in order to have the whole paddy threshed out in a day? (*Ans.* 30 coolies.)

8. Penning 4,000 sheep for a night is generally thought sufficient for manuring an acre. Only 500 sheep are available in a place. For how many nights should the flock be engaged in order that 12 acres may get sufficient manure? (*Ans.* 96 nights.)

9. To send rice bags from here to Samalkota the railway authorities allow us the use of wagons holding only

250 bags each. A merchant finds that 16 wagons have to be engaged for sending all the rice pounded in a mill. If wagons admitting only 200 bags each are spared, how many such wagons will have to be engaged?

(*Ans.* 20 wagons.)

10. With 10 ploughing pairs engaged each day, a ryot could have his lands ploughed in 8 days. If the ploughing should be finished in 5 days, how many ploughing pairs should he engage? (*Ans.* 16 ploughing pairs.)

11. To remove bricks from a kiln to a house under construction, 48 carts are engaged, assuming that a cart-load would come to 450 bricks. If a cart-load actually came to 360 bricks only, how many carts will have to be engaged extra? (*Ans.* 12 carts.)

12. Sowing at 20 kunchams of paddy seed for an acre, the ryots in those days found that the seed stored in some *mudikattus* could be used for sowing only 6 acres. If at the advice of the Agricultural Department he finds 5 kunchams will do for an acre, how many acres could he now sow with an equal quantity of paddy?

(*Ans.* 24 acres.)

13. Expecting that an acre of land yields at the rate of 6 puttis of ragi, a ryot calculates that the produce of ragi he may get from 2 acres during a panta would last his family for a year. If he actually finds that his lands yield only at the rate of 4 puttis an acre, the produce from how many acres should he set apart for the use of his family? (*Ans.* 3 acres.)

14. By selling jaggery at Rs. 3-8-0 a maund, a ryot finds that he would be able to pay up his dues to the sowcar by disposing of 30 maunds. If he finds the price to be Rs. 3 a maund at the time of the sale, how many maunds of jaggery should he then sell in order to clear his dues? (*Ans.* 35 maunds.)

15. From a *ganuga* could be extracted 20 seers of oil, if 3 bullocks are engaged to work at it during a day. If 100 seers of oil should be extracted in 3 days, how many bullocks should be engaged daily at the above rate?

(*Ans.* 5 bullocks.)

16. By planting 25,000 sugarcane sets an acre, a ryot found that the canes at his disposal would be sufficient only for 8 acres. If at the advice of the Agricultural

Department he finds that 10,000 sets will do for an acre how many acres may he expect to plant with the stock at his disposal? (*Ans.* 20 acres.)

17. In one ounce weight of "Mypali" paddy there are 1,040 grains and in one ounce weight of "Gunupur Sannelu" there are 1,501 grains. To sow plots of equal area so as to get the same plant-rate per acre, which of the seeds would you require in a larger quantity by weight, Mypali or Gunupur Sannelu, other conditions remaining the same such as germination, capacity, method of sowing, etc.?

18. At 20 kunchams per bag 32 gunny-bags are required to send some quantity of paddy. At 16 kunchams a bag how many bags will be required?

(*Ans.* 40 bags.)

19. At 300 palmyrah leaves for a "gathi" (a small room in a house) thatched roofing could be put up over 4 gathis in a house out of some cart-loads of palmyrah leaves. At 400 leaves a gathi, how many gathis could be covered over with thatched roofing out of the same number of palm leaves? (*Ans.* 3 gathis.)

20. B. Hanumantha Rao could get on lease the sugarcanes from 7½ acres with the money at his disposal paying at the rate of Rs. 500 for one acre. At Rs. 600 for an acre, how many acres of sugarcane could he get on lease for the same amount? (*Ans.* 6¼ acres.)

TOPIC 28.—DECIMALS—(*contd.*)

(**Scope of the treatment.**—*Multiplication and division of one decimal by another are expected to be treated here in an elementary form. Practical problems may be given, e.g., in finding the areas of rectangular fields in acres and cents when the dimensions are given in chains and decimal of a chain, in finding the areas of figures when their length and breadth are given in inches and tenths of an inch, in finding the yield per acre, when the yield per plot is given as is found necessary in the preparation of cultivation sheets in the farm, etc. As most of these problems could be worked even without a knowledge of the methods of multiplication and division of one decimal by another it may not be thought quite essential that the processes should be taught to the pupils at all. Still, as pupils may have later on to deal with the calculation of areas from lengths obtained from figures, the percentage composition of fertilizers, and the quantity of juice and jaggery in cane, etc., and as these calculations require a knowledge of decimal multiplication and division, these processes may be taught. After the multiplication and division of one decimal by another have been treated, the conversion of vulgar fractions into decimals and of decimals into vulgar fractions may be taken up, especially with fractions having the denominators,* 2, 4, 5, 8, 16. *The teacher will do well to restrict himself as far as possible to giving problems requiring products or quotients up to two or three places of decimals.*)

Decimal multiplication.

Propose :—A rectangle is 3·4 inches long and 2·7 inches wide. What is its area ?

Question pupils how they would proceed. Get them to note that the area will be given by 3·4 × 2·7 square inches, applying the formula giving the area when the length and breadth are known. Question how they would find the value of 3·4 × 2·7. Distribute among the pupils squared sheets of paper ruled into tenths of an inch, ask them to mark the corners of the rectangle 3·4″ by 2·7″

and get them to find its area. Point out to them that each small square in the squared paper is one-hundredth of a square inch. Let them count the number of complete square inches, find them to be 6 and then add the number of small squares, viz., 80 plus 28 plus 210 or 318 squares. Let them thus note that as these 318 small squares make up 3 square inches, and 18 hundredths of a square inch the entire area is 6 plus 3·18 or 9·18 square inches. Let them therefore deduce that the product of 3·4 and 2·7 is equal to 9·18. Tell them that the above result could also be obtained as follows :—

$$3{\cdot}4 \times 2 = 6{\cdot}8$$
$$3{\cdot}4 \times 7 = 23{\cdot}8$$

$\therefore\ 3{\cdot}4 \times {\cdot}7 = 3{\cdot}4 \times 7 \times 1/10 = 2{\cdot}38.$

$\therefore\ 3{\cdot}4 \times 2{\cdot}7$ is equal to 6·8 plus 2·38 or 9·18.

Here make pupils realize that the multiplication 3·4 × ·7 means the taking of a part of 3·4, the part here concerned being ·7 or 7 tenths.

Propose :—By the above method or by drawing rectangles on squared paper and counting the number of squares, find the areas of the following rectangles :—

1·8″ by 1·4″, 4·4″ by 2·8″, 1·4″ by 1·4″.

Let pupils write down the product in each case.

Let pupils next find the products of 18 & 14, 44 & 28, 14 & 14, 34 & 27. Let them compare those products with those of 1·4 & 1·8, 4·4 & 2·8, 1·4 & 1·4, 3·4 & 2·7, etc., already obtained. Lead them to note that the same digits occur in the same order in both the multiplicand and the multiplier and they are found to be so also in the product, and that a difference is noticeable in the number of decimal places. From the results got hitherto by multiplying one decimal by another, let pupils fill up the following table :—

Number of decimal places in the multiplicand.	Number of decimal places in the multiplier.	Number of decimal places in the product.

Get them to note that the number of decimal places in the product may on comparison be found to be equal to

the sum of the number of the decimal places in the multiplicand and of that in the multiplier. Hence let them deduce the following method of decimal multiplication :—

Treat both the multiplicand and the multiplier as if they are whole numbers, and then multiply ; after getting the product, insert the decimal point in the right place in it, remembering that the number of decimal places in it should be equal to the sum of the numbers of decimal places in the multiplicand and the multiplier.

Propose :—

1. Find the areas of the following rectangles :—

4·5″ by 1·6″; 5·4″ by 1·2″; 12·55 chains by 1·8 chains; 7·5 chains by 4·4 chains; 7·2 chains by 4·5 chains; 8·23 chains by 3·2 chains; 6·4 chains by 2·5 chains; 7·23 chains by 2·4 chains; 5·64 chains by 4·5 chains; 8·5″ by 8·4″; 7·5″ by 6·8″; 9·1″ by 1·9″; 4·5″ by 3·2″; 6·4″ by 5·8″ and 8·1″ by 1·8″.

(Change the data in the above if more questions are wanted for purposes of drill.)

(*Ans.* 7·2 sq. ins.; 6·48 sq. ins.; 2·259 acs.; 3·3 acs.; 3·24 acs.; 2·6336 acs.; 1·6 acs.; 1·7352 acs.; 2·538 acs.; 7·14 sq. ins.; 51 sq. ins.; 17·29 sq. ins.; 14·4 sq. ins.; 37·12 sq. ins.; 14·58 sq. ins.)

2. Experiments show that gold weighs 18·6 times as much as an equal quantity of fresh water, silver 10·4 times, sand 1·88 times and copper 8·6 times. If a certain quantity of fresh water weighs 2·3 chinnams, find the weight of an equal quantity of the above substances.

(*Ans.* Gold 45·78 chinnams, silver 23·92 chinnams, sand 4·324 chinnams, and copper 19·78 chinnams.)

3. It has been found that the quantity of nitrogen is ·003 lb. in 1 lb. of cowdung, ·0069 lb. in 1 lb. of sheep manure and ·076 lb. in 1 lb. of groundnut cake. Find the amount of nitrogen in a cart-load of each manure.

(Take a cart-load to be equal to 800 lbs. at Anakapalle.)

(*Ans.* 2·4 lbs. in cowdung, 5·52 lbs. in sheep manure and 6·8 lbs. in groundnut cake.)

4. In a map a length of 1 inch represents 2·5 miles. Find the actual length represented by a road which is 4·5 inches long in the map. (*Ans.*—11·25 miles.)

Decimal Division.

Propose :—In the Government Farm it has been found that a plot of 1·4 acres yields 336 kunchams of paddy. Find the yield per acre at this rate.

Get pupils to note that in this question they have to divide 336 by 1·4. Get them to estimate roughly how many kunchams they expect. Point out that as 1 acre is less than 1·4 acres they must expect something less than 336 kunchams.

Get them to work that 4 acres equals 140 cents, therefore the yield from 1 cent is 336/140 kunchams and that from 100 cents 336/140 × 100=33,600/140 or 3,360/14, *i.e.*, 240 kunchams.

Point out to pupils that the above results may also be obtained by expressing 336/1·4 as 3,360/14, *i.e.*, with the divisor an integer, by multiplying both the dividend and the divisor by the same number 10 and then working out the quotient.

Propose :—

From a plot 2·6 tons of cane was obtained and this gave ·325 ton of jaggery. How many tons of jaggery could thus be got from one ton of cane ?

Point out that here ·325 has to be divided 2·6, that this division may be replaced by the division of 3·25 by 26 and hence the quantity required may be worked out. Get pupils to work on the blackboard.

```
        ·125
   26) 3·25
       2·6
       ----
        ·65
        ·52
       ----
        ·130
        ·130
       ----
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Propose :—

1. The area of a rectangle is 2·88 sq. ins. If the length is 1·8 ins., what is its breadth ? (*Ans.* 1·6 ins.)

2. The area of a field is 2·56 acres. Its length is 32 chains. What is its breadth ? (*Ans.* ·8 chain.)

3. 6·5 tons of cane have been found to produce 1,820 lb. of jaggery. Find how many lb. of jaggery could be got from 1 ton of cane. (*Ans.* 280 lbs.)

4. 2·36 tons of cane give 3,172 lbs. of juice and ·295 ton of jaggery. Find to the nearest lb. the yield of juice and jaggery per ton of cane.

(*Ans.* 1,344 lb. of juice and 280 lb. of jaggery.)

5. The following particulars have been taken from the cultivation sheets of a farm :—

(a) Area of plot.		Name of crop.	Cost of production per plot.
			RS.
·8 acre	...	Pyru ragi	72
2·6 acres	...	Pyru gingelly	54
1·8 acres	...	Gingelly	22
1·1 acres	...	Sugarcane	117
1·4 acres	...	Maharajabogham	47
1·5 acres	...	Vazzanam paddy	49

From the above calculate to the nearest rupee the cost of cultivation of each crop per acre.

(*Ans.* Rs. 90, Rs. 21, As. 12. Rs. 106, Rs. 34, Rs. 33.)

(b) *Area of plot.*		*Yield of crop.*
(*a*) ·57 acre	...	1,755 lbs. of Vazzanam paddy and 3,110 lbs. of straw.
(*b*) ·24 acre	...	516 lbs. of Maharajabogham and 701 lbs. of straw.
(*c*) 1·25 acres	...	2,006 lbs. of paddy and 2,516 lbs. of straw.

From the above calculate to the nearest lb. the yield of crop per acre.

(*Ans.* (*a*) 3,079 lbs. of paddy 5,456 lbs. of straw; (*b*) 2,150 lbs. of paddy; 2,912 lbs. of straw; (*c*) 1,605 lbs. of paddy 2,013 lbs. of straw.)

(c) Area of plot.	Value of produce.
	RS.
2·66 acres of sugarcane	1,573
·75 acre of gingelly	38
1·46 acres of pyru gingelly	19
·57 acre of gingelly	24

From the above calculate to the nearest rupee the value of produce per acre.

(*Ans*. Rs. 591, Rs. 51, Rs. 13 Rs. 42.)

(*d*) Name of crop.	Weight in lbs. of a kuncham of crop.
Pyru gingelly	6·8
Pyru chodi	7·3
Bungara thiga	6·3
Navakoti sannellu	6·5
Ganti	7·5
Bobbili ganti	6·25

Find how many kunchams of each produce there will be in a busta holding 166 lbs. (Answer to the nearest tenth of a kuncham.)

(To express the results approximately as required, get pupils to compare the divisor and the remainder at the last step and apply the convention regarding approximation as to whether the remainder is equal to, greater than, or less than, half the divisor).

(*Ans*. 24	kunchams of	Gingelly
23	„	Chodi
26	„	Bungara thiga
26	„	Navakotisannellu
22	„	Ganti
27	„	Bobbili ganti.)

6. In crushing 16·5 puttis of cane 4·95 puttis of wet megass (Thadipippi) were obtained. Find to the nearest lb. the weight of megass given by a ton of cane.

(*Ans*. 672 lbs.)

7. Express $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{4}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{16}$, $\frac{3}{16}$, $\frac{5}{16}$, $\frac{9}{16}$, $\frac{11}{16}$, $\frac{15}{16}$, $\frac{7}{16}$, into decimals.

(*Ans*. ·5, ·25, ·125, ·75, ·375, ·625, ·875, ·0,625, ·1,875, ·5,625, ·6,875, ·9,375, and ·4,375.)

(Get pupils to adopt the process of decimal division and find the equivalents. Incidentally point out that some fractions like $\frac{1}{3}$, $\frac{2}{7}$ etc., would lead to non-terminating decimals and that if they have to deal with them it is enough if their values are expressed correct to 2 or 3 places of decimals).

8. Express ·75, ·125, ·375, ·95 as vulgar fractions.

(Get pupils to note that ·75 = 7 tenths plus 5 hundredths
= 7/10 plus 5/100
= $\frac{75}{100}$

In this way let them find the equivalents of the other decimals. (*Ans.* $\frac{3}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{19}{20}$.)

9. The price of a ton of fish guano is Rs. 120. It contains ·08 ton of nitrogen. Find how much of the fertilizer would contain 1 ton of nitrogen and calculate its worth at the above rate. (*Ans.* 12·5 tons, Rs. 1,500.)

10. A putti of cane contains 160 canes and weighs 500 lbs. Find the weight of a cane in lbs.

(*Ans.* 3·125 lbs.)

11. A putti of cane gives 5 pots of juice (*Aithu Kadavalu Rasam*). A pot holds 1·4 maunds. Find how many puttis of cane will give 2 maunds of the juice.

(*Ans.* ·28 putti.)

12. A length of 2·4 inches on a map has been found to correspond to 75 miles. How many miles will 1″ on the map represent? (*Ans.* 31·25 miles.)

13. A length of 6·5 miles is represented on a map by 1·9″. What actual length will 1″ on the map represent?

(*Ans.* 3·4 miles.)

TOPIC 29.—TRIANGLES AND THEIR CONSTRUCTION.

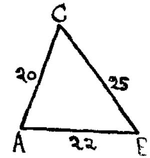
(**Scope of the treatment.**—*The pupils have already got notions of a triangle and of its angle-sum property. Here a few elementary cases of triangle construction may be taken up :—(1) by measuring the three sides, (2) by measuring two sides and the angles between them and (3) by measuring one side and the angles on either extremity of it. Special importance may be given to the first of the three methods of construction above specified as it admits of very easy and correct reproduction. The fact must be impressed that duplicate triangles would be got in each method of construction above. The pupils may be given some notions as to why three measurements are necessary in each case above. The formation of equilateral, isosceles and similar triangles and a study of their properties may be incidentally taken up. The method of constructing right-angled triangles may be dealt with as a special case of the method of construction (2) above. A few important practical bearings of the above constructions may be shown. It may also be described how triangles may easily be set out on the ground by measuring the perpendiculars to a side and the segments into which the side is divided by the foot of the perpendicular. Preliminary work of this kind would conduce to quick and correct understanding of the methods of plotting fields to be dealt with later on.*)

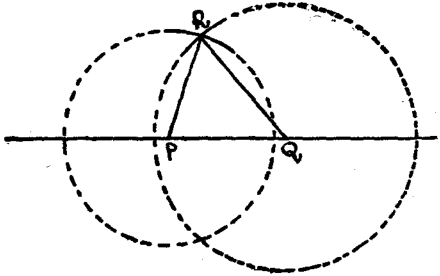
Triangles and their construction.

Propose to pupils that there is a triangular plot of ground adjoining Gourisam's house or field belonging to another and that Gourisam asks for the ground being given to him and in return promises to make over to its owner a plot of the same size and shape elsewhere. If the owner consents, question pupils what measurements Gourisam should take to mark out the plot required. Let them suggest various measurements being taken, e.g., of the sides, angles, etc. Tell them that the propriety of the measurements and their use in setting out the plot required will presently be considered.

(a) Setting out a triangle with the measurements taken for the three sides.

Mark out a triangular plot ABC on the school floor or in the open yard in front. Let pupils measure the length of its sides and note the measurements taken against the corresponding sides, say, as 22 links against AB, 25 links against BC and 20 links against CA. Propose to them that by using the measurements so taken they should try to set out another plot of the samo size and shape somewhere near. Question pupils how many corners will have to be fixed in order that the plot may be sot out. Get from them that three corners have to be set out and that the triangle could then be obtained by joining the corners by straight lines or by raising straight bunds, etc. Pointing out two of the corners, question pupils how they would lay them out on the floor. Lead them to stretch a chalked string straight on the ground, mark any point in it near one of its extremities and starting from there set off along the line a length PQ equal to AB. Question them how many corners have been fixed thereby.

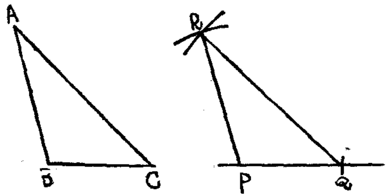




Let them mark out distinctly the two corners obtained thereby. Next question them how far the third corner should be from P and hence along what line they may expect it to lie. Remind them of the notion they have obtained while studying about circles that a point 20 links from P should be somewhere in the circumference of a circle drawn with

P as centre and with a radius of 20 links. Get them to describe the circle by means of a string and a stick-chalk. Question them if they could mark out any point in its circumference and choose that as the position of the third corner. Lead them to note that there is yet another condition to be satisfied, viz., that the corner must be at a particular distance, 25 links, from Q. Next question them where they would seek for the point which is 25 links from Q. Get them to realize that a point satisfying this condition should be somewhere in the circumference of a circle drawn with Q as centre and with radius 25 links. Ask them to describe the circle and find out where they would fix the position of the third corner. Lead them to infer that the place of inter-section of the two circumferences is at the specified distances from P and Q and that it may hence be chosen as the position of the third corner required. Let them name it R, join it to P and Q by stretching a chalked string as before and thus note that a plot has been set out with the help of the measurements taken. If they should suggest that the circumferences intersect in another point also, tell them that the second point of intersection may also be taken to represent the position of the third corner and another plot set out having the same measurements. Let pupils next compare each of the plots so set out and the original and examine if there is equality in size and sameness in shape. Get them to infer by intuition that the above properties exist. To convince them that the plot so set out is a duplicate of the original, adopt the following indoor-work:—

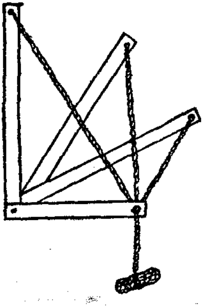
Ask pupils to draw any triangle ABC on the blackboard or on paper and propose to them to set out another triangle PQR whose sides are equal to those of the triangle ABC. Get them to adopt the method of construction



already indicated, viz., of drawing any straight line, stepping off with compasses a segment PQ along it equal to BC, drawing circles with centres P and Q and with radii equal to BA, CA, respectively, marking a point of intersection R and joining RP and RQ. Let pupils next trace the triangle PQR and place it over the triangle ABC and see if the two triangles would exactly coincide with each other.

Get them to note that the angles in the triangle PQR are found to be respectively equal to those in the triangle ABC and that the triangles are of the same size and shape, in other words, that the triangle newly constructed is a duplicate of the original triangle. Point out also that all of them adopted the same method and got duplicates of different triangles and that therefore the above method of construction would invariably give duplicate triangles. Thus make them realize that the triangular plot set out on the ground by the above construction may be taken to be of the same size and shape as the original.

Next propose to the pupils if they find it necessary to draw in full the circumferences of the two circles and let them suggest which part of each circumference they would consider to be quite essential. Thus lead them to note that it is enough if they draw parts of the circumferences where they expect the third corner to lie and that they would thereby save much time and space.



Taking a strip of cardboard hinged together at one end and with the other ends joined by means of a string stretched straight, show that several triangles, differing in size and shape, are possible of construction when only the lengths of the two sides are chosen for laying out the triangle and that it is only when the measurement for *the third side is also chosen* for setting out the triangle they obtain a fixed size and shape for the triangle.

Next propose to pupils to construct triangles by the above method :—

(*a*) with sides 2·5″, 2·2″, 2″

(*b*) with sides 2″, 4″, 3″

(*c*) with each side 2″

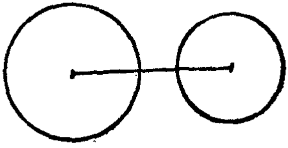
(*d*) with each side 1·5″

(*e*) with two sides, each 3″, and the remaining side 1″.

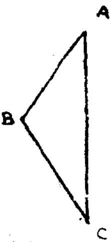
(*f*) with two sides, each 3·5″, and the remaining side 4″.

Let them measure the angles in each case. Get them to note that, in (*c*) and (*d*) above, the triangles have the three sides equal as also the three angles, that in (*e*) and (*f*) the triangles have two sides equal as also the angles opposite to them and that in (*a*) and (*b*) the triangles have the three sides unequal as also the three angles. Tell them that the triangles in (*c*) and (*d*) above are called "*equilateral*" triangles or triangles with all sides equal and those in (*e*) and (*f*) above are called "*isosceles*" triangles or triangles with two sides equal.

Propose to them to attempt the construction of a triangle, say, with sides 3″, 1·2″ and 1″. Let them actually attempt the construction and find out that there is no triangle formed. Question why the construction fails. Get them to note that the



circumferences of the circles they describe with a view to get the third corner of the triangle lie distinctly apart from each other without intersection. Point out to them that in order that the circumferences may intersect the lengths of two sides must be together greater than the length of the remaining side. Refer to the fact that if any one should go from a corner A to a corner C of a triangle ABC the shortest path is along the side AC and that he would be traversing a decidedly greater distance if he should choose to go from A to B and then from B to C.



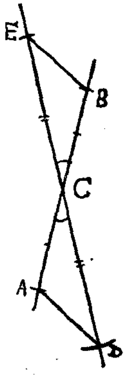
(b) Some alternate methods of triangle-construction.

Refer pupils to the triangles constructed up till now and get them to note that a triangle of the same size and shape as another has been constructed by taking the measurement for the three sides alone. Tell pupils that other measurements may also be chosen for triangle construction, say, (1) the lengths of two sides and the angle between them (2) the length of a side and the angle at its extremities and so on.

(1) Setting out triangles with the measurements for two sides and the angle between them being taken.

Show pupils the pair of narrow card-board strips hinged at one end already brought in as an illustration. Get them to rotate one strip around another and point out that if the angle between the two strips be known as well as their lengths the position of the moving ends could be fixed and thus a triangle of the given measurements obtained.

Ask pupils to draw two straight lines ACB, DCE intersecting at C. Get them to note that the angles ACD and ECB are equal, being vertically opposite angles. Let them step off equal lengths along CA and CB. Let them do like-wise along CE and CD and thus obtain triangles like ACD and ECB in the figure. Get them to compare the two triangles and express which sides and which angles are equal from the construction above adopted. Ask them to trace one of the triangles and see if it could be placed over the other so as to exactly coincide with it. Thus lead them to note that the two triangles are duplicates of each other. Point out also that if out of the three measurements chosen one be left out of account the triangles would not be duplicates of each other.

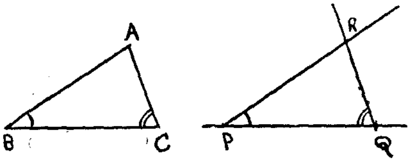


Referring to a triangular plot in the neighbourhood, question pupils whether the above method may be followed in setting out a plot of the same size and shape elsewhere. Lead them to grant

the possibility of the construction but point out that they would find it difficult to measure exactly the angle between two edges of the plot and to set them out correctly elsewhere.

(2) Laying out triangles with the measurement of one side being taken as well as of the two angles at each extremity of it.

Draw a triangle ABC on the board and propose to



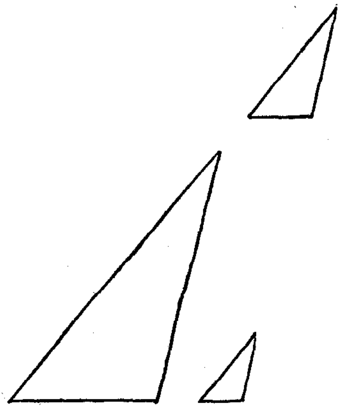
pupils to attempt the construction by measuring a side BC and the angles CBA and BCA at its extremities. Lead them to lay out a straight line hard by and stepping off along it a segment PQ equal to BC. Get them to note that two corners P and Q have been thereby fixed, corresponding to the corners B and C in the original triangle. Question them in what direction a person remaining at P facing PQ should turn so as to look towards the third corner required. Point out that he will have to turn through an angle equal to CBA. Set them to measure the angle CBA and lay out an angle QPR of equal size at P in PQ. Question if they could mark out the position of the third corner anywhere in the arm so set off. Get them to note that there is yet another condition to be satisfied, viz., that the angle PQR must be equal to BCA. Lead them hence to measure the angle BCA and to set off an angle of equal size at Q in PQ and on the same side of it as PR. Lead them then to decide the position of the third corner R to be the point of intersection of the arms newly set off. Get them to trace the triangle PQR and find by superposition that the triangle PQR is almost of the same size and shape as the triangle ABC. If any pupils should suggest that the triangles do not exactly coincide with each other,

point out that there has been some defect in the measurements of the angles and in setting them off and that still they may take the triangles to be very nearly equal.

Point out that by the above method of construction they could fix the position of the seat of any boy in the class-room or of any spot in the street or field by setting out angles at the extremities of a base-line chosen and also set out triangular plots of the same size and shape as another. Only get them to note that they would experience difficulties in measuring angles exactly and in setting them out correctly on the floor or on the ground.

Similar triangles.

Propose to pupils to construct triangles with two angles



given, say, to be 60° and 70° and choosing different lengths for the side forming the common arm. Let each pupils draw two or three triangles of the above type, measure the other sides and angles and note there is difference in the length of the sides but there is equality among the corresponding angles. Let them also note that the triangles are different in size but look like each other

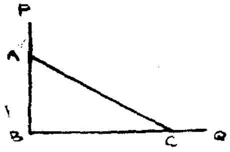
in shape. Tell them that such triangles are said to be '**similar**' to one another.

Next propose to pupils that there is a triangular plot of sides 250 links, 220 links, and 200 links, respectively, and they should get a representation of it made on paper. Get them to note that owing to want of space lengths like 250 links, etc., could not be set out in full size on paper and that they may be represented to scale. Get pupils to draw the triangle to different scales, taking 1″ for 100 links, 1″ for 50 links, etc. Let them compare the angles of the several triangles as obtained and find which angles are equal. If the pupils do not get quite accurate results, make them realize that there may be mistakes in setting out lines and angles and still they will find that corresponding angles are very nearly equal. Get them to compare the shape and thereby infer that the triangles are all similar in shape and each triangle drawn on paper is similar to the original triangular plot. Question pupils whether any relation may be established between the sides. Point out that a side of 25 links corresponds to a side of 2·5″ in paper, one of 22 links corresponds 2·2″ and one of 20 links to 2″. Lead them to realize that there is proportionate representation of the lengths of the sides, viz., of 1″ to 100 links. Let them next examine whether any proportion is observed between the sides of the similar triangles they have drawn. Get them to note that such a proportion exists.

Right-angled triangles.

Propose :—

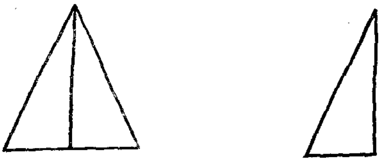
Construct a triangle with one angle a right angle. Get a duplicate of it taking the measurements for the sides forming the arms of the right angle.



Tell pupils that this construction is similar to (*b*) (1) above and that in setting out the right angle they may use

the set square or the try-square. After the pupils have drawn the figure, get them to note that they have all got duplicate right-angled triangles. Incidentally point out to them that the right angle is the greatest angle in the triangle and the side opposite to it the greatest side. Let them note this by comparing the lengths of the sides with a pair of dividers. Tell them that the longest side of the right-angled triangle is called its '**hypotenuse.**'

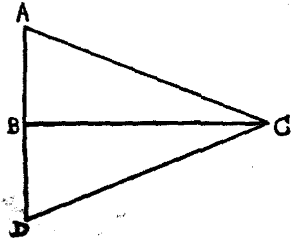
Cut out a sheet of triangle with its two edges



equal. Fold it along the corner common to these two edges so that one of the two sides falls along the other. Unfold the sheet. Into how many pieces is the sheet divided by the crease-line?

(Get pupils to note that the triangle appears to consist of two parts looking exactly like each other on either side of the crease-line, that the crease-line is perpendicular to the third edge and meets it at its middle point and that the whole sheet looks as if two duplicate right-angled triangles were placed side by side along one arm of the right angle.)

Hence get pupils to note that if two duplicate right-



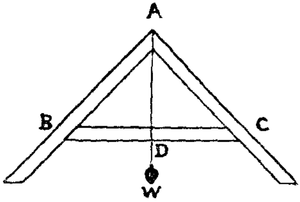
angled triangles ABC, DBC be placed along BC opposite

to each other as in the above figure they will find ABD in a straight line as well as AC equal to DC.

Some practical applications of the above properties.

(1) The method of testing levels.

Show pupils the "**Mattom**" used by the **masons.**

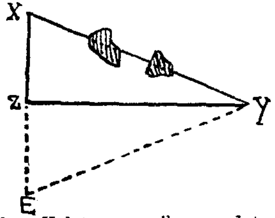


Let them note that the angle between its legs is a right angle. Let them place the mattom so that, the plumber (see figure) hangs freely with the plumb-line AW along the groove marked in the cross-piece. Get them to realize that the plumb-line is vertical and hence the cross-piece is horizontal, being perpendicular to the plumb-line, and also that the ends of its legs are in one and the same horizontal line.

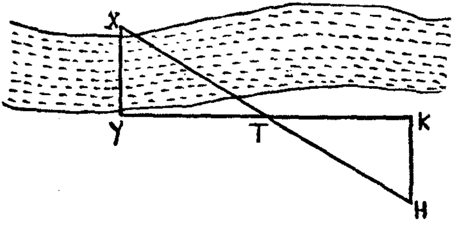
Explain to them how the mason tests whether the surfaces of two bricks he places while building a wall are on the same level or not, and how he judges of the correct horizontal position by seeing that the ends of the mattom held as above described fit in with the surface of the bricks without causing a changed position for the plumb-line. Let pupils place the ends of the mattom over stones, spots, etc., which are not on the same level, observe that the plumb-line is then disturbed from its position along the groove and made to occupy quite a different position, thereby indicating that the line joining the ends of the legs of the mattom do not occupy a horizontal position.

(2) Measuring out distances when there are obstructions.

(*a*) Suppose the distance between the two places X and Y in a garden has to be measured when there are obstructions like prickly pears, etc., in the straight line joining them. By using a cross-staff direct pupils to set out any right-angled triangle XYZ on that side of the garden where there is space. Starting from X let one pupil proceed to Z and count the number of paces he has walked. Let him next continue walking in the same direction until a place E is reached which is as far from Z as X. Direct a third pupil to measure the distance between E and Y. Get pupils to note that EZY and XZY are two duplicate right-angled triangles and hence they may conclude that the distance between X and Y may be derived by finding the distance between E and Y.



(*b*) Suppose the distance across a river has to be

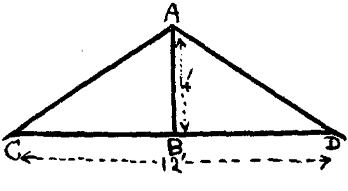


measured without crossing. Take pupils to the Sarada river close by. Let then pitch upon a tree or any spot X on the opposite bank. Let a boy stand at a place Y exactly opposite to X and walk straight along the bank, say, 100 paces from Y to T perpendicular to XY. Fix a flagstaff at T. Let him continue walking in the same

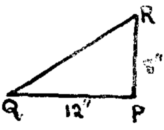
direction another 100 paces until a point K is reached. Let him next walk in a direction perpendicular to TK away from the river and pitch upon that spot wherefrom he is able to sight the flagstaff already planted at T in a line with the spot or tree X pitched upon in the opposite bank. Let him name this spot H and then measure the distance of that spot from K and take it to be the breadth of the river. Question why the length so measured may be taken to be the breadth of the river. Lead pupils to note that here again two duplicate right-angled triangles have been set out and hence they may conclude their procedure to be correct.

(3) Gauging the length of rafters required in putting up gabled roofing.

Propose to pupils how they are to ascertain the length of each rafter required for putting up a gabled roof. Roughly sketch out a diagram like the accompanying one on the blackboard. Get them to suppose that the width of the gable CD is 12 feet. Then tell them that the carpenter lays out the height AB to be invariably ⅓ of the width, 4 feet in the above case. Let pupils point out which line would then represent the rafter. Get them to note that if they should know the actual length represented by AC they would be able to get the length of the rafter in question. Point out that the triangles ABC and ABD are duplicate right-angled triangles, CB is half the length of CD, i.e., 6 feet, in the above case and that in order to find out the length of AC they have only to determine the hypotenuse of a right-angled triangle in which the sides containing the right angle are 6 feet and 4 feet. Point out that the length of AC may therefore be found out by setting out a right-angled triangle on the floor with the sides containing the right angle 6 feet and 4 feet in length

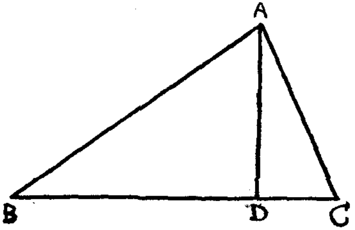


and then measuring the hypotenuse AC. Point out to them also that in case there is not sufficient space to set out the right-angled triangle in full size they may also set out a similar right-angled triangle to a smaller scale on sand, e.g., a right-angled triangle RPQ with PQ 12 inches, RP 8 inches and angle RPQ a right angle. Remind them of the proportionality between the corresponding sides. Get them to note that PQ represents CB to a scale of 2 inches to 1 foot and RP represents AB in the same proportion. Let them therefore conclude that RQ represents AC in the same proportion. Direct them to measure RQ from a figure. Let them note that RQ is nearly 14 inches in the figure and therefore AC should be 7 feet nearly. Thus lead them to note that the length of the rafter required may be found to be slightly greater than 7 feet.

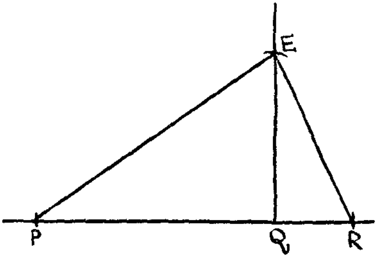


Setting out triangles by taking the measurements of the perpendicular to a side from the opposite corner and of the segments into which the side is divided by the foot of the perpendicular.

Draw a triangle ABC on the board and propose to pupils to draw an exact copy of it. Point out to them the ease with which they could set off right-angled triangles and get them to suggest how the triangle ABC could be split into two right-angled triangles. Lead them to note that by drawing AD perpendicular to BC the right-angled triangles ABD, ADC making up the triangle ABC may be obtained. Next question them what measurements should



be taken to set out the triangles ABD, ADC. Get them to suggest how the right-angled triangles ABD and ADC may be reproduced by stepping off segments PQ, QR equal to DB, DC, respectively, along a straight line, then



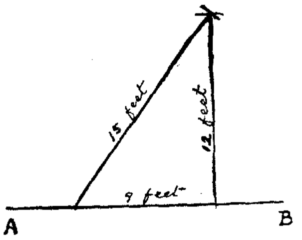
drawing a perpendicular QE to RP through the point Q corresponding to D, stepping off along the perpendicular a segment QE equal to DA and next joining ER, PE. If necessary, let pupils trace the triangle ERP thereby obtained, compare it with the original triangle ABC by superposition and derive that it is a duplicate of it. Point out also that as the duplicates of the right-angled triangles ADC, ADB have been obtained in EQR and EQP by virtue of the construction the entire triangle ERP may be taken as the duplicate of the triangle ABC.

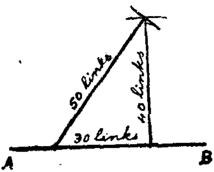
Mark out triangular plots in the school-yard or in the adjoining lands. Get pupils to set out plots of the same size and shape in the neighbourhood and suggest to them to use the cross-staff for finding out the point in RP at which QE will be perpendicular to RP and also for setting out perpendiculars to lines. Let them also use the chain for measuring the lengths of CD, DB, AD. After the plots required have been set out, point out to pupils that this method of setting out triangular plots is more convenient than the other methods described here to fore.

Setting out a right angle on the floor by the use of a measuring chain or tape.

Propose to pupils to construct a triangle with sides 3″, ″, 5″. Get them to measure its angles and thus note that

the angle between the sides of lengths 3″ and 4″ is a right angle. Next ask them to suggest lengths for the sides of triangles similar to the above, say 6″, 8″, 10″; 9″, 12″, 15″; 4½″, 6″, 7½″, etc. Let them construct triangles with sides of the above lengths, measure the angles and note that the angle between the sides of lengths 6″, 8″, or 9″, 12″, or 4½″, 6″, etc., is a right angle. Point out to them that as triangles constructed with sides of the above type are similar they must be right-angled. Tell them that this property may be made use of to set out right angles on the floor with a measuring tape or chain. Get them to suggest lengths like 9 feet, 12 feet, 15 feet, or 30 links, 40 links, 50 links for the sides and construct triangles on the floor with a side of 9 feet or 30 links set on a straight line AB drawn on the floor. Thus lead them to note how the straight line perpendicular to the given line AB could be drawn.





TOPIC 30.—PERCENTAGE.

(**Scope of the treatment.**—*The use of percentage as a means of comparison may be dealt with at the beginning with reference to problems on yield of jaggery from different varieties of cane, germination capacity, yield of field crops, etc. The facility afforded by chosing* 100 *as the basis for comparison in preference to other numbers may be clearly explained. Only simple, direct problems need be attempted, and problems of an inverse character especially on profit and loss may safely be avoided. Some of the common fractions and their equivalents in percentages must be brought home to the pupils. Problems on the calculation of increase or decrease per cent, gain or loss per cent, on commission, discount and brokerage may be given. Problems of an advanced character, etc., are given later on under " Revision Examples." It must be borne in mind that complicated problems on profit and loss would be far beyond the comprehension of the pupils and hence they need not be dealt with. As it is contemplated that the pupils should have only a working knowledge of percentage, the problems given are mostly such as could be solved orally and if the numerical data are changed by the teacher for repetition of the same type of problem or for some other reason the numbers may be so chosen as to admit of quick oral working.*)

Comparison by Percentage.

Propose :—A cultivator finds that by using one and the same method of crushing canes and preparing jaggery from cane he gets 15 puttis of jaggery from 125 puttis of J. 247 variety and 8 puttis of jaggery from 96 puttis of Bontha variety. Find which variety of cane gives a greater quantity of jaggery by weight.

Question pupils as to what weight of jaggery they get out of 1 putti of each variety of cane. Let them find out that from 1 putti of J. 247 the cultivator may expect 15/125 putti of jaggery, while from 1 putti of Bontha he may expect 8/96 putti. Point out that if they find which

of the two is greater, 15/125 putti or 8/96 putti, they could deduce easily which variety of cane yields a larger quantity of jaggery by weight.

Tell pupils that the comparison would be easier to make if the yield from 100 puttis is found out. Thus get them to work that from 100 puttis of J. 247 variety may be got 15/125 × 100 or 12 puttis of jaggery while from 100 puttis of Bontha may be got 8/96 × 100 or 8⅓ puttis of jaggery.

From the two methods above given, impress the fact that to make the required comparison they must find the yield of jaggery from equal quantities by weight of each variety of cane; *e.g., 1 putti, or 100 puttis, or 1,000 puttis* or 6 puttis and that the comparison can be deduced with less difficulty when the yield from 100 puttis is found out than when that from 1 putti or 6 puttis, etc., is known.

(If fromt he statement given in the exercise J. 247 variety yields 15 puttis of jaggery and Bontha variety 8 puttis the pupils suggest at the start that the former variety gives a better yield, point out that for comparison they ought to consider equal quantities by weight of each variety of cane and that as they have not done so the inference they have drawn is incorrect.)

Tell pupils that when from 100 puttis of J. 247 variety of cane the yield of jaggery is 12 puttis it is usual to say that the weight of jaggery obtained is 12 per cent of the weight of cane. Point out that the word "*cent*" simply means a "*hundred*" and that this expression "12 per cent" in the above instance indicates that out of every 100 puttis of the cane 12 puttis of jaggery may be expected or in other words 12/100 of the weight of cane may be taken to be the weight of the jaggery obtained.

Let pupils note that the numerator "12" in the fraction 12/100 gives the rate per cent, i.e., suggests how much out of a hundred puttis of cane is jaggery, and to denote that it is 12 out of a hundred it is usual to adopt a briefer form "12 per cent" (read 12 per cent).

Propose :—**1.** In the Anakapalle Agricultural Farm two plots were sown with two different kinds of paddy seed. In one of them 980 out of 1,000 grains sown sprouted. In

the second 1,940 out of 2,000 grains sown sprouted. What per cent of each seed sprouted? Which seed is the better one?

(*Ans.* 98 per cent of the first kind; and 97 per cent of the second kind. The first is the better of the two.)

2. From the following table find the percentage of germination capacity of the seed of each crop given below :—

Name of crop.		Number of grains sown.	Number which sprouted.	(*Answer*).
Vari	...	1,900	1,710	90
Ulava	...	1,000	970	97
Anumu	...	1,600	1,424	89
Jonna	...	10,000	9,400	94
Pesalu	...	15,000	14,105	91
Bobbara	...	350	336	96

3. What per cent of a rupee is a "pavala," a "beda" and an anna, respectively?

(*Ans.* 25 per cent; $12\frac{1}{2}$ per cent; $6\frac{1}{4}$ per cent.)

4. What per cent of a putti weight is a manugu? What per cent of a manugu is a visam? What per cent of a visam is a padalam? What per cent of a ton is a cwt.?

(*Ans.* 5 per cent; $12\frac{1}{2}$ per cent; 50 per cent; 5 per cent.)

5. What per cent of an Imperial seer is a rupee weight? What per cent of an Imperial maund is an Imperial seer? (*Ans.* $1\frac{1}{4}$ per cent; $2\frac{1}{2}$ per cent.)

6. The following table gives the value of produce obtained and the cost of cultivation incurred in regard to certain crops raised by a landlord in his lands :—

Number.	Name of crop.			Value of produce.	Cost of cultivation.
				RS.	RS.
1	Sugarcane	...	...	500	200
2	Cassava (Tapioca)		...	180	90
3	Plantains	...	...	690	230
4	Tobacco	...	...	160	84
5	Chillies	...	...	200	90
6	Turmeric	...	...	500	100

Number.	Name of crop.	Value of produce.	Cost of cultivation.
		RS.	RS.
7	Onions	120	69
8	Cambodia Cotton	126	42
9	Gogu	60	24
10	Groundnut	50	30
11	Castor	32	16
12	Jonna	30	18

Calculate what per cent of the value of the produce obtained has to be spent on the cultivation.

(*Ans.* 1. 40 per cent.	7. 57½ per cent.
2. 50 per cent.	8. 33 1/3 per cent.
3. 33 1/3 per cent.	9. 40 per cent.
4. 52½ per cent.	10. 60 per cent.
5. 45 per cent.	11. 50 per cent
6. 20 per cent.	12. 60 per cent.)

7. The weight of a certain quantity of paddy was 1,000 lb. immediately after the harvest. It was stored up and sold for sale three months hence, when it was found to weigh only 980 lb. owing to dryage. Calculate the loss per cent in weight by dryage of stored paddy.

(Point out that there is a loss of 20 lb. from the original weight of 1,000 lb. and therefore the loss is 20 per 1,000 or 2 per cent.)

8. When selling castor-cake a company charges a profit of 6¼ per cent on the cost. If it bought a bag of the cake for Rs. 7-4-0, for how much does it sell the bag ?

(Point out that as 100 × 1/16 = 6¼, the profit comes to Re. 1 for a cost of Rs. 16 or one anna per rupee and therefore the company sells the bag for Rs. 7¼ plus 7¼ annas, i.e. (for Rs. 7–4–0) + (0–7–3) or Rs. 7-11-3.

Let pupils find out and memorize the following equivalents :—

50 per cent = ½ ; 25 per cent = ¼ ; 75 per cent = ¾ ; 12½ per cent = 1/8 ; 40 per cent = 2/5 ; 6¼ per cent = 1/16 ; 3 1/8 per cent = 1/32 ; 33 1/3 per cent = 1/3 ; 66 2/3 per cent = 2/3 ; 20 per cent = 1/5 ; 40 per cent = 2/5 ; 60 per cent = 3/5 ; 80 per cent = 4/5 ; 10 per cent = 1/10 ; 5 per cent = 1/20 ; 15 per cent = 3/20 ; 30 per cent = 3/10.

In this connexion point out that in Tuni and perhaps elsewhere the Dulam (= 4 annas), Chowlam (= 8 annas), Pavu (= 1 rupee), Mada (= Rs. 2) and Punji or Pagoda (= Rs. 4), are used to denote percentages. Thus if a man wants to say that he is giving 6¼ per cent he will say he is giving a Dulam, *i.e.*, 1/16 of a Pagoda, and similarly for 12½ per cent, 25 per cent and 50 per cent, he would say he is giving a Chowlam, a Pavu and a Mada, respectively.

9. A ryot finds that 10 per cent of his usual paddy crop has been destroyed by stem-borer (*oorsathiri*). If the usual crop was 640 kunchams, find how much he lost by the attack. (*Ans.* 64 kunchams.)

10. A particular variety of sugarcane has been found to produce 60 per cent of juice and 10 per cent of jaggery. Calculate how much of juice and of jaggery could be obtained from 7 puttis weight of that cane.

(*Ans.* 84 manugulu of juice, 14 manugulu of jaggery.)

11. A broker promises to sell at the market rate all the jaggery you bring provided he is given a commission of 6¼ per cent on the sale proceeds. If on this condition he gets your jaggery sold for 256 rupees find how much you will have to pay him as commission. (*Ans.* Rs. 16.)

12. A landlord has the following green-manure crops in his fields :—

10,000 lb. of Sunnhemp (Tzanumu)
12,000 lb. of Daincha (Jilugu)
4,000 lb. of Kolinji (Vempati)
and, 8,000 lb. of greengram (Theegapesalu).

The percentage of nitrogen in the sunnhemp crop is ·2, that in daincha ·15, that is kolinji ·6, and that in greengram ·1. If the landlord applies the above green manure to 3 acres of his lands, find how many lb. of nitrogen he applies per acre. (*Ans.* 28⅓ lb. of nitrogen.)

13. B. Hanumantha Rao bought a Godavari pan for Rs. 60 and wants to sell it so as to gain a Chavulam profit. What per cent on his cost does he require as profit, and how much will he actually get by selling the pan at that rate ? (*Ans.* 12½ per cent ; Rs. 67½.)

14. A broker undertakes to get the plantains in your garden sold at Rs. 2 per hundred if he is allowed a commission of 6¼ per cent on the takings. He undertakes to supply a merchant with plaintains at Rs. 2–4–0 per 100

if he is allowed a commission of 4 as. per 100. How much will he realize if he gets 1,500 plantains bought and sold in the course of a market day? (*Ans.* Rs. 9–6–0.)

(Here point out that besides getting commission from both the parties the broker gets 4 as. profit for every hundred sold.)

15. You have to buy bullocks at a cattle fare. A broker promises to get you the two pairs of bullocks you select for Rs. 650 if you give him Dulam profit, i.e, $6\frac{1}{4}$ per cent on the price as commission. How much have you to pay him if he gets the bullocks for you at the price specified? (*Ans.* Rs. 40–10–0.)

16. A field produces 300 kunchams of paddy per acre. It is estimated that by better cultivation the yield would increase by 15 per cent. What would be the yield per acre by better cultivation?

(*Ans.* 345 kunchams per acre.)

17. Continuous green manuring with sunnhemp (Tzanumu) increases the yield from 260 kunchams of paddy per acre to 300 kunchams per acre. What is the increase per cent in the yield by using sunnhemp?

(*Ans.* 15·4 per cent.)

(Point out in this connexion that the increase is from 260 kunchams to 300 kunchams, that this comes to 40 kunchams for every 260 kunchams, and hence the rate works out as 15·4 per cent. Lead pupils to note that the tenth's place is put in only *approximately.*)

18. By wrapping and propping sugar-cane crop against wind and wild animals an acre of sugarcane which used to yield 12 puttis of jaggery without the above operations was found to yield 4 puttis more of jaggery. By what per cent has the yield of jaggery increased by wrapping and propping? (*Ans.* $33\frac{1}{3}$ per cent.)

19. $5\frac{5}{8}$ puttis of *pachchi pasupu* has been found to give only 1 1/8 puttis of *endu pasupu* when cured. What is the percentage of loss of weight in curing?

(*Ans.* 80 per cent nearly.)

20. The following details relate to two varieties of chillies grown in the Government Farm.

Variety of Chilly.	Green weight in lb.	Dry weight in lb.
1. Podugu Mirapakaya (Long chillies).	2,605	723
2. Potti Mirapakaya (Short chillies).	245½	55¾

Find, in each case above, what per cent the dry weight is of the green weight. Hence calculate the loss per cent in weight by dryage in each case.

(*Ans.* 27·8% ; 22·7% ; 72·2% ; 77·3%.)

21. 20 kunchams of paddy when unhusked is seen to give 9½ kunchams of rice. What per cent by capacity of the paddy is thereby removed as husk and bran ?

(*Ans.* 52½%.)

22. A cultivator got 25 puttis of paddy from his wet lands. If by the use of fertilizers as recommended by the Agricultural Department an increase of 15% in the yield is obtained find how many puttis of paddy he may thereby expect from his lands. (*Ans.* 28¾ puttis)

23. Find the percentage of deadhearts to earheads in the paddy varieties stated below :—

No.	Cultivation		Number of earheads.	No. of deadhearts observed.
1	Rasangi	...	1,280	4
2	Navakoti	...	1,150	19
3	Peshanam	...	850	15

(*Ans.* 1 ·3 ; 2 ·1·7 ; 3 1·8).

24. A tenant has signed a ***Kadappa*** to the effect that he would pay his landlord Rs. 500 after selling jaggery in the harvest time. The landlord makes over the *Kadappa* to M. Thathiah who undertakes to pay cash after deducting 12½% from the amount specified in it. How much cash may the landlord expect to get ?

(*Ans.* Rs. 437–8–0.)

25. J. 247 variety of sugarcane yields 12% by weight of jaggery. A ryot promises to supply 6 puttis of jaggery to a sowcar. What weight of cane must he cut ?

(*Ans.* 50 puttis.)

TOPIC 31.—SIMPLE INTEREST.

(Scope of the treatment.—*The system of lending and borrowing money may be referred to and the ethics of levying interest discussed in the class. The method of calculating interest according to the prevalent rates in the market may be explained and the pupils given sufficient drill in the calculation of interest. Of the two methods of calculating interest, the English and the indigenous, the latter may be specially insisted on to be used by the pupils. Problems on the levy of penalty interest (gujisthu vaddi) on the payment of interest in kind as in the " phaida " and the " namu " systems and those on the advance contract systems like the " Zetti ", etc., may be given in an elementary form. The method of calculating compound interest need not be dealt with as a pro-note once executed appears to be renewed at the end of a year and therefore the people do not appear to find any necessity for adopting the method of calculation by compound interest, all necessary calculations being made by the simple interest method alone. The chit fund systems are very rare and hence calculations based on them may be safely avoided. It should be brought home to the pupils that with reference to the culti, vator, the value of the land corresponds to the principal, and that of the produce to interest. Problems on interest calculation on deposits made in the Post Office Savings Bank or other banks, on loans made by Government for effecting agricultural improvement, etc., may be given. The teacher must bear in mind that, as far as possible, exercises on interest calculations should be done by the pupils quickly and orally by the Dokudalu method quite in preference to the formula method. Problems of an inverse character need not be dealt with, except in the case of calculating the rate per cent in a few cases.*)

Propose:—A cultivator (Boddedu Deviah) wants Rs. 300 for cultivation expenses. He has not cash on hand. How is he to get the money ?

Elicit that he may have to sell the jaggery, paddy, or ragi, etc., he may have and if there be no such article at home which he could afford to sell, he may have to borrow money from a sowcar, or from any other friend who could spare him money. Question whether the sowcar would

give him the money on pure trust. Impress the fact that unless the sowcar is given some surety, e.g., some jewel pledged, or land mortgaged or at least a pro-note to the effect that the money would be repaid when he wants it he may not be willing to part with the money. Question if it is desirable that B. Deviah should give him such a security. Discuss the several cases when the borrower may dupe the lender, the lender may refuse the re-delivery of a pledged jewel, etc., and let pupils suggest the precautionary measures to be taken by both the lender and the borrower in such cases lest they should be duped.

Talk about the several trades, industries, etc., and elicit the fact that people invest money in them just with a desire to make more money out of them.

Propose :—B. Gourisam has plenty of cash with him. G. Appadu thinks that if he has money he could run a shop and earn about Rs. 50 a month from it. He requests Gourisam to lend him Rs. 200 to start the shop and executes a pro-note that he would repay the money some time hence. Question pupils if the debt would be cancelled if Appadu pays back Rs. 200 only sometime hence. Let them express their views and the teacher also his, say, as follows :—

(1) If the money remained with Gourisam, he might have started some jaggery trade and thus earned more money out of it, say Rs. 300.

(2) Oh, no, not so much. He might have received only Rs. 220 if he had spent the money in the trade.

(3) He might have had a loss as well.

(4) If Appadu did not get the money from Gourisam, he could not have started the shop and consequently, he would not have earned the profits which enabled him to pay back the loan, etc.

By inviting discussions of the above kind, impress the fact that G. Appadu should not only pay back the sum of Rs. 200 he borrowed, but he must also give some extra amount as interest to make up the gain that would have accrued to Gourisam, had he kept the money with him and invested it in some trade. Let pupils note that it is justifiable to demand interest just as landlords require produce like ragi, paddy, jaggery, etc., or their money value being paid to them by the tenants for the use of their lands. If

the ethics of the question be further discussed, let them conclude that a high rate of interest should not be exacted. From the above discussion, lead pupils to note that interest is the extra money given for the use of another's money over and above the original sum borrowed and that the total amount to be repaid to cancel the debt equals the original sum borrowed, plus interest.

Next propose :—

G. Gourisam lent B. Deviah Rs. 200 and the latter is prepared to repay the debt at the end of 6 months. How is the amount to be repaid to be decided ?

Point out to pupils that B. Deviah would have previously come to an understanding with Gourisam as to *the rate per hundred rupees per month at which the interest is to be charged*, for example, " muppavala " per hundred rupees per month. Let pupils find at this rate the interest on Rs. 200 as follows :—

The interest on Rs. 100 for 1 month is 12 annas.
∴ " " 200 " 1 " 12×2 annas.
∴ " " 200 " 6 months is $12 \times 2 \times \frac{6}{16}$ rupees or Rs. 9

Get pupils to note that Rs. 200 plus 9 or Rs. 209 will have to be repaid to cancel the debt. Tell them that the amount repaid is generally spoken of as the " Amount " and the sum borrowed as " Principal ". Let them note that (Amount= Principal plus Interest.)

Propose :—

Boghmal Sowcar lends Rs. 300 to Bapanna at 1 rupee per Rs. 100 per month and Rs. 300 to Reddy Paiadaiah at Rs. 1-8-0 per Rs. 100 per month. If both Bapanna and Paidaiah repay the loan at the end of 6 months, from whom does the sowcar get a greater amount ?

Let pupils compare the rates of interest charged by the sowcar from each of the borrowers and note that while the *" Principal "* and the *" time "* are the same, the *rate* alone differs and that the sowcar would get a greater amount from Paidaiah as the latter pays a higher rate of interest.

Propose :—

Bapanna borrowed Rs. 100 from Boghmal Sowcar and took 6 months to repay the debt with interest.

B. Buchiah borrowed an equal sum of money at the same rate of interest, but took 12 months to repay the debt. Who would be paying more to the sowcar at the time of repayment of the debts?

Here, get the pupils to realize that as the period for which interest is to be charged is alone different, the man who has been keeping the money for a longer period will have to pay a greater amount. Thus, get pupils to infer that the longer the period for which the money is retained, the greater would be the interest that would have to be paid.

Propose two questions of the following type:—

1. K. Malliah invested Rs. 1,000 in jaggery trade and got a profit of 20 per cent. How much did he get back from the trade?

2. D. Suryanarayana Razu lent a cultivator Rs. 1,000 at Rs. 1-8-0 for Rs. 100 for one month. If the loan was repaid at the end of 6 months, what sum may he expect to get back?

Let pupils compare the conditions given in the two questions above and infer that both are questions on percentages. Question in what respects the two differ. Point out that in the latter question requiring the calculation of the interest *the element of time is involved* while in the former question on percentage, *the element of time is not involved.*

Exercises.

Propose the following exercises:—

1. Find the amount to be repaid at simple interest on Rs. 400 for 8 months at 12 per Rs. 100 per month.

(If the pupils find the interest on Rs. 400 for one month to be Rs. 3 and hence that for 8 months to be Rs. 8 × 3 or Rs. 24 and thus calculate the amount to be repaid to be Rs. 400 plus 24 or Rs. 424, tell them that the method they adopt is not quite so prevalent in the locality as the following one known as *the dokudalu method*; and that they may find it more convenient to adopt the latter method. The interest on Rs. 400 for 8 months is the same as that on Rs. 400 × 8 or on Rs. 3,200 for one month. The interest on Rs. 100 for one month is "muppavala."

Rs. 3,200 is 32 times Rs. 100. Therefore, the interest required is 32 muppavalalu or Rs. 32 × ¾ or Rs. 24.

Tell pupils that the above working is taken in mentally as follows :—

400 × 8 gives 3,200. This gives 32 dokudalu by division of 3,200 by 100. Therefore the interest to be paid is 32 muppavalalu or Rs. 24. Hence the amount to be repaid is Rs. 324).

2. Find the amount to be repaid at simple interest in the following cases :—

Sum borrowed RS.	A.	P.	Rate of Interest.	Time.	Answer. RS.	A.	P.
50	0	0	1% per month	1 year	56	0	0
250	0	0	do.	5 months.	262	8	0
25	0	0	1½% do.	6 do.	27	4	0
300	0	0	do.	5 do.	322	8	0
750	0	0	do.	6 do.	817	8	0
300	0	0	2% do.	15 do.	390	0	0
1,500	0	0	½% do.	6 do.	1,545	0	0
30	0	0	3 pies per rupee	2 do.	30	15	0
85	0	0	2 do.	6 do.	90	5	0
25	0	0	1½% per month	8 do.	28	0	0
150	0	0	3% do.	6 do.	177	0	0
75	0	0	3% do.	10 do.	97	8	0

An artifice.—When the interest is at 6 per cent per year, that is, ½ rupee per Rs. 100 per month, lead pupils to note that it is equivalent to a rate of 1 pie per rupee per month roughly, and that the deduction from the interest got by such calculation should be at the rate of 3⅓ annas per thousand rupees, i.e., four pies or a dabbu per Rs. 100 per month. In a question like that of finding interest on Rs. 48 for a month at 12 annas per Rs. 100 per month, point out that to have the calculation made *orally* the artifice may be applied as follows :—

½ a rupee per Rs. 100 per month equals 1 pie per rupee per month less 4 pies per Rs. 100 per month. Therefore the interest on Rs. 48 for 1 month at ½ a rupee per Rs. 100 per month equals 48 pies less 48 × 4/100 or 192/100 pies equals 4 annas less 192/100 pies. Therefore the interest on Rs. 48 for 1 month at Muppavala per Rs. 100 per month equals 4 × 3/2 or 6 annas less (192 plus 96)/100 or 288/100 pies, i.e., 6 annas less 3 pies to the nearest pie, i.e., 5 annas 9 pies.

Applying the above artifice, let pupils find the interest on Rs. 58 for 2 months at Re. 1 per Rs. 100 per month (*Ans.* Rs. 1–2–7.

Change the numbers in this exercise and propose a dozen exercises. Point out that the above method helps a finding quickly a fair approximation of the interest due.

Propose :—

Find the interest on Rs. 485 for 8 months at 12 annas per Rs. 100 per month. Let pupils work by the *dokudalu method* as follows :—

485 × 8 gives 3,880 dokudalus.

Reckoning 12 annas for 100 dokudalus we get for 3,800 dokudalus 38 half rupees plus 38 quarter rupees, that is 19 plus 9½ or Rs. 28½ ;

for 75 dokudalus 12 × ¾ or 9 annas as 75 is ¾ of 100 ;

for 5 dokudalus 7 pies at 144 pies (12 annas) per 100 dokudalus.

(Here point out that as 20 times 5 equals 100, division by 20 is involved) and therefore for 3,880 dokudalus Rs. 28½ plus 9 annas plus 7 pies or Rs. 29–1–7.

Point out to pupils that in the calculation of interest fractions are likely to occur in the pies' column, and then they must apply the convention regarding approximation they have already learnt and express the result to *the nearest pie*, as done in the above exercise.

Propose :—

3. D. Suryanarayana borrows Rs. 250 from Bodazr Narayanamurti on the 15th November 1922 at 12 annas per Rs. 100 per month and returns the money with interest on the 24th April 1923. How much should he return to cancel the debt ?

(Here point out that the days of borrowal and repayment are specified and the period for which interest is to be calculated should be expressed in days. Let pupils calculate the number of days from the 15th November 1922 to the 24th April 1923. If they commit mistakes point out that whenever the period to be calculated is from one specified date to another, it would be rather hard to charge interest for both the days and hence one only of the day

specified need be calculated and also that in expressing the number of days in the period as a fraction of the year it is usual to reckon always 365 days for the year. Let pupils work the question as follows :—

For	November	1922	15	days	
„	December	„	31	„	
„	January	1923	31	„	
„	February	„	28	„	
„	March	„	31	„	
„	April	„	24	„	160 days.

Therefore the interest is to be calculated for 160 days.

Twelve annas per Rs. 100 per month works out to Rs. 9 per Rs. 100 per year.

Therefore, the interest on Rs. 250 for 365 days is Rs. 9 × $2\frac{1}{2}$ or Rs. $22\frac{1}{2}$.

Therefore, the interest on Rs. 250 for 160 days is Rs. $22\frac{1}{2}$ × 1/365 × 160 or Rs. 9–13–10 to the *nearest pie.*

Hence the amount to be repaid comes to Rs. 293–13–10.

Get pupils to note that the interest on Rs. 250 for 160 days is equal to that on Rs. 250 × 160 for 1 day, also the interest on Rs. 100 for 365 days equals that on Rs. 365 × 100 or Rs. 36,500 for 1 day and hence the interest required equals 9 × 250 × 160/36,500 at Rs. 9 for Rs. 36,500 for 1 day.

Propose :—

4. A person invests Rs. 500 in a bank on 11th November 1922, withdraws Rs. 200 on 25th December 1922 deposits again Rs. 250 on 14th January 1923. If, from the bank, he withdraws the whole amount due to him on 11th February 1923, how much will he draw as "principal," and how much will the bank be paying him as interest at the year end at $\frac{1}{2}$ rupee per cent per month ?

(Lead pupils to note that from 11th November 1922 to 25th December 1922 there are 44 days and investing Rs. 500 for 44 days is the same as investing Rs. 22,000 for one day.)

From 25th December 1922 to 14th January 1923 he has Rs. 300 to his credit in the bank, and investing Rs. 300 for that period of 20 days is equivalent to investing Rs. 6,000 for one day ;

and from 14th January to 11th February 1923 he has to his credit Rs. 550 in the bank and investing Rs. 550 for this period of 28 days is equivalent to investing Rs. 15,400 for one day; and therefore, on the whole, the interest due to him should be what the bank would give for investing Rs. 43,400 for one day.

Let pupils thus find that the interest due equals Rs. 43,400/36,500 × 6 or Rs. 7–2–2 as ½ per cent per month is equal to 6 per cent per year. Inform pupils that some bankers calculate as follows :—

Investing Rs. 100 for a month is taken as equivalent to investing Rs. 3,000 for one day, assuming a flat rate of 30 days for the month, and hence, the interest at 1 per cent per month is taken as Rs. 43,400/3,000 at one per cent per month, i.e., as Rs. 14-7-6, and hence, the interest due at ½ per cent per month is Rs. 7-3-9. Point out to pupils that strictly speaking, 365 days ought to be counted for the year, that the above method is, hence, not mathematically correct and that it aims at a greater interest being charged than what is equitable. Tell pupils that in banks the principal may be withdrawn at any *time* and that the interest is generally given only at the end of six months or one year, and therefore, the depositor may withdraw from the bank Rs. 550 on 11th February 1923, and could expect the interest of Rs. 7-3-9 to be repaid only at the end of the bank year.

Propose :—

5. Find by the Dokudalu method the amount at simple interest on :—

(*a*) Rs. 1,250 for 6 months at Muppavala vaddi (¾ per cent per month).

(*b*) Rs. 750 for 5 months at 1½ per cent per month.

(*c*) Rs. 985 for 11 months at 1½ per cent per month.

(*d*) Rs. 225 for 6 months at ¾ per cent per month.

(*e*) Rs. 22 for 3 months at 1 rupee per 100 for one month (Nilavu vaddi.)

(*f*) Rs. 60 for 18 days at 2 annas per day (Rosuvari vaddi.)

Ans. (*a*) Rs. 1,306–4–0; (*b*) Rs. 806–4–0 (*c*) Rs. 1,147–8–5; (*d*) Rs. 235–2–0; (*e*) Rs. 66–10–7; (*f*) Rs. 22–4–0.

6. A cultivator borrowed Rs. 200 from a co-operative bank at $1\frac{1}{2}$ pies per rupee per month to buy a pair of bullocks and he repaid the loan with interest in 6 months. Another cultivator bought a pair of bullocks for Rs. 200 on credit from a cattle dealer and agreed to pay the price amount with 10 per cent of it extra as interest at the end of 6 months. What was the total amount paid by each and what is the difference between the two amounts?

(*Ans.* Rs. 209–6–0; Rs. 220; Rs. 10–10–0.)

7. The Government lends money at $6\frac{1}{4}$ per cent per annum for agricultural purposes, if a fourth of the money borrowed by the ryot is repaid by him with interest at the end of each year. A cultivator borrowed Rs. 850. How much should he pay at the end of each year?

(*Ans.* Rs. 265–10–0 at the end of first year.
„ 252 –5–6 „ second year.
„ 239 –1–0 „ third year.
„ 225–12–6 „ fourth year.)

8. A cultivator borrowed Rs. 250 from a village co-operative bank at Rs. $1\frac{1}{2}$ per Rs. 100 per month to buy a carting pair and he repaid the loan with interest in 6 months. How much did he pay then?

(*Ans.* Rs. 62–8–0.)

9. The Post Office Savings Bank gives interest at 3 per cent per annum on every complete rupee I am credited with in my pass-book, the lowest balance to my credit between the fourth and the final days of the month being reckoned as principal for the month. My pass-book shows the following entries:—

			RS.
17th May 1918.	Deposited	... ...	100
16th July 1918.	Do.	... ...	18
1st November 1918.	Withdrawn	...	25
6th March 1919.	Deposited	... ...	100

Assuming that the pass-book was newly opened on 17th May 1918 and that there were neither deposits nor withdrawals up to the end of 30th April 1919, calculate the interest that would accrue to my share up to 30th April 1919.

(*Ans.* Rs. 8–4–6.)

(While working this question, let pupils write down as follows :—

	Principal.			Interest.		
	RS.	A.	P.	RS.	A.	P.
5th month of 1918 ...		...			...	
6th " " ...	100	0	0	0	4	0
7th " " ...	100	0	0	0	4	0
8th " " ...	118	0	0	0	4	9
9th " " ...	118	0	0	0	4	9
10th " " ...	118	0	0	0	4	9
11th " " ...	93	0	0	0	3	9
12th " " ...	93	0	0	0	3	9
1st " 1919 ...	93	0	0	0	3	9
2nd " " ...	93	0	0	0	3	9
3rd " " ...	93	0	0	0	3	9
4th " " ...	193	0	0	0	3	9
Total ...	1,212	0	0	3	0	9)

Tell pupils that the interest is calculated in the Post Office month by month as shown above, and that in the calculation, the following rates are adopted as equivalents.— 4 annas per Rs. 100 = 12 pies per Rs. 25, or 2 pies for Rs. 5, approximations being duly effected.

11. In a village bank a ryot deposits Rs. 250 on 11th April 1922, withdraws Rs. 100 on 15th June 1922, deposits Rs. 450 on 25th September 1922, withdraws Rs. 180 on 15th December 1922, deposits Rs. 500 on 18th March 1923. How much would stand to his credit in the bank on 7th April 1923, if the interest due is then added on to the sum outstanding at Re. ½ per Rs. 100 per month ?

(Assume that there were neither deposits nor withdrawals from 18th March 1923 to 7th April 1923, and that for interest calculation 12 months of 30 days are alone reckoned for the year instead of as 365 days.)

(*Ans.* Rs. 921-10-4.)

12. Golla Appadu borrows Rs. 200 from B. Hanumantha Rao on 1st July 1922 and promises to return the loan on 1st January 1923 with interest at 12 annas per Rs. 100 per month, and if the money be not paid on that date, he binds himself to pay penalty interest (*Gujistu vaddi*) at Rs. 1-8-0 per Rs. 100 per month with retrospective effect, that is, from the date of borrowal up to the date of repayment. If the loan was repaid on 1st March 1923,

how much was paid back on that date? How much less might have been paid if the debt was cleared on 1st January 1923? (*Ans.* Rs. 224; Rs. 15.)

13. One year a ryot got a net profit of Rs. 1,500 by cultivating sugarcane on his lands. The lands are valued at Rs. 10,000. Find what rate of interest per month the ryot realizes from the cultivation during the year as calculated on the value of the lands.

(Let pupils adopt the following working:—

The interest on Rs. 10,000 for one year is Rs. 1,500.

Therefore, the interest on Rs. 100 for one year equals Rs. 1,500/100 or Rs. 15.

Therefore, the rate of interest which he gets from his lands is 15/12 or $1\frac{1}{4}$ per cent per month.

Point out to the pupils that this rate of interest could be secured only once in alternate years as cheruku cultivation can in practice be had in one and the same land only in alternate years. Now, impress the fact that *for the land-owner, the value of the land corresponds to capital and that of the produce to interest and also that a higher rate of interest would be obtained by a greater outturn of crops by intensive as well as extensive cultivation.*

14. A land-lord leasing out an acre of land for paddy cultivation is given half a garisa of paddy for one year. He has only to pay an assessment of Rs. 15 per year and has not to incur any extra expenditure. If an acre is worth Rs. 1,000 and a garisa of paddy sells at Rs. 150, what rate of interest per cent per month does the land fetch him during the year? (*Ans.* 6 per cent per year.)

15. For an acre of garden lands, worth Rs. 600, leased out for sugarcane cultivation, a landlord is given Rs. 50 cash by the tenant for a year. If the landlord is to incur no expenditure other than paying an assessment of Rs. 9–8–0 for the year, find the rate of interest per cent per month the land fetches him for the year.

(*Ans.* As. 9 per Rs. 100 per month.)

16. A landlord has Rs. 4,000 worth of dry lands (*mettu*) fetching him Rs. 200 net income for a year, and Rs. 6,000 worth of wet lands (pallamu) fetching him Rs. 180 net income for the year. What rate of interest does he hereby get on the value of each kind of land? Which of the lands fetches him a higher rate of interest?

(*Ans.* 5 per cent per year; 3 per cent per year; the former.)

17. A ryot knows that lands fetching Rs. 120 by cultivating chillies from July to March (both included) cost Rs. 2,000 to buy. If there is no cultivation during the rest of the year, find what rate of interest he could realize from chilly cultivation on the land.

(*Ans.* ½ per cent per month.)

(Here point out that the net profit from the cultivation for nine months should be taken as the interest for the whole year.)

18. A cultivator borrows Rs. 50 at the time of transplantation in Uttara Karthikai in Sravanam (July 19th) for giving wages to coolies and executes a pro-note to pay Rs. 2 per 100 rupees per month as interest. When he actually pays back the money after the harvest in the month of Pushyam (February 19th) he pays interest in kind by paying back Rs. 50 plus 50 kunchams of paddy. Reckoning the price of paddy then to be 1¼ kunchams per rupee, calculate what rate of interest per cent per month he actually gives. (*Ans.* Rs. $11\frac{3}{7}$ per Rs. 100 per annum.)

(Tell pupils that this system of paying interest in kind is known as "*Phaida*").

19. I give on December 1st Rs. 10 to a ryot for the payment of kist. At the end of January he returns Rs. 10 principal, along with 1 kuncham of paddy as interest. If a kuncham is then worth 8 annas, calculate what rate of interest I am given. (*Ans.* Rs. 2½ per Rs. 100 per month.)

20. A merchant advances 100 kunchams of paddy to a ryot for purposes of maintenance and cultivation. During the cropping season which is to take place six months after the advance is made, the merchant expects to get back 150 kunchams of the paddy as repayment of the loan. If the price of a kuncham of paddy was 7½ annas when the advance was made, and 6¼ annas at the cropping season, find what rate of interest the merchant is actually given.

(*Ans.* 50 per cent per year.)

(Tell pupils that this system of levying interest is known as "*namu*").

21. Before the crop is harvested, a sowcar advances Rs. 600 to a cultivator on the understanding that the principal alone need be returned to him during the harvest time, reckoning Rs. 50 only as the price of a putti. If a period of 4 months elapses between the date of advance

and the date of supply, and if paddy sells at Rs. 60 a putti during the harvest time find what effective rate of interest the sowcar realizes for the advances given.

(*Ans.* Rs. 5 per Rs. 100 per month.)

(Tell pupils that this system of advance contract is known as "*Zetti*").

22. A cultivator is not allowed to cut a crop from a field unless he pays the Government tax, his kattu padi to the zemindar, etc. So on December 15th the kist payment day, he feels the necessity to borrow, goes to a sowcar and gets a loan of Rs. 300. At that time a garisa of paddy sells at Rs. 100. The sowcar converts the sum lent at its grain equivalent and expects the borrower to return 3 garisas of paddy on the 15th of March of the ensuing year when paddy will be removed from the threshing ground. If paddy sells at Rs. 200 a garisa then, find what rate of interest per cent per month does the cultivator pay on the money he borrowed.

(*Ans.* Rs. 0–5–4 pies per rupee per month.)

(In the above question, a confusion is likely to arise as to whether the price of paddy at the time of removal from the threshing ground is likely to be higher than that at the time the crop is cut and laid on the ground. Point out with reasons that the actual conditions are so.)

23. A sugarcane cultivator who borrows Rs. 90 has to supply the sowcar six months hence with jaggery which may be valued at Rs. 90, charging only $\frac{3}{4}$ of the market price. At the time of so repaying the debt, jaggery sells at Rs. 60 a putti How much of jaggery will the cultivator have to give? What is its value in the market then? What rate of interest does the sowcar actually get?

(*Ans.* 2 puttis; Rs. 120; Rs. $5\frac{5}{9}$ per Rs. 100 per month.)

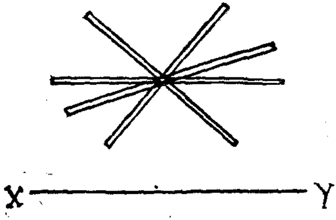
24. With a view to cancel a debt a cultivator delivered to a sowcar all the available produce from his lands. Still he found that owing to shortage of produce he had yet to give Rs. 56–8–0. If during the forthcoming year he had to pay up the arrears with interest thereon at $13\frac{1}{2}$ per cent per annum find how much he would have to pay back at the end of 18 months to cancel the debt.

(*Ans.* Rs. 67–15–1.)

TOPIC 32.—PARALLELS.

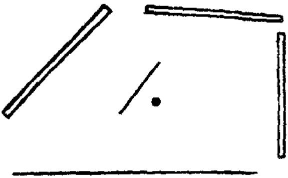
(**Scope of the treatment.**—*A conception of parallels hould be given as indicating sameness of " lie " of lines by ppealing to the pupils intuition as far as possible. By omparing the positions of various straight lines drawn hrough a point they must be led to note that through a articular point only a single parallel is possible to a given ine. Notions of lines tending to meet and of parallel lines ot so tending to meet, of the uniform distance between arallels, of the equality between sets of corresponding angles nd alternate angles may all be given in an elementary orm. As far as possible, too much of terminology should be voided, just to avoid confusion in pupils. The use of the T-square and of the set square for drawing parallels may e explained. The theory regarding parallels and proportionality is not intended to be dealt with here.*)

Notion of parallels.—Refer pupils to a nail fixed n a wall. Point out to them the ground-line XY where



the wall meets the floor surface. Give them a narrow strip of card-board and ask them to place its straight edge in different positions over the nail so as to rest on the wall surface. (See the above figure for guidance.) Question in which position the edge appears to lie in the same way as the ground-line. Point out the position and get pupils to note that in that position there is the sameness of " lie " between the ground-line and the straight edge. Tell pupils that the above position which indicates the sameness of

" lie " is known as a " parallel position " to the ground-line and that the straight edge held in that position is said to be parallel to the ground-line.



Draw a slant line on the wall surface and get pupils to adjust the straight edge along the nail so as to be parallel to the line. Repeat the exercise with horizontal, vertical and slant lines specially drawn on the wall surface.

Next let pupils choose other spots in the same wall marking points to represent the nail, turn a straight edge round and round the points in succession and ascertain by intuition in which position the edge will be parallel to the ground-line. Let pupils draw several lines to indicate the various positions the edge then assumes and mark out distinctly the position when it is parallel to the ground-line. Point out that in all other positions the lines drawn show a tendency to meet the ground-line if produced far enough, either to the right or to the left, but in the particular position of sameness of " lie " no such tendency is noticeable. Let pupils next repeat the above exercise with other points and lines chosen on the wall surface and examine if the above inference is true.

Direct pupils to mark a point on the blackboard and hold a straight edge through the point parallel to each edge of the board in succession. Set them next to draw lines through the point so as to be parallel to the edges of the board.

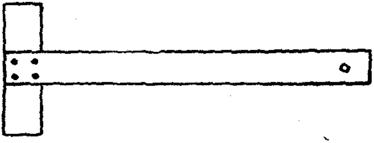
Mark a point on the blackboard and also draw a straight line not passing through it. Ask pupils to hold a straight edge through the point in a position parallel to the line drawn. Let them next draw a straight line through the point parallel to the given straight line, adjusting the straight edge as may be found necessary.

Next bring home the fact that in assuming a position

shifted through the same distance and in the same direction all along its path, that is to say, each point in it appears to have moved through the same distance and as measured in the same direction. (Actually shift a straight ruler or pointer along the surface of the blackboard and show.) Illustrate also by sliding a lamina having straight edges along a fixed line on a plane. Let pupils measure the distance between two lines assuming parallel positions and infer by intuition that a line drawn perpendicular to one of them appears to be also perpendicular to the other and that the distance between the two is uniform throughout. Show pupils in this connexion how a piece of paper may be folded successively round a thick strip of uniform width and thus a series of crease-lines obtained which are parallel. Drive a Sampson harrow straight along the ground surface. Point out that the distance between any two traces is the same throughout and hence that the traces are parallel. Show pupils a six-tined gorru and let them explain why the seed sown with it falls along parallel rows. Let pupils suggest edges and lines in the school-room which are parallel, e.g., the opposite edges of desks or tables, of the blackboard, of the doorway, etc. Refer them also to the rafters placed in parallel positions in the roofs of houses, the rails placed in parallel positions along a railway, the parallel bunds in fields, etc. Draw some figures on the blackboard having some parallel edges, e.g., a square, an oblong, a trapezium, a rhombus, a parallelogram, a regular hexagon, etc. Let pupils recognize which of the boundary lines are parallel.

CONSTRUCTION OF PARALLELS.

(a) With the T-square.

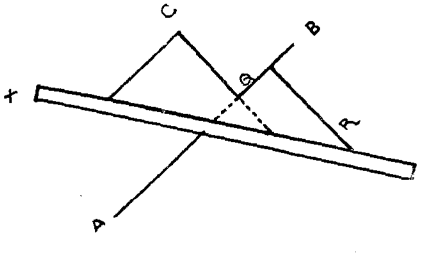


Tell pupils that though they know how to guess the position of a parallel they would find it convenient to use instruments for drawing parallels quickly and correctly.

Show a T-square. Hold it with its head pressing against the edge of the blackboard. Next draw a line along the leg of the T-square. Direct pupils to shift that edge to parallel positions and draw lines along them in these positions. Caution them to press the head of the T-square against the edge of the blackboard tight while drawing the lines. Point out that in the various positions the leg has been shifted through one and the same distance measured in the same direction and that hence the edge of the leg indicates sameness of " lie " in all the positions it assumes.

(b) With the set squares.

Draw a slanting straight line on the blackboard. Question pupils if they could use a T-square for drawing a parallel to it. Get them to note why they find it difficult. Tell them that the set squares may be used for drawing parallels to such lines, as described below :—



Name the line drawn on the board as AB. Mark a point C outside it. Propose to pupils that the line should be drawn parallel to AB which would pass through C. Ask them to suggest along which line the parallel is likely to fall and allow them to use a straight edge, if necessary, to indicate its position. Next direct them to place a set square in such a position that one of its perpendicular edges falls along the line AB. Let them next place a straight edge against the edge of the set square, i.e., the one opposite to the right angle. Direct them to slide the set square

up or down the straight edge until the edge of the set square that lay along AB appears to lie in the position in which a straight line drawn along it would pass through C. After adjusting the set square in that position, ask one of them to draw a straight line along that edge. Lead pupils to note that in so using the set square they have shifted the position of the edge of the set square that lay along AB to a new position through C having the same "lie" and that hence a parallel has been obtained to AB passing through C.

Some properties of parallels.

1. Draw some straight lines on the blackboard and ask pupils to draw parallels to them. Next ask pupils to draw lines on paper and draw a series of parallels to them. Get them to note that *any number of parallel straight lines could be drawn to one and the same straight line.*

2. Next ask them to draw a straight line on paper and mark a point outside it. Get them to draw a parallel to the line through the point given and see if more than one parallel could be drawn. Get them to note that *a single parallel alone is possible to a given straight line through a given point.*

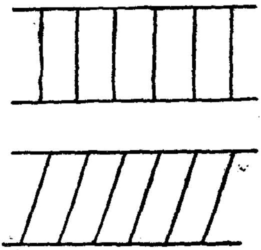
3. Ask pupils to draw two parallel straight lines on the blackboard with a T-square. Let another draw a straight line to cut the two parallels. Point out that eight angles are formed. Get the eight angles measured with a protractor. Let the pupils examine which angles are equal. Showing them the corresponding angles and alternate *angles in pairs, get them to note that the angles in each* pair are equal. (To avoid the confusion caused by the use of too much of terminology the teacher would be advised not to introduce the terms "corresponding angles" or "alternate angles." It is enough if the pupils have a notion as to which of the angles are equal.)

Let pupils repeat the above exercise with some more parallel lines, and arrive at the same conclusion.

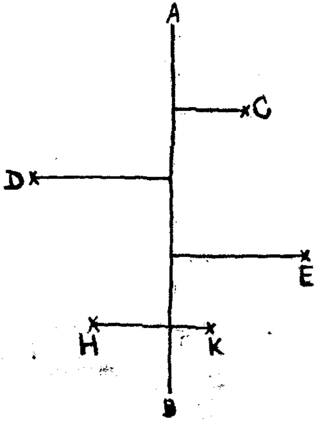
With reference to pairs of parallels already drawn, ask them to draw a line to cut them. Question which angles they may expect to be equal. Get them to verify by measurement if they are so.

4. Let pupils next draw a pair of parallels with the T-square or the set square as above directed, draw perpendiculars to one of them and produce them to cut the other.

Let them note that the lines so drawn are also perpendicular to the other parallel line. Set them to measure the distance between the parallel lines along these perpendiculars and get them to realize that the distances between them are equal all through.



Get them also to note that the distances between two parallels will be equal even if measured along a series of parallel lines as shown in the accompanying figures but that the distance measured along the perpendicular is the shortest of them. Let pupils actually compare with a string and note the above truth. Hence get them to realize that when *the distance between two parallel straight lines* is referred to, the shortest distance *is* meant and hence the distance *along a common perpendicular* should be measured.



From the above properties point out also that if perpendiculars should be drawn to one and same straight line AB from various points C,D,E, etc., it is enough if one of the perpendiculars be drawn and ***parallels to it be then set out*** **through the other points.**

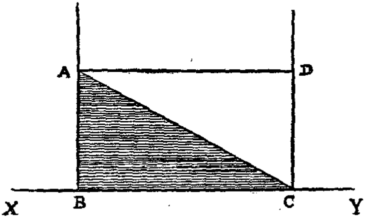
TOPIC 33.—AREAS OF IRREGULAR FIGURES AND PLOTTING OF FIELDS.

(**Scope of the treatment.**—*The pupils should be able to find the areas of plots of grounds they may have to deal with in after life as agriculturists. They should hence be taken through the elements of area calculation which are quite essential for the purpose. They must at first be taught the methods of finding the area of (1) a right-angled triangle, (2) of any kind of triangle, and (3) of a trapezium with two angles right angles. The principles underlying the triangulation and field book methods may next be explained to them. The formulae for the areas of parallelograms, quadrilaterals, trapeziums of the general type and rhombus need not be dealt with as the areas of such figures can be found by the methods of triangulation. Only simple, direct problems need be given and problems of the type of finding the length of the base of a triangle when the area and the corresponding altitude are given need not be attempted. As much time as possible should be saved so that the essentials may be done in full. Special importance should be laid upon outdoor work in connection with measuring lines and offsets for finding the areas of fields. The method of counting squares may be dealt with as applied to finding the areas of irregular figures drawn on paper with rectilineal as well as curvilineal boundaries. Also, as some of the fields the pupils may come across may have one or more curvilineal boundaries the mid-ordinate method may be explained in its fundamentals along with the field-book method in cases where it is found necessary. The Simpson method of finding the areas of irregular figures need not be dealt with as pupils are likely to find it cumber some. Plotting fields to scale may be explained and practice given in finding areas of plots from plans. How the plans given in the village survey register would help one to replace missing field stones may also be treated briefly.*)

The area of a right-angled triangle.

Ask pupils to draw any rectangle ABCD on paper. Draw also one on the blackboard. (In order that the rectangle may be drawn by the pupils without spending

much time, let them draw any straight line XY, draw any two perpendiculars to it, e.g., BA, CD, and then a parallel to XY to cut the perpendiculars at A and D). Let them



next join the diagonal AC. Do likewise on the black-board. Question into how many parts the rectangle is split thereby. Let them shade one of the two parts so obtained. Point out to them that each part is a triangle with one angle a right angle, and is hence a right-angled triangle. Taking a sheet of tracing paper, let them next trace the triangle ABC and see if the traced triangle can be placed to fit in exactly with the other portion of the rectangle, viz., the triangle ACD.

(The teacher will do well to note that the use of tracing paper in this and other demonstrations which follow helps at quick execution of the work and hence the method of tracing and superposing is strongly recommended in place of cutting and superposing. If no tracing paper could be had, oiled, translucent, paper may be got ready for the purpose, and if even this could not be had, the method of cutting and superposing may be adopted as a last resort.)

If necessary, show how the triangle ABC can be turned upside down and placed so as to coincide exactly with the triangle ADC. Thus elicit from the pupils that each of the triangles ABC and ACD is one half of the rectangle ABCD. Get pupils to realize that as the area of the rectangle ABCD is given by AB × BC, the area of the right-angled triangle ABC may be calculated from "½ AB × BC." Let pupils thus record *that the area of the right-angled triangle is given by half the product of the two sides containing the right angle.*

Propose :—

1. Find the area of a right-angled triangle ABC, with angle B a right angle, AB 3″ and BC 4″.

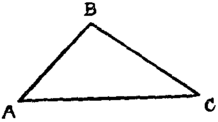
(*Ans.* 6 square inches.)

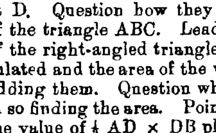
2. In a right-angled triangle AED, with angle E a right angle, AE = 300 links, and ED = 230 links. Find the area of the triangle. (*Ans.* 84 cents.)

Change the numerical data in the above and give six more exercises of a similar type.

The area of any kind of triangle.

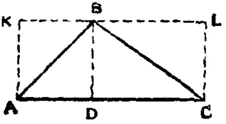
Draw on the blackboard a triangle ABC as shown *in the margin* and let pupils draw a triangle like it on paper. Question if any of its *corners are right-angular.* Tell them that in order to find the areas of such triangles a rule will now be derived. *Question* if the triangle ABC could be split into right-angled triangles. Get pupils to note that one of the easiest ways *to* do this will be to draw the perpendicular to a side from the opposite vertex, say from B to AC. Ask them *to* draw the perpendicular from B to meet AC at D. Question how they would then calculate the area of the triangle ABC. Lead them to note that the areas of the right-angled triangles ADB and CDB may be calculated and the area of the whole triangle ABC found by adding them. Question what lengths have to be measured in so finding the area. Point out that with a view to finding the value of $\frac{1}{2}$ AD × DB plus $\frac{1}{2}$ CD × DB three lengths have to be measured, AD, DC and DB. Tell them that there is a *simpler* expression for the area of the triangle, which would require only two lengths being measured. (Most of the pupils are likely to find it hard to generalize that $\frac{1}{2}$ AD × DB plus $\frac{1}{2}$ DC × DB = $\frac{1}{2}$ (AD plus DC) × DB, even if a *series of numerical* data be exhibited in a fine





order on the blackboard. Therefore, the following metho is recommended to be of easier comprehension).

On the blackboard complete the rectangle ACLK a

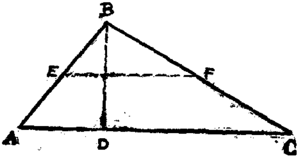


shown in the figure, by draw ing the perpendiculars AI and CL to AC and the par allel KBL to AC. Lead pu pils *to note that the triangl* ADB is half the rectangl ADBK, the triangle BDC i half the rectangle BDCL and that, therefore, the whol triangle ABC is half the rectangle ACLK. If they sug gest that the area of the rectangle will be given by AC × CL or by AC × AK, point out that DB, CL and AK are al of equal length, representing as they do the distance be tween the two parallels KL and AC, and hence that th area of the rectangle ACLK may be taken to be given b: AC × DB. Thus lead pupils to note that the area of th triangle ACB is equal to $\frac{1}{2}$ AC × DB. Set them to measur the necessary lengths on the blackboard to find the are of the triangle ABC drawn there. Point out that tw measurements will do, viz., the length of a side AC an that of a perpendicular to it from the opposite vertex Hence, let them record that *the area of a triangle is equal t half one side × the length of the perpendicular to it fror the opposite corner.*

Let pupils draw some more triangles on the blackboar or on paper and find their areas in this way.

The indigenous method.

Set pupils to draw a triangle ABC, roughly mark th



middle points of it sides AB and B(and name them I and F. Let then next measure th distance between I and F and compar it with AC. Get then to note that EF i equal to ½ AC very nearly. Tell them that the local peopl roughly mark out the positions of EF and BD in triangula plots, measure the lengths EF (*Thula*) and BD (*Arulsu*

and get the area of the triangle from EF × BD. Point out that if the positions of the middle points E and F of the sides AB and BC be correctly marked, the above method gives the accurate area.

Propose :—

3. Find the areas of triangular plots in which—

(*a*) one side is 30 yards and the perpendicular to it from the opposite vertex is 12 yards ; (*Ans.* 180 sq. yds.)

(*b*) one side is 200 feet and the perpendicular to it from the opposite vertex is 20 feet ; (*Ans.* 2,000 sq. ft.)

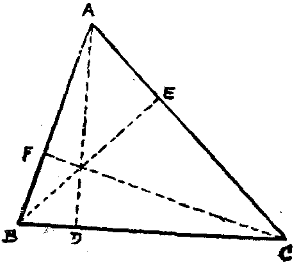
(*c*) one side is 200 links and the perpendicular to it from the opposite vertex is 150 links ; (*Ans.* 15 cents.)

(*d*) one side is 500 links and the perpendicular to it from the opposite vertex is 250 links. (*Ans.* 62½ cents.)

Before the pupils proceed to find the area, insist on their making a rough sketch of the triangle in each case above. Let them choose a side, roughly draw the perpendicular to it from the opposite vertex and note against the side and the perpendicular the lengths given. If the pupils forget multiplication by ½ in the calculation of the area of any triangle, point out to them then and there that the error they commit is one of giving "*double*" the required area.

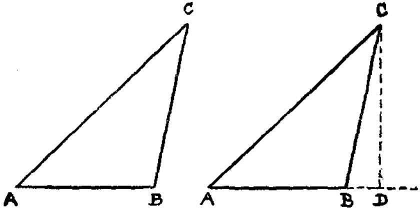
Propose :—

4. Draw a triangle ABC with each angle less than a right angle. Find the area of the triangle first by measuring AB and the perpendicular to it from C, next by measuring AC and the perpendicular to it from B, and then by measuring BC and the perpendicular to it from A. Verify if the results obtained from these three methods tally with one another.



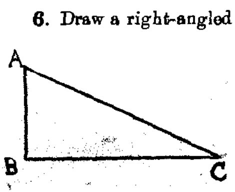
(If the results disagree, point out that the discrepancy is due to the errors in measurement and that, in spite of it, the results may be declared to be very nearly equal.)

5. Draw a triangle ABC with one angle B greater than a right angle. Which side and which perpendicular would you choose for measurement in order to find the area of the triangle? Choose AB as the side to be measured, draw the perpendicular to it from C and then find the area of the triangle.



(A common error noticeable in this connection is that the pupils would choose a line lying within the angular space ACB as the perpendicular from C to AB. If the mistake is noticed, point out that the line so drawn cannot be perpendicular to AB. Show how AB may be produced and the perpendicular drawn from C to meet the produced part at D. Impress the fact that the distance from A to B is the length of the side to be measured and that from C to D is the length of the corresponding perpendicular. This fact must be repeatedly pointed out as the pupils are likely to commit mistakes of the type of measuring AD and DC or AB and BC, etc., for finding out the area.)

6. Draw a right-angled triangle ABC with angle B a right angle. If BC be chosen as the side for measurement, which line should be measured as the perpendicular? Find the area of the triangle from these two measurements.



(Point out that, in the case of the right-angled triangle ABC above drawn, ½ AB. BC represents half the product of the sides containing the right angle as well as half one side multiplied by the perpendicular to it from the opposite corner, and hence, the use of both the formulæ would give one and the same area.)

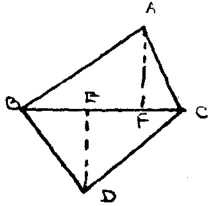
The base and altitude of a triangle.

Tell pupils that the formula "*The area of a triangle is equal to ½ one side × the perpendicular to it from the opposite corner*" is also expressed as follows:—"*The area of a triangle is equal to ½ the base × the altitude.*"

Point out that the triangle is then considered to rest on one of its sides, that this side is then referred to as the base, and the perpendicular to it from the opposite vertex as the "altitude" or "height" of the triangle. Get pupils to realize that any side of a triangle may for the purpose of the above calculation be chosen as the base and the perpendicular to it from the opposite vertex would then represent the altitude or height of the triangle.

Finding the area of polygons by triangulation.

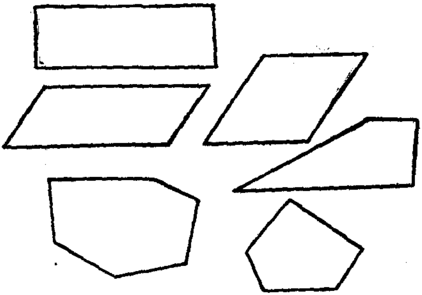
(i) Draw a quadrilateral on the blackboard. Name its corners A, B, D, C, as shown in the figure. Propose to the pupils that they will have to find the area of the portion of space bounded by the four straight lines drawn. Lead them to note that it has four sides, that its corners are named A, B, D and C and hence that as in the case of the rectangle the figure may be named ABDC.



Suggest to them, if necessary, that they may apply their knowledge of the method of finding out the areas of triangles to finding out the area of the figure in question. Lead them to note that the joining of a diagonal BC or AD would divide the quadrilateral into two triangles. Let pupils measure a diagonal and the perpendiculars to it

from the opposite vertices, and then calculate the area of the whole quadrilateral.

(ii) Draw on the blackboard a parallelogram, a rhombus, a trapezium, a pentagon, a hexagon, etc., as shown below :—



Propose to the pupils to find the area of each figure. Lead them to note that if one corner be joined to each of the other corners by means of straight lines, the figures would be split into triangles. Question what measurements they would take to find out the areas of the triangles. Lead them to take the requisite measurements and find out the area of each figure.

(The teacher need not dwell on the properties of the parallelogram or the rhombus, etc.) It is enough if the pupils know how to split the figures into triangles.)

Propose :—

1. In a quadrilateral ABCD, AC is 20 feet, the perpendiculars from D and B to AC are respectively 10 feet and 8 feet. Find the area of the quadrilateral.

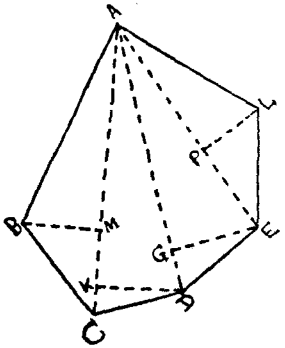
(*Ans.* 180 sq. ft.)

2. In a quadrilateral ABCD, AB = CD = 5 feet. The perpendicular from D to AB is 3 feet and that from B to CD is 4 feet. Find the area of the quadrilateral.

(*Ans.* 17½ sq. ft.)

3. Draw any four-sided figure ABCD. Join AC. Draw the perpendiculars from C to AB and from A to CD. Measuring these perpendiculars, and the corresponding sides, find the area of the figure.

4. In the accompanying figure AE represents 600 links, LP 320 links, EG 222 links, AD 700 links, AC 800 links, DK 100 links, and BM 200 links. What is the area ? (*Ans.* 2 acres, and 94 cents.)



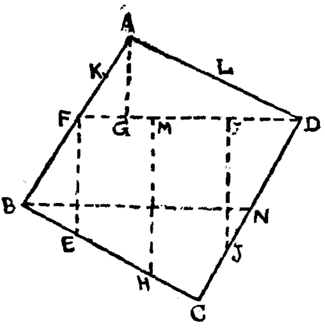
(Tell pupils that figures having many sides will hereafter be termed *"Polygons"* and that as in the case of a rectangle or a four-sided figure, the polygon above named as ALEDCB, or ABCDEL, or CDELB, that is, by giving the names of the corners in the order in which one walking along the boundary would be passing them.)

5. If, in the figure above, AC represents 250 links, BM 200 links, DK 200 links, AD 400 links, EG 250 links, AE 200 links and PL 100 links, find the area of the polygon. (*Ans.* 1 acre and 10 cents.)

Insist on the pupils' making a rough sketch when they attempt each exercise above and on their entering the measurements given against the corresponding lines in the sketch.

The indigenous method of finding the areas of polygons.

If the area of a quadrilateral plot ABCD has to be found, tell pupils that the people in the locality draw some north to south or east to west lines like DF, BN from the corners, find the area of the triangular plot AFD from KL × AM, draw three parallel lines like FE, MH, IJ through the points of trisection F, M, I of FD, take the average of these lengths to get the breadth (*Arulzu*) of the rectangle, equivalent to the part BCDF coresponding to BNchosen as its length (*Thula*), thus find the area of the part BCDF from BK × $\frac{FE + MH + IJ}{3}$ and thus get the area of the whole plot. Point out that this gives only an approximate result and in order to be fairly accurate, they must learn and apply the other methods mentioned in this chapter.

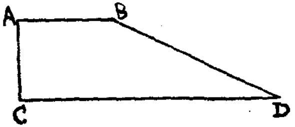


The area of a trapezium with two angles right angles.

Tell pupils that there is an alternate method of finding the areas of polygons which would be found handier in many cases and that in order to be able to apply that method they must derive a rule for finding rapidly the area of the figure to be presently described.

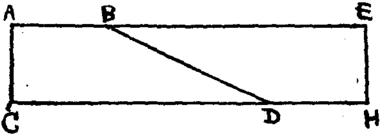
Ask pupils to draw a straight line AC and two perpendiculars to it AB and CD of different lengths, and lying on one and the same side of it. Let them join DB.

Lead pupils to note that a quadrilateral ABDC is then formed and in it two of the angles, viz., CAB and ACD are right angles. Lead them to note that AB and CD are parallel and AC and BD are not parallel.



Tell pupils that such a figure will hereafter be referred to as a *trapezoid* :

Propose to them to find out the area of such a figure. If they suggest that the figure may be split into triangles by joining a diagonal and its area obtained by finding the areas of the triangles so obtained, tell them that there is a handy rule for obtaining the area and get them to derive the rule by the following method. Ask them to produce AB and CD towards B and D. Using the compasses, let them set off, along AB produced, a length BE equal to CD, along CD produced a length DH equal to AB, and then join EH. Question what sort of a figure is thereby obtained in ACHE. Point out that it is a rectangle. Ask



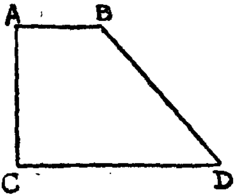
them to trace the trapezoid ACDB on tissue paper and place the traced figure in such a manner that the trapezoid ABDC coincides exactly with the trapezoid BDHE. If they are unable to place the trapezoid ABDC as above described, show how it may be done and let them do likewise. Let them note that in that position AB lies along DH, AC along HE, DC along EB, and BD along itself. Then get them to realize that the trapezoid ACDB is half the rectangle ACHE. Next lead them to note that the area of the rectangle is given by CH × AC, and as CH = (CD plus DH) = (CD plus AB) the area may be expressed as given by the sum of the parallel sides AB, CD, multiplied by AC. Hence bring out the fact that the area of

the trapezoid ACDB is equal to half the sum of the parallel sides AB, CD multiplied by AC.

Let pupils record that *the area of a trapezoid is equal to ½ the sum of the parallel sides × the side which is perpendicular to them, i.e., which represents the distance between them.*

Propose :—

In a trapezoid ABDC the sides AB and DC are parallel and the side AC is perpendicular to them. Find the area :—



(*a*) when AB = 5″, CD = 9″, AC = 4″. (*Ans.* 28 sq. inches).

(*b*) when AB = 200 links, CD = 280 links, and AC = 150 links. (*Ans.* 36 cents.)

(*c*) when AB = 240 links, CD = 160 links, and AC = 85 links. (*Ans.* 17 cents.)

(*d*) when AB = 100 links, CD = 150 links, and AC = 40 links. (*Ans.* 5 cents.)

(*e*) when AB = 200 links, CD = 250 links, and AC = 60 links. (*Ans.* 13½ cents.)

Impress on pupils the necessity for drawing a rough sketch of the trapezoid in each case above, and note the measurements against the corresponding sides and then proceed to calculate the area.)

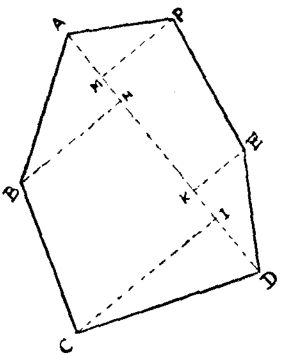
Alternate method of finding the areas of polygons. (The "Field Book" method.)

Draw on the blackboard polygons of five, six, eight, etc., sides.

Question what method the pupils know already for finding out the area of each figure. Thus remind them of the triangulation method. Tell them that they may in many cases find the following alternate method more convenient in which a knowledge of the formulæ for

calculating the areas of right-angled triangles and trapezoids will have to be applied :—

Draw on the blackboard a figure like ABCDEP. Let pupils choose the longest diagonal AD. Draw the perpendiculars PM, EK, CI, BH, to it from P, E, C, and B respectively. Get them to suggest that the area of the whole figure would be obtained from the sum of the areas of right-angled triangles AMP, DKE, CID, ABH and of the trapezoids KEPM and CIHB. Let pupils measure the necessary lengths and find the areas of the above triangles and trapezoids and then obtain the area of the entire polygon.

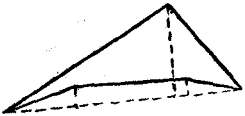


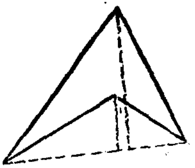
Propose measurements for the lengths measured in the right-angled triangles and trapezoids above and let pupils calculate the area of the polygon. Give a series of exercises of the above type by changing the numerical data.

Draw polygons of various shapes, regular and irregular on the blackboard and set pupils to find out their areas by the above method.

(In order to facilitate rapid reading of the measurement, the pupils may be directed to draw the polygons on squared paper or on the graph-board and then find out their areas.)

Draw on the blackboard figures of the accompanying type with one or two reflex angles. Ask pupils to find their areas by splitting them into right-angled triangles and trapezoids, or by





splitting them into triangles alone. Show how trapezoids and triangles may be formed as indicated by the dotted lines in the figure drawn in the margin and point out that instead of adding the areas of certain triangles they have to subtract the areas of one or more triangles.

Field measurement.

Tell pupils that they were hitherto finding the areas of many-sided figures as drawn on paper or on the blackboard and that they must apply a knowledge of the methods they have learnt to finding out the areas of polygonal fields and plots appearing on the surface of the earth. As a preliminary to such calculation tell them that they will have to learn how to measure lengths with a chain along uneven and sloping ground and how to set off perpendiculars to lines and measure them.

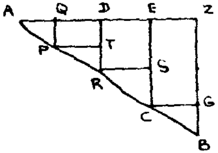
(a) Measurement along sloping ground.

Talk to the pupils about the nature of the land surface they see around them. Point out to them that in some places the land appears to be flat and in other places sloping, e.g., near the hillocks, etc. Get them to realize that, if in measuring they should be placing the measuring tape or chain along rugged, uneven ground they would be getting lengths and distances longer than the actuals. Illustrate this by measuring the distance between two spots, first by placing the chain or tape along uneven ground, next holding the chain or tape horizontally, and then by showing the disparity between the distance obtained in the two cases. Illustrate this further by pointing out that if they should be measuring the distance of the gateway near the Farm-office from a field on the other side of the neighbouring Sarada channel the measurement they would get by placing the tape or chain along the bed of the channel would be decidedly greater than that obtained by measuring the horizontal distance across the bunds, and hence,

that it is the latter measurement which must be preferred from the point of view of accuracy. Point out also that paddy plants, *ragi*, *turmeric*, plantains, sugarcane, etc., grow vertically and hence, that when people express the extent of lands, etc., they refer to the space, only as extending horizontally, i.e., along quite a level and uneven surface, e.g., the surface of still water in a lake. Tell pupils therefore that when the Government authorities measure and record the extent of lands possessed by the landlords they measure distances only horizontally. Give plenty of exercises to pupils in measuring distances horizontally along uneven ground.

Next take pupils to a sloping spot, e.g., at the foot of a *hillock or on the ground lying between the Sarada channel* and the Sarada river hard by. Propose to them that they must measure with a chain the distances (horizontal) between two places A and B.

The teacher may be guided by the accompanying figure for the nature of the slope required. If pupils hold the chain nominally horizontal but really sloping, point out to them the error, viz., of getting a length larger than the correct one, and tell them that they might get the requisite measurement correctly as follows :—



Ask one pupil (tho follower) to hold the chain at A. Let another pupil (the leader) stand at a place P down the slope so that he may in that position hold at least some part of the chain horizontally, e.g., along the distance AQ. (See figure.) Let him drop an arrow vertically and make a mark on the ground where it falls, say at R. Here point out that the distance PT represents the horizontal distance between P and R. Let the follower next go to R and the leader to a place C still further down the slope. Let the follower and the leader hold the chain horizontally, as mentioned already, and measure the horizontal distance, RS between R and C. In the same way, let them measure the horizontal distance CG between C and B. Next point out that the total horizontal distance between A and B is equal to the sum of the distances AQ, PT, RS and CG. Let pupils note that the sum of AQ, PT, RS and CG equals

that of AQ, QD, DE and EZ, i.e., equals AZ and hence gives the horizontal distance between A and B. Give plenty of exercises to pupils in measuring the horizontal distance between two places chosen along sloping and uneven grounds. If, in the course of such measurement, any pupils attempt to measure from down-hill to up-hill, point out that the measurement up-hill would be more difficult than that down-hill and hence get them to measure down-hill.

Testing a chain before measurement and removing any defects noted in it.

Talk to the pupils that even though a chain represents a length 22 yards, it often happens that the chain actually used is a bit shorter or a bit longer than the above length. Hence, impress on them the necessity for testing a chain before measurement. Tell them that one way of testing the chain would be as follows :—

On level ground let pupils stretch a chain straight, holding it moderately tight. Let them next take the *offset pole.* (The teacher need not here mention the name of the pole, but merely use the pole, making pupils realize at the same time that the length of ten such poles make up a chain.) Direct them to measure the chain by means of the pole and examine if ten such poles cover the chain from end to end. If the length of the chain falls short of the above measurement point out that the chain is defective in length. Let them then suggest how the defect in length may be made up. Get them to straighten any bent link or add spare rings to make up the defect. If, on the other hand, the chain covers a length greater than that measured by the ten offset poles, point out that there is some excess length in the chain. Let pupils suggest how the excess length may be cut short. Get them to remove some rings so as to suit the case, but never allow them to bend any of the links as thereby the chain would get spoiled. Next question what sort of error is likely to arise when lengths shorter than a chain are measured with a chain which have some rings added to it solely in some particular portion. Point out that the addition of rings to a defective chain or their removal from a chain to cut it short should be distributed uniformly, at least between the two halves of the chain, if not among the ten equal parts of it separated by

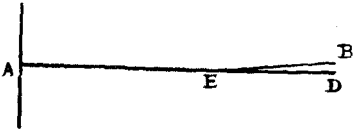
brass discs. Thus see that whenever a chain is taken up for field measurement the pupils test it beforehand as indicated above.

The use of the cross-staff in taking offsets.

Refer pupils to the various measurements taken in a polygon in order to find its area. Point out to them that the perpendiculars required were easily drawn on paper or on the blackboard with a set square and straight edge. Question pupils how they are to set off perpendiculars to the diagonal of a field from the vertices on either side of it so that their lengths may be measured for purposes of area calculation. Remind them of the use of the cross-staff to set off perpendiculars along the ground as described already when dealing with perpendiculars. As the pupils are likely to have only a faint remembrance of the method, deal with the method briefly as follows :—

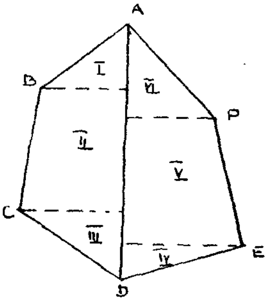
Take them to the open yard, say near the guava gardens. Choose a quadrilateral plot of ground for measurement. Question pupils which lines form the diagonals of the plot. Let them choose one of the diagonals as the chain or the base-line (preferably the longer of the two). Let them next roughly mark the place in the diagonal from which an opposite vertex appears to be in a perpendicular line. Ask a pupil to stand on the spot guessed. Let him plant the cross-staff on the base-line and turn its head so that looking along one of the two grooves in it, he may sight the flag-staffs at the end of the base-line. Let another pupil look through the perpendicular groove on the head of the cross-staff and test if he is able to see in the line of his sight the flagstaff stationed at the opposite corner. If not, let him try the cross-staff likewise at several spots along the base-line until the spot is reached from which he is able to sight the vertex in question in a direction perpendicular to that of the base-line. Fixing the spot arrange for the distance being measured between it and the opposite corner. As this distance is generally short, point out to the pupils that they may measure it with the ten link-pole. Here inform them that as this pole is used for measuring such perpendicular distances and as these perpendiculars are known as "offsets" to the base-line from the vertices in question. the pole is called "the offset-pole." Impress on them the fact that the above method of finding the length of the offset would give reliable results only for

short lengths of the offsets and errors are sure to arise for lengths of offsets exceeding three chains. Illustrate this further by drawing a diagram as in the accompanying figure and showing how the marking out of offsets is likely to be made along the lines shown as emerging from one and the same spot.



Field measurement and area calculation.

Choose a polygonal plot of ground in the neigbourhood. Take pupils there and propose to them that they have to make the necessary measurements which would help them to calculate its area. If without forethought the pupils measure any diagonal, get them to explain their procedure. Ask them to cast their eyes on the bounding lines and corners of the polygon. Let them suggest with reasons which line they would choose as the base-line and what measurements they would take in order to find out the area of the plot. Get them also to make a rough sketch on paper of the plot in question, draw in it the diagonal representing the base-line chosen, as also the lines that would be representing the offsets to the diagonal from the different corners. Set them next to handle the cross-staff and measure with the offset pole the length of the offset required. As soon as each offset is measured, insist on them to make the entry regarding the measurements against the corresponding lines in the rough sketch drawn already. Point out to them that as they have to find out the areas of trapezoids and right-angled triangles, they may have to measure not only the lengths of the offsets, but also the portions of the

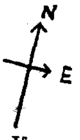


base-line which represent the distance between the parallel sides of the different trapezoids, like MH, HK, KI, etc., in the figure above. Hence, as each offset is taken, impress on them the necessity for finding out the distances of the spot *from the nearest end of the base-line or from the spot where*-from the previous offset was taken. Direct them to express the lengths so obtained, not in chains and decimals of a chain, but in links only. After the measurements have been so taken and recorded, let them examine whether they have got all the necessary measurements for finding the area. Let them then calculate the area of the whole plot by the method already known to them.

Select some more plots of the above type and give pupils plenty of exercises in field measurement and area calculation. The method of recording the measurement according to what is commonly called the field-book method need not be explained to them as it may involve more time and also cause some confusion.

Plotting of fields.

After the pupils have finished the area calculation described above, propose to them to have a clearer examination of the field so that when they go to their class-room, they may be in a position to sketch an accurate plan of the field with the measurements taken. If pupils should think that they have taken down the necessary measurements point out to them that they have to care for the direction of the boundary lines when they prepare the plan. Remind them of the conventions that in making sketches on paper or in drawing maps the top is taken to represent the north, the bottom the south, the right hand side the east and the left hand side the west. Point out to them that there is also an alternative method of representing the cardinal points by means of pointers as shown in the margin.



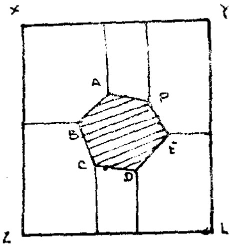
Ask them to walk once again along the boundary lines of the field, observe the corners, etc., and note which bunds and diagonals run in a north to south line or in an east to west line so that they may be guided by them in representing the directions aright while preparing the plan. If any of them should suggest the desirability of measuring the angles which the bunds or the diagonals would make with a north to south line chosen for the

purpose, tell them that their suggestion is acceptable but they have not then got the requisite apparatus for measuring accurately the angle referred to. Next take them to the class-room and there let them discuss the question of the scale to be adopted. Point out that the chóice of the scale should be made according to the measurements they have to represent and that they may in the above instance adopt a scale of one inch to 50 links. If pupils should suggest scales like 1″ to 30 links or 70 links point out that then they have to divide the number of links in the distances by 30, 70, etc., for getting the lengths to be represented on paper. Let pupils discuss which divisors may be chosen for the purposes of the above division 10, 20, 40, 50, 100 or 30, 70, 90, etc. Get them to realize that the adoption of the latter type of divisors would render the division less easy, it leads to fractional values like $2\frac{2}{3}$, $1\frac{3}{7}$, $5\frac{5}{9}$, etc., being obtained for the quotient and hence, lend to more cases of approximate calculation of the quotient. Thus, lead them to infer that the adoption of scales like 1″ to 20 links, 1″ to 50 links, or 1″ to 10 links is far more convenient and desirable than that of scales like 1″ to 30 links, 1″ to 70 links, 1″ to 130 links, 1″ to 170 links, etc. With the help of the measurements taken already, direct pupils to draw a line to represent the base-line, indicate in it the points representing the spots chosen for taking offsets, draw perpendiculars through these points to the diagonal, setting them off on one side or on the other side of the diagonal as the location of the corners of the field requires, set off to scale the lengths of the offsets along them and then join the corners so obtained by means of straight lines. In order that a person looking into the plan may have a clear idea of the directions of the field-bunds, etc., keep in view the line in the plan representing a bund or a channel in the plot running east to west or north to south, and draw short parallels to such lines so as to get the pointers indicating the cardinal points in the plan.

Obstruction to measurement of diagonal or offset.

Draw on the blackboard a figure like ABCDEP as shown in the margin. Propose to pupils that it represents a plot of sugar-cane cultivation or cassava or a bushy growth or prickly-pear. Question how they would measure the diagonal or take offsets when there is such

obstruction. Get them to realize that measurements across the field could not then be taken. Tell them that when the above area has to be determined, they may adopt a method like the following :—

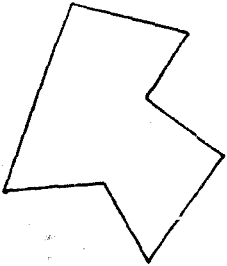


Let pupils fix four places in the garden X, Y, L and Z so that a rectangle may be formed outside the area. (A triangle may also be formed). Let pupils measure along the sides of the rectangle and take the offsets to them from the corners of the fields as shown in the above figure. Let them next calculate the area of the whole rectangle as well as that of the non-shaded portion which is *composed of right-angled triangles and trapezoids*, and obtain the area under cultivation by subtracting the area of the non-shaded portion from that of the whole rectangle.

A study of the plans in the Village Survey Register.

(*a*) *Recognition of plots from plans.*

Place at the disposal of the pupils a plan like that shown in the accompanying figure. Ask them to mark out a plot of ground roughly resembling it in shape and taking care of the directions. Let them point out which corners in the plan and in the plot correspond, which lines, etc. Next get a loan of a plan of a field as recorded in the Village Survey Register. Take pupils to the field for which the plan is secured. Set them to recognize the several bends, corners, etc., in the field corresponding

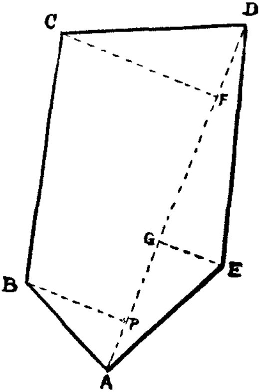


to what is seen in the plan. Remind that the actual lengths are laid down in the plan only proportionately and that the angles are shown as they actually are between the sides of the field.

Repeat the above exercise with some more plots for which plans could be secured.

(b) The use of the plans in the Village Survey Register for replacing missing field stones.

Show pupils a plan like that in the accompanying figure. Ask them to mark out a plot of ground roughly resembling it in shape and taking care of the directions. Point out where the survey stones, the corner stones, etc., are likely to be laid in the plot. Propose to them how they are to determine the correct position of these stones if they are removed from their positions, e.g. by the encroachment of neighbouring landlords, etc. Let pupils explain how bunds like AB, BC, CD may be relaid in their correct positions if the stones laid at B and C are missing. Lead them to suggest the several methods that may be adopted, e.g.,



(i) Setting off in the field angles DAB, ADB equal to the corresponding angles as found in the plan and then determining the correct position of B from the intersection of AB and DB.

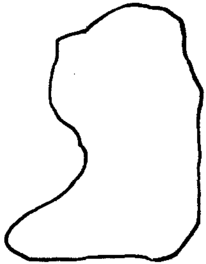
(ii) Finding the actual lengths of AB, DB from the lengths obtained from the plan, then describing circles with radii, AB, DB and determining the position of B from the point of intersection of the circumferences.

(iii) By drawing in the plan the perpendicular BP to AD, then finding the actual lengths of AP, PB from the

corresponding lengths obtained from the plan and then fixing the position of B, etc.

Let pupils next compare the methods above and find which is the most suited in practical measurements. Refer them to method (3) as the best suited. After the correct position of the field stone B has thus been marked out, lead them to proceed likewise to find the correct position of C. As a further check about the correctness of the positions of B and C thus obtained, direct pupils to measure the actual distances say, AB, EB, DB and AC., EC, DC and verify if they tally with those obtained from the plan. If they do not tally, point out that the stones B and C may be slightly shifted until the measurements are read aright. Let pupils note that the method described above, however correctly attempted in replacing a missing field stone, is sure to involve some error and that they should only see that the error is as small as possible.

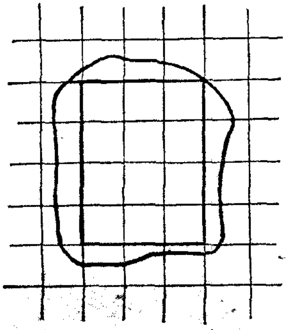
Areas of curvilineal figures.



Draw on squared-board a curvilineal figure like that shown in the margin. Propose to pupils to find its area. Point out that the formulæ which they know for finding the areas of triangles, trapezoids, etc., are not of direct application as the boundaries are curved and hence they may adopt the following method, among others which may be related later on.

Ask pupils to count the number of complete square inches in it, row after row, and question what they should do with areas less than a square inch. Refer them to the convention for effecting approximations, viz., that fractions greater than, or equal to, ½ may be counted as " one " and those less than ½ may be neglected. Point out that in applying this method of approximation, two types of errors are possible, namely :—(1) overstating the area due to counting parts equal to or greater than half a square inch

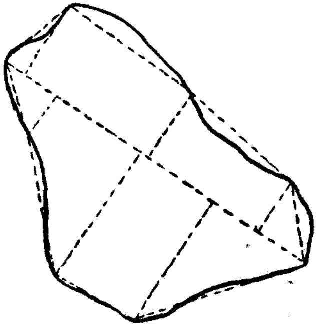
as one square inch, and (2) understating the area due to neglecting parts less than half a square inch. Let pupils note that these errors from their very nature counteract each other. If pupils betray ignorance in recognizing parts greater than, equal to or less than half a square inch, divide (an inch square) horizontally, vertically and diagonally into halves, draw curved and other irregular lines covering parts of the inch square apparently greater than, equal to or less than, half of it and thereby instil the required notion. In this way let pupils find out the areas of curvilineal figures drawn on the squared board or on squared paper. When the figures are drawn on squared paper point out to them that they have a smaller unit in terms of which the area may be expressed, viz., a hundredth of a square inch, and the convention regarding approximation should then be applied only to that unit. Get pupils to note that if the inch square alone be counted, and the convention regarding approximation applied to that square as the unit, they would be getting a result far from the accurate one. In order that they may count the squares rapidly, show how the figure given may be split into rectangles and squares as shown below, how the areas of these may be obtained by calculation and those of the remaining parts alone need then be counted. Let pupils next draw some irregular rectilineal figure on squared paper and find out their areas by the above method of counting the squares as well as by other methods. Let them test if the area found by counting the squares tallies with that obtained by calculation. If the areas do not tally, point out that the area found by counting the squares is only approximate and not accurate and hence there is some disparity between the results obtained, and nevertheless, the method of



counting the squares gives quite a fair approximate value of the accurate result.

Propose to pupils how the above method may be applied to find the areas of such irregular figures, curvilineal, or rectilineal, when they are drawn on plain paper. Point out that the area may be determined if the figures could be transferred to squared paper. Let pupils suggest how this may be done and lead them to note that the use of carbon paper, tracing paper, tissue paper or any other greased or oiled, translucent paper would be useful. Let pupils trace the figures, place them over squared paper and then find out the area of each by counting the squares as above mentioned. Or, let them oil some squared paper and place it over the figures whose areas have to be found. Let them next count the number of small squares in each area and thus express it approximately in square inches and hundredths of a square inch.

Next propose how they are to find the area of a curvilineal plot as marked on the ground. Point out that in many of the fields the plots have at least two of the opposite boundaries curvilineal, if not all. Question if they could set out a polygon on the ground roughly covering the entire plot. (Illustrate by means of a figure like that

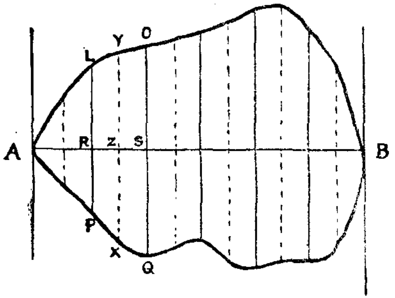


shown above.) If that be possible point out that by choosing the longest diagonal as the base-line and by

measuring the offsets to it from the corners, they may be able to find out the area of the entire plot, or they could adopt a method like the following :—

The mid-ordinate rule.

Draw on the blackboard a figure like that shown below and tell pupils that it represents a field. Propose to them

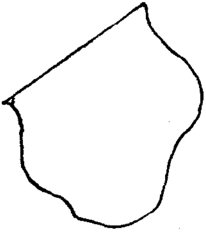


to find out its area. Direct them to mark two spots on the boundary which are most remote from each other in the field, e.g., like A and B in the above figure. Taking AB as the base-line, set them to divide it into a convenient number of equal parts, say, four, ten or sixteen, and draw perpendiculars from the points of division to AB until they reach the boundary line in the figure. Choose a strip like that between LP and OQ and ask them to show the two parts into which it is divided by RS. Question what sort of figures its two parts PRSQ and LRSO look like. Thus get them to deduce that each part is a trapezoid in its ultimate form and hence the area of the whole strip could be calculated by measuring PR and QS as well as RL and SO, taking half their sum and then multiplying by RS. Point out that half the sum above required is half the sum of LP and QO and hence that the area of the strip in question will be given by RS × half the sum of PL and QO. Next get pupils to mark the middle point Z of RS

and draw a perpendicular XZY to RS. Ask them to measure XY, PL and QO and compare XY with half the sum PL and QO. Thus lead them to infer that XY is very nearly equal to half the sum of PL and QO and hence that XY × RS will give the area of the portion LRSO. Similarly get them to infer that the area of each of the other strips can be obtained from "the width of the strip corresponding to RS × the length of the line in the strip corresponding to XY." Let pupils note that as all the strips are of equal width the work may be abbreviated by first finding the sum of the lengths like XY in all the strips and then multiplying it by RS. Point out that the area so obtained would be only approximate and that as the error is generally so slight as to be overlooked, a fair approximation to the actual area is considered to be given by the above method. Let pupils note that the accuracy of the result depends on the shortness of the distance between the consecutive perpendiculars. Take pupils to plots with curvilineal boundaries. Set them to find out their areas by the above method.

Finding the area of a figure from a plan drawn to a scale.

Draw on the blackboard an irregular figure as shown in the margin and tell pupils that it represents the plan of a field as found in the Karnam's register drawn to a scale of one inch to two chains. Question pupils how its area may be found by counting the squares. Suppose they find the approximate area to be 5 square inches. Question what actual lengths would be represented by the sides of a square 1″ by 1″ appearing in the above plan. Let them hence find out that a square, one inch by one inch, really represents a square two



chains by two chains, i.e., one square inch on the plan or the map represents an actual area of four square chains. Question therefore how many square chains would be represented by five square inches in the plan, and let them

obtain the real area of the field to be 20 square chains, or 2 acres. Let pupils find out the area represented by the above figure taking one inch as equal to 4, 5, 10 and 20 chains respectively. Point out that a square, one inch by one inch represents an actual area of 4 chains by 4 chains, 5 chains by 5 chains, 10 chains by 10 chains and 20 chains by 20 chains, i.e., of 4×4, 5×5, 10×10, and 20×20 square chains.

Draw some irregular figures on the blackboard, state that they represent fields to scale, mention the scale, get pupils to find out the areas of the figures and then derive the areas of the fields.

TOPIC 34.—VOLUME-MEASUREMENT.

(**Scope of the treatment.**—*Space-notions may at first be given of volume and a few exercises set on the comparison of volumes at sight. The necessity for a unit in volume-measurement and the choice of a cubic inch as a unit may next be dealt with. The formula for the volumes of rectangular blocks may be derived and the table of British cubic measure built up. How people in the locality measure brick-work, earth-work and timber may next be explained and a few exercises set on them. That volume-measurement is closely allied to the capacity-measurement already dealt with may be well impressed and the application of the principle of volume-weights in recognizing weight by volume and vice versa, especially in timber-measurement, may be shown. In the case of measurement of round logs, etc., the method of quarter-girth measurement need alone be dealt with. The teacher may find that he is hard up for time to deal with the formulæ giving the volume of a cylinder or of a prismoid in general and besides the pupils of this school are not likely to find very useful practical applications of the formulæ. Plenty of drill may be given in estimating the volumes of beams, masonry, etc., at sight.*)

Space notions about volume and the necessity for a unit in volume-measurement.—Place before pupils two vessels. Question them which vessel could hold more. If pupils suggest that they would place one vessel within another, direct them to try the method and decide which vessel could hold more.

Next place before pupils two vessels, one broad-mouthed but short in height and another narrow-mouthed and very tall. Question them how they would decide which of the vessels could hold more. Get pupils to note that if they should measure oil, water, sand or anything with one vessel and empty the contents in another they could decide which could hold more. Remind them that they may use the measures, "kuncham," "adda," etc., and by measuring with them express the capacity of the vessels and compare their contents.

Next propose which could hold more grain, either the central room in the store house or the corner room there (Choose for the purpose two rooms which are apparently equal.). Get pupils to note that if they should be measuring in kunchams and filling up the rooms with grain they could decide which can hold more but that such a method would take an unduly long time, would be tedions and tiresome and besides to fill up the whole space with grain until the room is quite full is almost an impossibility. Point out to them that in the above cases the comparison would be convenient to make if the quantity of air-space in each room is expressed in terms of a fixed unit.

Get two small walls of bricks raised in front of the school-yard. Take pupils to them and get them to find which wall occupies more space and by how much. Lead them to note that the number of bricks may be counted, the amount of space occupied by each wall may be expressed as so many brick-spaces and then the comparison required may be easily made. Incidentally point out the advantage of using a brick-space as a unit in that the bricks by virtue of their rectangular shape fit in with each other without leaving wide gaps as inter-spaces.

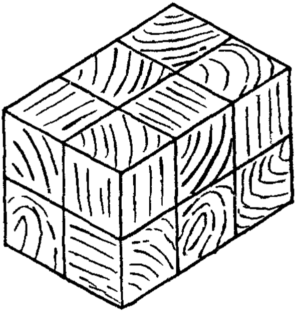
Place before pupils two rectangular blocks of different dimensions, one of them appearing decidedly bigger than the other. Question which of the two blocks occupies a greater amonnt of space and how much one block is bigger than the other. Repeat this exercise with a short stont block and a long narrow block. Point out that to express definitely the quantity of air-space occupied by each wall and to make the comparison it would be convenient if the air-space occupied by each block is expressed in terms of some chosen unit corresponding to the brick-space above used in comparing wall-spaces.

The adoption of the cubic inch as a unit in volume-measurement.

Show pupils a wooden cube with each edge 1 inch long and question what its length, thickness and height are. Point out the amount of air-space which it occupies and get pupils to note that the space could easily be set out and recognized and remembered by every one who has an idea of the length of an inch. Tell pupils that the

amount of space covered by the cube is hence adopted as a unit in volume measurement and is known as "*a cubic-inch*" of space.

Place before the pupils some inch-cubes. First arrange them so as to get a rectangular block 3 inches long, 2 inches wide and 1 inch high. Question how much of space the block occupies. Point out that in indicating the amount of space the solid may be shown as extending along its length, breadth and height. Get pupils to note that it contains 6 cubic inches.



Next form a rectangular block 3 inches long, 2 inches wide, and 2 inches high. Question pupils how much of air-space the block occupies. Let them count the inch-cubes and find out that it occupies 12 cubic inches.

Next place at the disposal of pupils a rectangular tin vessel. Question them how many cubic-inches of material it will hold. Let pupils put in inch-cubes into it, one after another and count how many inch-cubes it would hold. If they should suggest that there is some extra space in the vessel which could not be covered by thrusting the inch-cubes, tell them that they will presently learn how to measure that space too and express it in cubic inches. Point out that by so expressing the amount of space in terms of a chosen unit, say, in cubic-inches, they will be able to express how much of space a vessel contains or a block covers and thus compare the amount of space covered by different blocks or contained in different vessels, etc. Point out that this method of expressing the space-contents is closely allied to the measurement of capacity or holding and that when the term capacity is used, reference

has been made to a vessel, bag, basket or room which could be filled up with materials and from which the materials could be removed after so filling up. Tell pupils that hereafter the term "volume" will be used to denote the quantity of air-space covered by any object irrespective of its form, the material of which it is made, etc.

Prepare an inch cube of clay or plasticine or cut it out of plantain stems, plantain fruit or guava fruit. Split it into irregular pieces or mould the clay and the plasticine to assume different forms. Point out that in all these forms the amount of air-space covered is only 1 cubic inch and so bodies which are not exactly cubical or cuboidal in form may also have a volume of one cubic inch.

The formula giving the volume of a rectangular block in terms of its dimensions.

Place some inch-cubes at the disposal of the pupils and let them arrange them so as to get rectangular blocks of various dimensions, e.g., 3″ long, 2″ wide and 1″ high, 6″ long, 3″ wide and 2″ high, 5″ long, 4″ wide and 2″ high, etc. Let them exhibit in a tabular form the dimensions and their volumes as follows :—

Length of block in inches.	Its breadth in inches.	Its height in inches.	Its volume in cubic inches.
3	2	1	6
6	3	2	36
5	4	2	40

Direct pupils to examine the numbers and suggest a short way of finding out the volume instead of counting the number of inch-cubes. Lead them to note that the product of the number of inches in the three dimensions above specified would give the number of cubic inches in the volume.

Cut out an inch-cube of wet clay, fruit, or some other substance across the middle of a face and get two equal rectangular pieces. Ask pupils how much of space each piece occupies. Get them to note that each piece is then 1″ long, 1″ wide and ½″ high and its volume is half a cubic inch, the piece being half of an inch cube. Point out that the volume of half a cubic inch may also be obtained from the product 1 × 1 × ½ as indicated by the above rule. Similarly cut out this piece into two equal rectangular

pieces ½" long, ½" wide and 1" high. Let them derive the volume to be ¼ cubic inch by considering that the piece is ¼ of an inch-cube. Let them next examine whether the above rule of finding the volume from the product $\frac{1}{2} \times \frac{1}{2} \times 1$ would hold good and see that it does. Again, split the piece into two halves so as to get a block ½" long, ½" wide and ½" high. Get them to note that the block is ⅛ cubic inch. Thus let pupils generalize that the above method of finding the volume would hold good for all lengths chosen for the dimensions of a rectangular block. Tell them that it is usual to remember the rule shortly as follows:—

Volume of a rectangular block = its length × its breadth × its height.

Point out to pupils that in the case of a cube the three dimensions are equal and hence the volume would be given by (length) × (length) × (length).

Propose to pupils to find out the volume of a rectangular block 2½" long, 2" wide and ¼" thick. Let them work as follows:—

$$\text{The volume required} = 2\tfrac{1}{2} \times 2 \times \tfrac{1}{4} \text{ cubic inches.}$$
$$= 1\tfrac{1}{4} \text{ cubic inches.}$$

Tell pupils that hereafter a rectangular block, 4" long, 3" wide and 8" high will be referred to as one 4" by 8" by 8" and also that the term "Cubic inches" will be shortly written as "C. ins."

The adoption of bigger units in volume measurement.

(*a*) *The cubic foot* :—Refer pupils to a long rectangular beam of wood and question them how they would find its volume. Get them to measure its dimensions and calculate the volume in cubic inches by applying the above rule. Point out that a very big number is obtained in the result and that they would find it convenient to adopt a larger unit for volume measurement. Tell them just as they formed a cube 1" long, 1" broad and 1" high and name the amount of air-space occupied as one cubic inch, they may also form a cube 1 ft. long, 1 ft. broad and 1 ft. high and name the amount of air-space occupied by it as one cubic-foot. Get pupils to express the dimensions of a foot-cube in inches and find its volume in cubic inches.

Let them thus derive that 12 × 12 × 12 or 1,728 *cub inches make one cubic foot.* Point out to them that th equivalence would help them to express a volume given i cubic inches in terms of a cubic foot and vice versa.

Show them rectangular blocks prepared out of clay havin a volume of half a cubic foot, one-fourth of a cubic foo etc. Lead them to note that the rule already derived fo finding out the volume may also be applied in the case o rectangular blocks whose dimensions are expressed in fee Impress on them to remember that the number of cubi feet in the volume of a rectangular block will be given b multiplying together the number of feet in the lengtl breadth and height.

Propose:—A rectangular beam of wood is 18′ long wide and 9″ thick. Find the volume of wood in it.

Let pupils work as follows:—

Length	=	18′
Breadth	=	1′
Thickness	=	$\frac{3}{4}$′
∴ Volume	=	18 × 1 × $\frac{3}{4}$ c.ft.
	=	13$\frac{1}{2}$ c. ft.

The adoption of the cubic yard as a unit.

Refer pupils to big solid structures, huge mounds, larg quantities of earth dug out in levelling fields, etc., and tel them that it is also usual to express their volumes in cubi yards. Point out to them that a cubic yard is taken as th amount of space covered by a cube 1 yard long, 1 yar wide and 1 yard high.

Show pupils how much that space would come to. Ge them to note that as the dimensions of the cube are 3 ft. b 3 ft. by 3 ft. the volume may also be expressed as 3 × 3 × 3 or 27 cubic feet. Point out that if they remember th above equivalence they may be able to express fractions o a cubic yard easily in cubic feet and vice versa.

Point out that in practical life that they would neve have to deal with the amount of air-space occupied b solids 1 chain long, 1 chain wide and 1 chain high nor b solids 1 mile long, 1 mile wide and 1 mile high and henc units bigger than a cubic yard are not prevalent.

Exercises.

Propose:—

1. Find the volumes of rectangular beams 18 ft. by 9 ins. by 6 ins.; 18 ft. by 6″ by 6″; 10′ by 1′ by 10″.
(*Ans.* 6¾ c.ft., 4½ c.ft., 8⅓ c.ft.)

2. A wall is 24 ft. long, 12 ft. high and 16″ thick. Find the volume of masonry in the wall. (*Ans.* 384 c. ft.)

3. There are two rooms in a granary—one 15′ by 10′ by 10′ and another 16′ by 12′ by 8′. Which could hold more of grain? (*Ans.* The latter.)

4. A rectangular block of wood is 12′ long, 1′ 2″ wide and 26″ thick. Find its volume and estimate the cost at Rs. 1-2-0 per cubic foot. (*Ans.* Rs. 11-13-0.)

5. A rectangular block of teak is 15′ long, 10″ thick and 1′ high. Find its cost at Rs. 4 a cubic foot.
(*Ans.* Rs. 50.)

6. For laying out the foundation for a wall a basement 24 ft. long, 2 ft. wide and 10 ft. high has been raised. If the contractor supplies rubble at Rs. 1-8-0 a cubic yard find the cost of rubble for the basement.
(*Ans.* Rs. 80.)

7. Find how much of air-space may be added to a hall by removing a partition wall in the middle 12 ft. long, 11 ft. high and 13 ft. thick. (*Ans.* 143 c.ft.)

8. A room should contain 1,800 c.ft. of air-space. What dimensions would you choose for it?
(*Ans.* 15′ by 12′ by 10′ or 20′ by 10′ by 9′, etc.)

9. A plot is 150′ long and 120′ wide. It is irrigated so that water stands to a height of 3″ in it. Find in cubic feet the quantity of water then found in the plot. (Here explain incidentally what is meant by an inch of rainfall.)
(*Ans.* 4,500 c.ft.)

Change the numerical data in the above and propose a series of exercises.

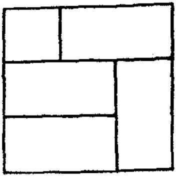
The local method of measuring brick-work.

Propose to pupils that a wall is 11′ long (podavu), 11′ high (ethu) and 13″ broad (thalasari). Question them how they would find the number of bricks required for erecting it. Show them the ordinary brick used in the locality and

get them to note that it is 8″ long, 4″ wide and 2″ thick. Leave pupils to work as follows :—

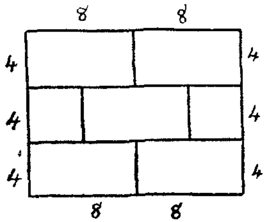
The length is 11′ or 132 inches and so the number of bricks that could be placed lengthwise is given by 132″ ÷ 8″ or 16½. The breadth is 13″ and so if bricks be placed breadthwise, 13/4 or three rows of bricks will be required to cover 13″, so altogether in a layer of the wall there will be 16½ × 3 or 49½ bricks. As the thickness of a brick is 2″, to cover height of 1 ft., 12 divided by 2 or 6 layers are required and so for a layer 1 ft. high, 49½ × 6 or 297 bricks will be required, and so for covering up the entire height of 11 ft. 297 × 11 or 3,267 bricks will be required.

Point out in practice the mason leaves some interspace between one brick and another and covers it up with mud or sunnamulu and hence the number 3267 calculated above does not give the actuals required. Point out that the actual number of bricks required may be calculated by knowing how the bricks are laid in building a wall. Describe to pupils that the thickness of walls is generally 2 brick-lengths, *i.e.*, 16 inches or 1 brick-length plus 1 brick-breadth, *i.e.*, 12″. Get them to note that if the walls are 1 ft.

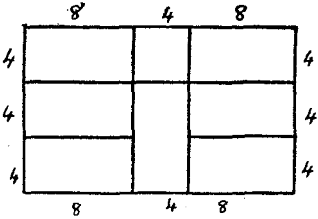


thick, 4½ bricks are counted for a layer, 1 ft. long, 1 ft. wide (see the figure in the margin) where three bricks are placed length-wise and 1 brick breadthwise and ½ a brick at the corner. Tell them also that 5 such layers alone are reckoned to cover a square foot of wall surface. Explain this by showing that even though they ought to cover only up to a height of 5 × 2 or 10 inches it is found that in practice they cover up to a height of 1 foot owing to the inter-space being covered by a thin layer of mortar. Get them to derive therefore that for one cubic foot 5 × 4½ or 22½ bricks may be reckoned in raising such walls.

Next describe to pupils how in some houses the walls are laid 16″ thick. Get them to arrange bricks as in the margin and therefrom to infer that to raise a part so as to cover a foot square of surface on the wall on a layer 16″ by 12″, 5 layers of bricks 2″ thick are alone required, because of the mud covering the inter-space. Let pupils work out that 5 × 6 or 30 *bricks are required for covering up a foot square of wall—surface,* hence to raise a wall so as to cover 1 square foot of wall-surface on a layer 12″ by 12″, the number of bricks required is 30 × 12/16 or 22½ and thus let pupils reckon 22½ bricks as before for a cubic foot of wall of the above type.



Next describe to pupils how in some houses the outer walls are laid 16″ thick and the inner walls 20″ thick. Propose to them to find the number of bricks that may be reckoned for every square foot of wall-surface. Get pupils to arrange the bricks as shown in the margin and thus work out that if the walls be 20″thick the number of bricks required for a layer 20″ by 12″ is 7½ and counting 5 such layers as before for covering a square foot of wall-surface the number of bricks required for covering a square foot of wall surface may be estimated as 37½. Let pupils work out that this also comes to 22½ bricks for a cubic foot.



Measurement of earth-work.

Propose to pupils that wages have to be calculated for levelling land on hill slopes and other places. Show them

how when the coolies dig out the earth they leave ridges in the middle, measure the height of the ridges, take the average and multiply by the other dimensions of a rectangular block thus formed and calculate wages at 9 pies to 24 pies for a cubic yard of earth removed.

Propose :—**1.** A pit-well about 20 ft. wide, 20 ft. long and 18 ft. high has been dug out of earth. Calculate the cost of digging at 2 annas a cubic yard. (*Ans.* Rs. 33–5–4.)

2. In levelling a field the height of the ridges left in the middle have been found to be 5 ft., 4 ft., 3 ft. and 2½ ft. respectively and the rectangular space levelled out found to be 240 ft. by 180 ft. Calculate the wages for levelling the land at 1 anna 4 pies a cubic yard. (*Ans.* Rs. 483–5–4.)

3. If 7 earthen pillars of height 2½, 3, 4, 5, 5¼, 4¼, 3¼ ft. are left on a ground 50 yds. long and 45 yds. wide when levelling, find the wages for levelling at 4 dabbus a cubic yard. (*Ans.* 250.)

Measurement of timber.

Refer pupils to rectangular beams of wood used in house-building. Get them to suggest how they would find out their volumes. Point out that the length, breadth and thickness may be measured and the volume found by the method already mentioned. Remind pupils also of the theory of volume-weight, of recognizing weight by volume and of volume by weight. Tell them that in some places the weight of teak, in particular, is expressed in tons, thus, 1 ton would mean a volume of 100 c.ft. and ½ ton would mean a volume of 50 c.ft.

Propose :—A beam of teak-wood, 18 ft. long, 1 ft. 3 inches wide and 1′ 4″ high, is bought at Rs. 250 a ton. Find the cost of the beam.

Let pupils assume that the cost of 100 c.ft. is Rs. 250 and work out the result. (*Ans.* Rs. 75.)

Show pupils round logs of wood. Question if they would take them to be exact cylinders. Let them examine their ends and note that though they look like circles they are not exact circles, satisfying all the geometrical properties. Point out to them that in some cases the ends may be oval too. Get them also to note that in many cases the two ends of the logs are not exactly equal, etc. Next

refer pupils to how such timber is measured in the timber-yard. Tell them that the people measure the girth in three or more places and thus calculate the average girth from the measurments and that they then refer to a pocket book which gives out the number of cubic feet in the volume when the girth and the length are known.

Point out to pupils that the following method-underlies the readings given in the table. Suppose the average girth is found to be 3 feet 6 inches. Then the quarter girth comes to 10½". If the length is 10 feet, they find out the volume from the expression (length in feet) × (quarter-girth in feet) × (quarter girth in feet). Thus they get the volume of the above log to be 10 × (21/2 × 1/12) × (21/2 × 1/12) or 245/21 c.ft., i.e., 721/32 c.ft. Inform pupils that the volume of wood given by the above calculation is only about ¾ of the actual quantity in the log and that such a measurement is allowed because of the wastage involved in cutting out the log into scantlings, beams, etc., fit for use.

Propose :—

1. A dhoolam is found to be 18 feet long and 48 inches in average girth. Find the cost of the wood at Rs. 1–14–0 a cubic foot. If such a log is to be bought of teak-wood it would be valued at Rs. 4 a cubic foot or of Nallamadhi at Rs. 1–12–0 a cubic foot. Find the cost of the log of the same dimensions (1) of teak used (2) of Nallamadhi.

(*Ans.* Rs. 33–12–0, Rs. 72–0–0, Rs. 31–8–0.)

2. A round log is 15′ 9″ long and 5′ 4″ in girth. If the wood be of Thelavalama and costs Rs. 1–2–0 a cubic foot, find its price. (*Ans.* Rs. 31–8–0.)

Estimating volume at sight.

(*a*) Show pupils certain rectangular blocks of wood, masonry, etc., and propose to them to estimate their cubical contents. Get them to measure by the eye the length, breadth and height and estimate the volume by multiplying roughly.

(*b*) Next show them round logs of wood, etc., and propose to them to estimate the volume. Remind them that the girth may be taken to be slightly greater than the diameter. Get them to estimate at sight the diameter

of an end and also the length of the log and thus obtain roughly the volume of log by the quarter-girth method. In the case of masonry work point out that the above method gives only ¾ of the actual volume and that therefore they may add to the result obtained 1/3 of it and thus get at a fairly correct estimate.

(*c*) **Quantity of straw in stack.**—Take pupils to some straw-stacks in the neighbourhood and propose to them to estimate the quantity of straw in the stacks. Get *them to note that the stacks which the ryots arrange to* have in the locality are generally 6 feet in height and are seldom more than 8 feet in height, the width is generally 4 yards at the bottom and the lengths alone differ. Tell them that the people in the locality judge of the quantity of straw to be 1½ cart-loads in a stack 5 yards long, 6 feet high and 4 yards wide, each cart-load containing 30 small bundles, two of which they see in a kavadi. Thus get them to work out that a stack 7½ yards long would contain 1½ × 1½ or 2¼ cart-loads and that they may similarly *increase the quantity proportionately to the increase in* length and in other dimensions if any and thereby get at approximate results.

Give plenty of practice to pupils until they are able to gauge at sight the approximate cubical contents of structures.

TOPIC 35.—SIMPLE MACHINES.

(**Scope of the treatment.**—*At first the boys may be given some idea about the transmission of force and the necessity for using mechanical contrivances in the transmission of force. The principle of the lever may next be explained and along with it the mechanical advantage or other conveniences afforded by using levers of the first, second and third order. Ideas about the use of fixed and movable pulleys may next be dealt with. Elementary notions about the inclined plane, the wedge, the screw and the toothed wheels may next be imparted. As the object is only to impart very elementary notions about the simple machines, too much of example-grinding need not be attempted. It is enough if the fundamental principles are made familiar to the pupils and a few of the exercises given worked out.*)

Section (I).—The use of mechanical appliances in the advantageous transmission of force.

(*a*) *The necessity for such appliances.*

Refer to pupils' experience as to how a massive beam or stone lying on the ground is shifted from one place to another, how a large quantity of water is baled out from wells from irrigation purposes, how a heavy beam, stone or pillar is carried from a lower to a higher level in house-building, how a heavy log of timber lying on the ground is placed in a country cart, etc.

Question whether they can do such work single-handed. Point out that there are some logs of wood, blocks of stone, etc., which cannot be moved directly by them even if some or all of them join together and attempt to shift them. Get the pupils to describe some of the mechanical contrivances with which they are familiar, like the crowbar in heaving up stones and timber, the picottah in baling out water to fields, the pulley for drawing water from a well, an inclined pair of planks in rolling rice-bags, etc., from boats to canal banks, pumps of various kinds, etc. Let them realize that in the absence of such contrivances a limited number of workmen will not be able to cope with the work, that there are some pieces of work, e.g., the pressing of bales of cotton, palm-fibre, etc., which cannot

be done even by many workmen and that these mechanical contrivances enable a piece of work being performed quickly and efficiently with a saving of much time and labour. Tell pupils that they would presently study in greater detail about the advantages of using the contrivances and ascertain exactly wherever possible what advantage there is.

(b) *Elementary Ideas of Force and Work.*

Take pupils to the open school-yard, choose a fairly big sized block of stone and propose to them that it should be lifted from where it is and removed elsewhere. Let one of the pupils attempt to lift it. Point out that he exerts some force. When they find that he could not lift it, lead them to realize that in virtue of its weight it successfully resists the force he applies. Here point out that the weight of the stone is also a force. Ask another pupil, and, if necessary two or three more, to help him to get the block of stone lifted and removed as required.

Question why they are then able to lift it. Point out that the force due to its weight has been overcome by the total force applied by the four pupils and that the force applied by each person engaged in lifting it gets less as a larger number join hands. Point out also that the force exerted by a person individually when four are employed may reasonably be expected to be a fourth of the total weight (force) of the stone. Let pupils work out the fraction of the total weight falling to the share of each, when a different number, say five, six, eight and ten are put to the task and drill the fact that the total weight tends to be divided fairly equally among them. Make them realize that there is similar distribution of force between the two bullocks engaged in drawing a cart or in dragging a country plough and between the two men who are lifting water by means of swing baskets over a low embankment for purposes of irrigation.

Next, suggest to pupils the case of a farm cooly carrying a basket of ragi weighing, say, a maund to the top-floor of the farm store-house, 12 feet high. Point out that in carrying it the force which is due to the weight of the ragi is continuously acting downwards and that in carrying it up the cooly is counteracting that force. Lead the pupils to realize that if the cooly carries two such basket-loads of ragi he will be doing exactly twice the amount of work.

Again suggest the cooly carrying a single block once and another block again and get from the pupils that then he does twice the amount of work. Vary the examples, e.g., a man hauling up water in buckets of various sizes and thus lead pupils to generalize that the amount of work that a person does depends on two factors—(i) the force which he overcomes, generally due to the weight and (ii) the distance through which he overcomes the force.

Section (II)—A—The lever.

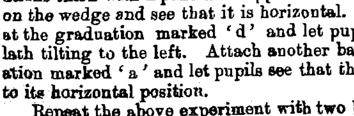
(i) *The principle of the lever.*

For performing the following experiments, get ready the apparatus stated below :—

An ordinary rectangular lath of wood about a yard long, 1 inch wide and ¼ inch thick with a groove at its centre across the lath : a wooden wedge about a foot high to support the lath : a number of small cloth bags (of the type used for storing sample seed in the Samalkotta Farm), some quantity of paddy, a common balance and a set of weights : and some pieces of string.

Experiment I.

Get ready a dozen bags of paddy, each weighing ¼lb. Divide the lath, say, into 6 equal parts about 6 inches apart either side of the central groove and mark the graduations thick with a pencil.



Support the lath at the centre on the wedge and see that it is horizontal. Attach one bag at the graduation marked 'd' and let pupils observe the lath tilting to the left. Attach another bag at the graduation marked 'a' and let pupils see that the lath is restored to its horizontal position.

Repeat the above experiment with two bags, three bags, etc., and get from pupils that *equal weights at the graduations marked 'a' and 'd' keep the lath horizontal,* in other words, *"balance" each other.*

Try the bags at 'e' and 'b', 'f' and 'c' and let pupils verify if the above statement would hold true.

Thus, lead them to ennunciate the principle that *equal weights attached at equal distances from the point of support balance each other.*

Ask pupils to examine a common balance. Let one of them measure the arms of the balance and the class observe that the arms are equal in length. From the principle deduced above, lead them to see that, when the weights in the pans of the above balance keep the beam horizontal, those two weights must be equal.

Experiment II.

Suspend one bag at 'e' and two bags at 'a'. Let pupils examine the position of the lath and note that it then remains horizontal. Similarly, try two bags at 'e' and four bags at 'a' and let pupils note that these keep the lath horizontal. Point out that in these positions the weight at 'e' is half that at 'a' and the distance of 'e' from the point of support is double that of 'a'.

Suspend one bag at 'f' and three bags at 'a'. Let pupils examine the position of the lath and note that it then remains horizontal. Similarly, try 2 bags at 'f' and six at 'a' and let pupils note that these keep the lath horizontal. Point out that in these positions the weight at 'f' is a third of the weight at 'a' and the distance of 'f' from the point of support is thrice that of 'a'.

Similarly, try 3 bags at 'b,' 6 bags at 'd,' and so on. Lead pupils to deduce that *a less weight attached at a greater distance from the point of support balances a larger weight at a smaller distance.*

Record the results of the above observations in a tabular form on the blackboard as follows :—

Left side of lath.		Right side of lath.	
(a)	(b)	(a)	(b)
Weight attached.	Distance of weight from the point of support.	Weight attached.	Distance of weight from the point of support.
(i) 1 bag	1 division.	1 bag	1 division.
(ii) 1 bag	2 divisions.	2 bags	1 do.
(iii) 1 bag	3 do.	3 bags	1 do.
(iv) 2 bags	3 do.	3 bags	2 divisions.

Ask pupils to work out in each case the products of the numbers in the columns (*a*) and (*b*) above as well as of those in columns (*c*) and (*d*). Let them express the products in the form of a table as follows :—

—	I. Product of the numbers in the columns (a) and (b).	II. Product of the numbers in the columns (c) and (d).
(i)	1	1
(ii)	2	2
(iii)	3	3
(iv)	6	6

Let the pupils examine the figures in columns I and II above and note that they are identical. Lead them to formulate the principle that, in the case of the lath supported at the middle and remaining horizontal under two weights placed suitably on either side of the place of support, the relation between the weights and the respective distances may be given by

Weight on left side × Distance of weight on left side from the point of support.	=	Weight on right side × Distance of weight on right side from the point of support.

Inform the pupils that the same results would hold good with the lath pivoted about a point or suspended, or the weights applied as a push from above or a pull from below. Tell them that such an arrangement as a lath so supported as to turn *freely* about a fixed point in its length is called a '*Lever*', the point about which it turns is called the '*Fulcrum*', one of the weights the '*Load*', the other the '*Effort*' or '*balancing load*', the portion of the lever on either side of the fulcrum is called an '*Arm*' of the lever, the arm at which the load is attached is called the '*Load-Arm*' and that at which the effort is applied is called the '*Effort-Arm*'. Point out to pupils that the term '*Arm*' of the lever means the distance between the point of support and that at which the load or effort acts.

(Intimate to pupils that the principle of the lever above stated may be easily remembered in the form),

Load × *Load-Arm* = *Effort* × *Effort-Arm*.

(The teacher will do well to note that the principle of the lever is also ennunciated as follows :—

$\frac{\text{'Load'}}{\text{Effort}} = \frac{\text{'Effort-Arm'}}{\text{Load-Arm.}}$ This form of ennunciation is no doubt an improvement. Still the former method of ennunciation may be adopted as it is found by experience to be easier of comprehension by the pupils and the inverse ratio involved in the latter method of formulating the principle, *in particular, causes a good deal of confusion in* them).

Make the pupils understand very clearly that the *load* and the *effort* though having different names are in reality *forces* due to *weights*.

Also, draw the pupils' attention to the fact that in the practical use of the lever the weight of the lever is a factor to be reckoned with and point out that in the above experiment, as the lever has been supported at its middle, the weight of one arm exactly balances that of the other.

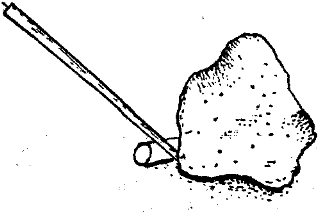
(ii) *Kinds of Levers.*

Let the pupils note that the lever used in the above experiment is supported, that is, has its fulcrum, at the middle and that the load and effort are applied on either side of it.

Instances.

(*a*) *The Crowbar.*

Refer pupils to the crowbar used in lifting a heavy block of stone. Take them out and direct them to handle the crowbar to lift a fairly big-sized stone. Lead *pupils* to note that a small stone or peg is put near the big block to be lifted, the crowbar is placed



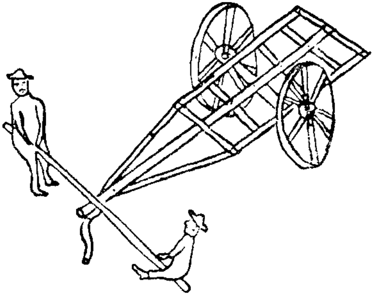
so as to rest on the small stone, one end of the crowbar rests firmly up against the bottom of the big block and a fairly easy push given to the other end is able to heave the stone to a certain height. Get from the pupils that the small stone on which the crowbar rests is "*The Fulcrum*" of the lever, the heavy stone which is being heaved up is "*The Weight*" and the push given by them at the other end is the "*Effort*". Lead them to see that the length of the load arm is very short when compared with that of the effort arm, that in this case the load and the effort are both directed downwards and the fulcrum is between the load and the effort. Get them to apply the principle of the lever they have learnt and explain that as the effort arm is far longer than the load arm, a very easy push given to the former is able to dislodge the stone. Point out that by making the effort arm longer, smaller effort would be sufficient to move a greater load and that thus there is certainly an advantage in the matter of the force applied to overcome a given force. Tell pupils that as this advantage is derived by the use of a machine it is called a *mechanical advantage* of the machine and is represented by Load÷Effort. Illustrate this further by pointing out that if a load of 10 lb. is lifted by an effort of 5 lb., the ratio of the load to the effort, viz., 10 lb., 15 lb., or 2 is the mechanical advantage gained by the use of the machine.

Propose to pupils to consider how the weight of the lever acts. Tell them that the weights of bodies such as the crowbar, the roller and the lath which are of uniform thickness are supposed to act through their centre. Point out that the lath used in the experiments abovementioned was supported at the middle to keep it horizontal when no weight was attached on either side and that similarly the weight of the crowbar in the above instance acts downwards through its middle. Elicit from them that the weight of the crowbar so acting at its centre aids the labourer in heaving up the stone.

(*b*) *The See-Saw.*

Refer pupils to their experience at playing the see-saw. Let them describe the ordinary yoke—pole of the country cart and show how it is a lever. Lead them to see that the fulcrum of the lever is in the middle and the weights of, the two boys playing at it, the forces on either side. Get them to apply the principle of the lever and lead them to

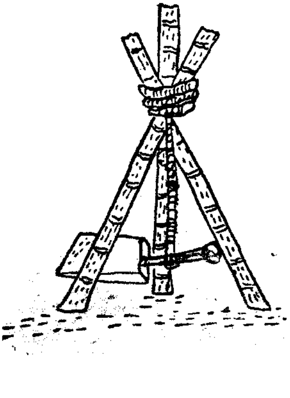
infer that if, of the two boys playing at it, one is heavier



than the other, the heavier boy would have to sit nearer the fulcrum than the lighter. Take pupils outside the class-room, choose from them a thin boy and a heavy boy and let them balance the see-saw in the farm-cart or in the country carts resting near the inn close to the farm office and verify the truth of the deduction made.

(c) *The common water-shovel.*

Refer the pupils next to the common water-shovel used for lifting water over a low embankment for irrigating fields and ask them to describe it. Lead them to interpret it as a lever. Let them see (i) that there is an instance of a suspension at the fulcrum, (ii) that the effort and load are on either side of the fulcrum, (iii) that both act downwards and (iv) which arm is longer and why?



(d) The simple balance.

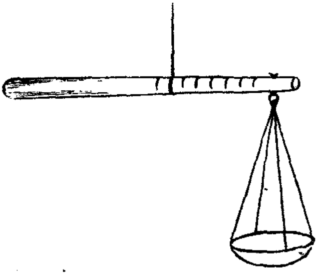
Remind pupils next of the simple balance and a pair of scales already studied as a lever. Impress upon them the necessity for having the arms of the balance of equal length and the scale-pans of equal weight. Refer to the false balances and show them a specially constructed one. Explain how if one arm is made longer than the other the beam may remain horizontal with unequal weights thrown into the scale-pans, and how one can easily detect fraud by just exchanging the weights in the pans and seeing whether the beam is then horizontal.

Weighing by substitution.

Propose to pupils how they could manage to weigh correctly when the false balance above described is alone available. Explain to them how if a visam weight in one scale-pan balances some stones in the other scale pan, the visam weight may be removed and the article to be weighed, say jaggery, may be placed instead in the scale-pan and thus an actual visam of jaggery may be weighed.

(e) The country Steel-Yard.

Refer pupils to the country steel-yard (*Thunika Katta*) and lead them to interpret it as a lever. Point out that



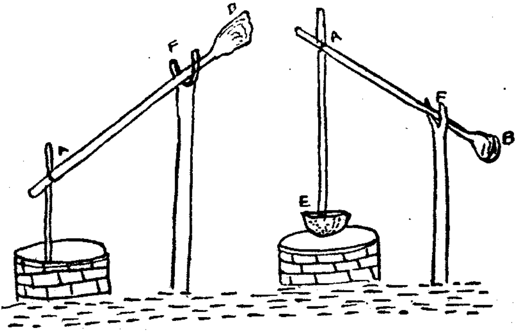
the fulcrum here is the place where the string is attached to the yard for being held up when balancing, and that when the article to be weighed is put in the pan, the

balancing load is the weight of the yard itself acting through the fixed balancing point (*Yethugethu*) and that this point could be found by moving the fulcrum or the string along the rod till it is horizontal, the pan being kept empty. Point out also that the fulcrum is moved to various positions along the yard to balance different loads, that as the pan is attached to the yard at a fixed point—the end of the yard—a change in the position of the fulcrum increases the effort-arm and diminishes at the same time the weight-arm, or increases the weight-arm at the same time diminishing the effort-arm, the total distance between load and effort being constant. Get them to infer that the graduations on the yard mark the position of the fulcrum for different loads and that the distance of these graduations measured on the side of the scale-pan from the balancing point, i.e., the *Yethugethu Mark* increases for increasing loads.

Nevertheless let them see that the fulcrum in any of its positions lies between the points where the effort and the load act.

(f) The counterpoise water-lift.

Next refer pupils to a counterpoise water-lift and ask



them if it could be called a lever arrangement and if so, why. Point out the position assumed by the beam of the

lift when it is not worked, that is, is at rest. Ask pupils to describe how the lever oscillates as the bucket is pulled down and the water is lifted. If the pupils have not had experience in handling the lift, take them to some place where such a lift is used, let them pull down the empty bucket to the level of the water, fill the bucket with water and lift the full bucket. Question them which requires greater effort, to pull the empty bucket down, or, to raise the full bucket. Point out that they have to exert greater effort to pull the bucket down than to raise the full bucket, and that in spite of the bucket being full they find that even the slightest force is enough to lift it. Question why they have to exert greater effort to pull the empty bucket down than to raise the full bucket. Get them to note that it is due to the weight of stone attached at the further end of the beam. Question why a weight is attached there. Point out that if without using the lift above mentioned they lifted the bucket full of water they will have to pull upwards and that then they will find the work very hard. Lead them to note that by attaching the weight at the other end they only arrange to see that it almost balances the bucket when full of water, so that the bucket is raised without much of the difficulty above mentioned. Get them also to note that the difficulty they experience consists in exerting effort to pull the empty bucket down and that it is far easier to pull down than to pull up. Lead them also to observe that the arms of the lever on either side of the pivot are unequal, that the bucket is attached to the longer-arm and a weight is attached to the shorter-arm, so that the effort required to pull down the bucket might be less than the weight attached to the shorter-arm.

(g) *A set of shears.*

Next, let pupils examine a pair of scissors used in cutting sheets of tin, wire, etc. Point out to them that it is a double lever, having as its fulcrum the joint of the two levers, the handle as the point of application of the effort and the cloth or a sheet of tin or wire as the resistance to be overcome. Applying the principle of the lever, get them to deduce that the nearer the resistance is to the joint the easier it is to work the scissors, etc. Let pupils actually use the scissors or shears to cut cloth, wire, tin

plates, etc., and verify the truth of the inference above made.

Cite other instances, if possible, within the actual experience of the pupils of levers of the above kind, e.g., the Karim or the trough lift used in the Gōdāvari district, the *Kavadi* used for carrying 'straw', the *picottah* used in the Vizagagatam district, etc.

Let the class compare the several instances cited above of lever application and realize that all have certain common features :—

(i) The fulcrum is between the points at which the load and effort are applied, i.e., to say, the load and effort are on either side of the fulcrum.

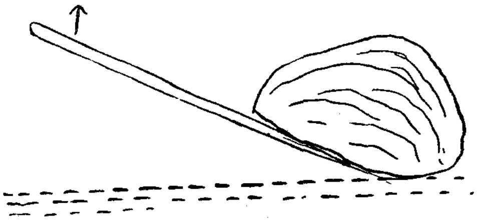
(ii) Both the load and the effort act in the same direction.

Inform pupils that such levers are said to belong to the *first class or order.*

B.—Levers of the second kind or order.

(a) The Crowbar.

Refer pupils to a crowbar used to shift a load without

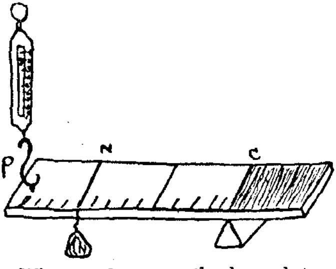


allowing it to rest on a stone or a peg but with one end pressing against the ground, the bar near that end made to rest on the stone to be shifted and the effort applied at the other end. Let the pupils see that the fulcrum here is the end of the bar which rests on the ground and about it the bar turns, the load is the weight of the stone acting near the fulcrum and the pull exerted by the man is the effort. Point out that the fulcrum here occupies one end, the effort the other end,

that the weight acts in the middle and that the weight or load acts downwards while the effort acts upwards. Let pupils compare the positions of the fulcrum, the load and the balancing load in this order of lever with their positions in the first kind or order of lever and note the difference. Recall the idea of load-arm, effort-arm, ask pupils to point these out and compare their lengths. Question pupils which is the greater of the two, the load shifted or the effort exerted in shifting. Hence lead them to grant the possibility of the principle of the lever deduced from the first-class of lever holding true here too. Tell them that it is so and that here too they may apply the equality.

Load × Load-arm = Effort × Effort-arm.

(If possible, the following experiment may be performed, the additional apparatus required being a spring balance. Arrange apparatus as in the figure. The shaded portion can be left out of consideration as it serves only to balance off effects due to the weight of the lath used. Let the pupils read the spring balance with the bag or bags of paddy each weighing a quarter pound at different places on the lever between C and P and tabulate the values obtained as in the last experiment. Lead them to note that CN is the load-arm and CP the effort-arm and deduce from the tabulated values that *Load × Load arm = Effort × Effort arm.*



Get pupils thus to see that the lever principle formulated in the case of the lever of the first kind or order would hold true, even if the fulcrum be at one end and the load and the effort act in opposite directions.

Impress on the pupils that as in the above order of lever, the effort and the fulcrum are at the ends, the effort-arm is nearly equal to the length of the lever and the

load-arm is always less. Hence let them deduce that the effort is always less than the load. Incidentally refer to the fact that in a lever of the first order the effort may be equal to, greater than, or less than the load. Get pupils to derive that as load × load-arm = effort × effort-arm,

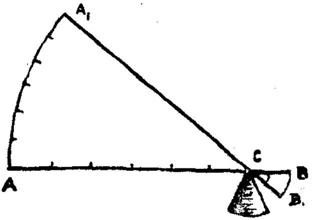
$$\frac{\text{Load}}{\text{Effort}} = \frac{\text{effort-arm}}{\text{load-arm}},$$

Point out to them that as $\frac{\text{load}}{\text{effort}}$ gives the mechanical advantage of the machine it may as well be calculated from its equivalent

$$\frac{\text{effort-arm}}{\text{load-arm}}.$$

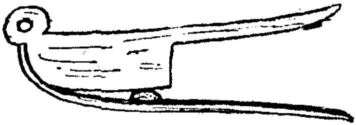
Suggest to pupils that by the simple dodge of making the effort-arm long enough and the load-arm shorter it may be possible to gain enormous mechanical advantage by having the second order of lever. • But point out that the lever in such a case might become inconveniently long and unwieldy.

(The teacher will do well to note the following piece of information for his own knowledge though he need not impart it to pupils. Though corresponding to increased length of the effort-arm there is increased lifting power gained, there is also a disadvantage. Thus, in the accompanying diagram, we have the load-arm only a sixth of the effort-arm but the effort has been applied through a distance six times as great as the distance through which the load moves; in other words, a certain weight has moved through a distance in virtue of another weight only 1/6 of it having moved through 6 times the distance and what has been gained in power has been lost in the distance traversed. Remembering the definition of work, one may easily realize that the work done in the effort being raised is the same as the work done on the load.)



(*b*) *The nut-crackers.*

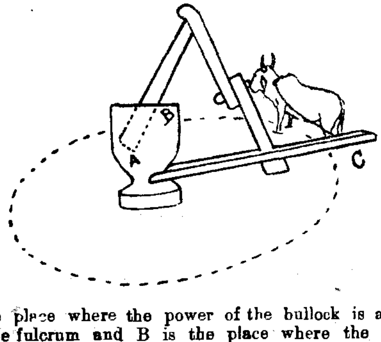
Suggest to pupils a set of nut-crackers and let them



ompare it with a pair of scissors and thus identify the nut-rackers as a double lever. Question as to where the ulcrum is, where the effort is applied and where the esistance, viz., the nut, is held. Lead them to identify it s a lever of the second class. Point out in this connexion hat the machine used for squeezing juice from lime fruit losely corresponds to the nut-crackers and is a lever of he second class.

(*c*) *The country oil-mill.*

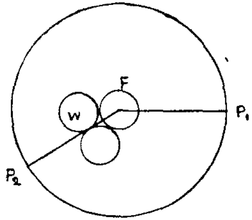
Refer pupils to the country oil-mill (see figure) where C



s the place where the power of the bullock is applied, A the fulcrum and B is the place where the seed is pressed by virtue of weight and oil is extracted. Lead pupils to thus interpret it as a lever of the second kind or rder.

(d) Sugar mill.

Refer pupils to the sugar mill at work. Point out how it



acts as a lever of the second class or order when the bullocks dragging the pole F P_1 come to the position F P_2. Point out also that W is the place where the cane is then crushed and that the force with which the cane is crushed × F W = the effort with which the bullocks turn the lever F W P_2 × F P_2.

(e) Other instances of levers of the second kind or order.

Lead pupils to suggest some more familiar types of levers of this class, e.g., the knife used to cut guava or mango fruit with its sharp end thrust into the fruit and then turned on its blade, the claw-hammer used in pulling out nails while unpacking cases, or a door turning about the hinge when some one resists its being closed, the oar of a boat as the rower uses it for rowing, where the fulcrum is the place where the oar dips in the water, the "load" is the resistence of the boat acting through the row-locks and the power is applied by the rower at the other end of the oar, etc.

C.--Levers of the third class or order.

Recapitulate the two orders of levers already studied and get from the pupils that in the first class the fulcrum is in the middle and in the second the load is in the middle. Ask pupils to suggest examples of a third kind of lever in which the effort might be applied in the middle,

the fulcrum being at one end and the load applied at the other. If they fail, propose for their *consideration the goldsmith's fire-tongs* or the watch-maker's forceps. Get from them where the fulcrum is, where the effort and load are applied and lead them to identify these as levers (double) of the third class. Intimate to pupils that here too the lever principle would hold good and point out that as the distance of power from fulcrum is shorter than the distance of load from fulcrum we *have to apply a greater force to lift a* smaller weight but there is personal convenience gained by using the lever, *e.g.*, a *firm grip* in spite of our *inability* to touch the embers, to pick very small objects, etc.

Propose and work out some model questions on the application of the principle of the lever as the following:—

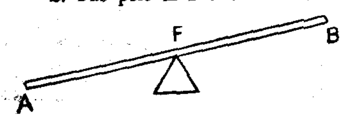
1. A block of granite weighing 80 lbs. requires to be lifted. The end of the bar striking against the block is $\frac{3}{4}$ ft. from the piece of stone on which it rests. If the hand pressing down at the other end of the bar is 4 feet from the stone, find with what force the hand must press.

(Point out that the effort may also be expressed as so many pounds just like "load" or "weight." Question pupils about the law giving the relations between "effort" and "load" as already derived. Get them to note that as effort $\times$ effort-arm $=$ load $\times$ load-arm, the effort required $\times 4 = 80 \times \frac{3}{4}$ lbs. or 60 lbs.

Thus let them find out that the effort required $=$ 60 lbs. /4 or 15 lbs.

(Therefore the handle requires to be pressed with a force of 15 lbs.)

2. The pole A F B is a see-saw yoked to a pole in the country-cart at the place F and when at rest it is horizontal. A boy 75 lbs. in weight



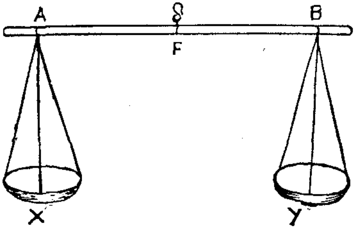
sits in the arm F A at a distance of 2 feet from F. At what distance from F should another boy. 90 lbs. in weight sit so that the see-saw may remain horizontal ?

(*Point out that the weight of the second boy* × *the* distance at which he sits from F = the weight of the first boy × the distance at which he sits from F.

∴ 90 × the distance at which the second boy sits from F = 75 × 2 feet or 150 feet.

∴ The distance of the second boy from F = $\frac{150}{90}$ feet 1 foot 8 inches.)

3. The accompanying figure shows a balance. F is the place at which the beam is supported. A F and F B



are respectively 10 inches and 12 inches. The weight placed in the scale-pan X = 20 tolas. What should be the weight placed in the scale-pan Y in order that the beam may remain horizontal ?

(Let the pupils work out as follows :—The weight in the scale-pan Y × F B= the weight in the scale-pan X × A F.

Hence the weight in the scale-pan Y = $\frac{10}{12}$ × 30 or 25 *tolas.*)

Propose the following exercise on "**Levers**":—

1. What must be the length of a crowbar used to lift a weight of 150 lbs. acting at one end of it and resting on a peg 1 foot 6 inches from that end, if the cooly can exert a force of only 60 lbs. at the other end ?

(*Ans.* 5 feet 3 inches.)

(Point out that the length of the effort-arm is 45 inches and hence that of the bar 45 inches plus 18 inches or 63 inches.)

2. Two boys weighing 54 lbs. and 72 lbs. are playing at see-saw at the ends of a light bar 8 feet long. If the heavier boy sits at 3 feet from where the bar is supported, find where the other boy should sit in order that the beam may be horizontal in that position.

(*Ans.* 4 feet from where the bar is supported.)

3. The weight of water which a water shovel could carry in a single lifting is 80 lbs. The distance of the bucket from the point of suspension of the handle bar is 2 feet. The distance of the handle from the point of suspension is 5 feet. With what force must the handle be pressed down to bale out a bucketful of water with the shovel? (*Ans.* 32 lbs.)

4. The weight of water which a shovel could hold is 70 lbs. The distance of the bucket from the suspension point is 1 foot 6 inches. Of what length should its handle be so that an ordinary man who could freely exert with ease a force of 28 lbs. could use the shovel for baling out water? (*Ans.* $3\frac{3}{4}$ plus $1\frac{1}{2}$ or $5\frac{1}{4}$ feet.)

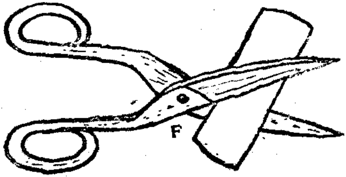
5. The left arm of a balance is 15 inches long. What must be the length of the right arm if a weight of 40 tolas in the left scale-pan balances a weight of 30 tolas in the right pan? (*Ans.* 20 inches.)

6. Find the weight which when placed in the right scale-pan of a balance would counterpoise a weight of 10 tolas in the left scale-pan, given that the arms of the balance are respectively 10 inches and 12 inches.

(*Ans.* 12 tolas.)

7. In cutting a sheet of tin plate or wire placed at 2 inches from the rivet F, a bazaarman presses the shears at 6 inches from F. If he squeezes with a force of 8 lbs. what force would be exerted on the plate or wire?

(*Ans.* 24 lbs.)



8. The length of the longer arm of the beam of a counterpoise water-lift is 12 feet and that of the shorter arm is 5 feet. A weight of 120 lbs. is attached to the end of the shorter arm. With what force should the man working at the lift pull down the empty bucket?

(*Ans.* 50 lbs.)

9. A man could pull down the empty bucket of a counterpoise water-lift with a force of only 30 lbs. The longer arm of the beam of the lift is 14 feet and the shorter arm is 6 feet. What weight should be attached at the end of the shorter arm so as to enable the man to pull down the bucket? (*Ans.* 70 lbs.)

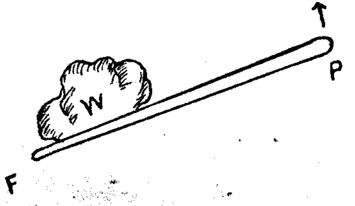
10. The longer arm of the beam of a counterpoise water-lift is 15 feet. The force with which a man pulls it for lowering the bucket is 40 lbs. The weight attached to the shorter arm is 100 lbs. What should be the length of the shorter arm? (*Ans.* 6 feet.)

11. The force with which a man can pull the bucket down in a counterpoise water-lift is 28 lbs. The shorter arm of the beam of the lift is 4 feet 6 inches and the longer arm is 18 feet. Find what maximum weight may be attached at the end of the shorter arm which would admit of the bucket being pulled down by the man.

(*Ans.* 112 lbs.)

12. In a counterpoise water-lift the longer arm of the beam is 15 feet and the shorter arm is 5 feet. The weight attached at the end of the shorter arm is 84 lbs. With what force must the man pull it in order to lower the bucket? (*Ans.* 28 lbs.)

13. One end of a crowbar rests on the ground at a place F and acts as a fulcrum about which the bar is turned to shift a heavy stone. If an edge of the stone is pressed by the lever at a place W 8



nches from the fulcrum and if the weight to be lifted is

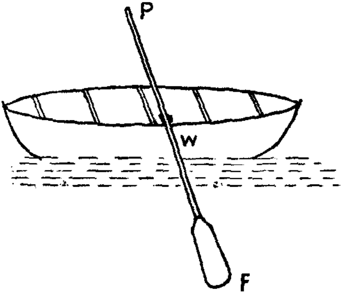
1,320 lbs. what force must be applied at the other end P, when the length of the crowbar is $5\frac{1}{2}$ feet? (*Ans.* 160 lbs.)

14. The power available to lift the weight in the last question is only 120 lbs. The same crowbar is used. How far from the weight should the fulcrum then be kept? (*Ans.* 6 inches.)

15. In a fruit-squeezer the fruit is placed at a distance of 4 inches from the rivet and the man pressing at the ends at a distance of 10 inches from the rivet applies a force of 5 lbs. With what pressure would the fruit be squeezed? (*Ans.* $12\frac{1}{2}$ lbs.)

16. In using the nut-crackers the nut is placed at a distance of 3 inches from the rivet and the man pressing at the ends uses a force of 5 lbs. at a distance of $7\frac{1}{2}$ inches from the rivet. What force would be exerted on the nut? (*Ans.* $12\frac{1}{2}$ lbs.)

17. A boat offers a resistance of 126 lbs. to be



counteracted before it could be moved in still water. A person places the oar F W P so that one end F presses against the water and he exerts the necessary force at P so that the boat moves. If from F the distance of W is 4 feet and that of P 9 feet, what force should the boatman exert at B in rowing? (*Ans.* 56 lbs.)

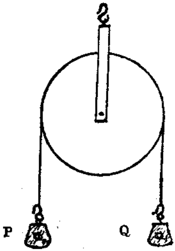
18. In a ganuga (the country oil-mill) the bullock drags with a force of 80 lbs. at 12 feet from the fulcrum. If the gingelly is pressed at a distance of 8 inches from the fulcrum, with what force would it be squeezed in oil extraction ? (*Ans.* 1,440 lbs.)

19. Calculate the mechanical advantage of a machine which enables us to lift an engine weighing 10 tons by applying a force of 200 lbs. (*Ans.* 112.)

Section III.—The Pulley.

The fixed pulley.—Refer pupils to the ordinary pulley used to draw water from wells. Ask them how it is used and elicit from them that it seems easier to haul up a bucket of water over a pulley than without it. Let them next describe the pulley. Point out to them that it consists essentially of a wheel or cylindrical block of wood, with a groove cut along its edge or in the middle of it just sufficient to keep within bounds the rope passing round it, that it is capable of rotating on a horizontal axis more or less freely and that the axle is mounted on a rigid wooden or metal frame secured to a bar. Tell pupils that as the axis of the pulley is thus fixed to a block and as it does not move, such a kind of pulley is called a fixed pulley.

Experiment.—(For this experiment an ordinary wooden pulley mounted on stout iron-frame, a strong but thin piece of string, a balance with a set of weights and some cloth bags containing paddy will be required). Mount the pulley on some stand or tie it fairly to a horizontal beam and pass the string through its groove. Attach a bag to each end of the string and put some quantity of paddy into one. Ask pupils to load the other with paddy until the first one begins to move up. Then direct them to weigh each bag and compare the weights. Let them repeat the above experiment with different quantites of paddy put into the first bag and tabulate the results on the blackboard as follows.



The weight of bag P	Weight of bag Q

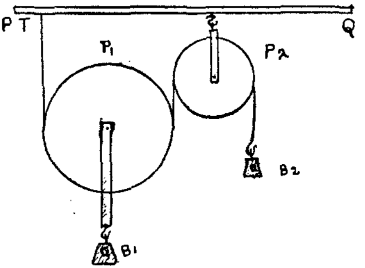
Lead them thus to infer that both the bags are of equal weight. Impress on them that 'P' can be called the load, and Q the effort and hence elicit from them that the mechanical advantage is '1'. Let them hence interpret that there is neither gain nor loss in the force employed. Question pupils which is easier, to haul up water from a well by means of a pulley or do it without a pulley. Ask them to explain why it seems so easy to haul up water from a well by means of a pulley and so difficult to do it without the aid of the pulley. Point out that in the former case we pull downwards and in the latter we pull upwards and also that we are able to bring to our help the weight of our body in the former case and hence the advantage. Impress on them that in a pulley arrangement of the above type the load and the effort are equal, hence there is no special mechanical advantage and that the only advantage in using such a pulley is that it enables us to apply effort in a direction convenient to us.

Refer pupils to their experience of punkah pulleys which are all pulleys of the above type and that they serve merely to render a certain force applied in a certain direction act in some other direction.

The movable pulley.

Suggest to pupils that a pulley may not be fixed to a beam at all as in the type described already but that it may be capable of moving upwards and downwards with the block over and above mere rotation about an axis. Tell them that such a kind of pulley is called a movable pulley and propose to them that they may next ascertain if there is any mechanical advantage in using such a pulley.

Arrange a movable and a fixed pulley as in the figure, say, thus:—

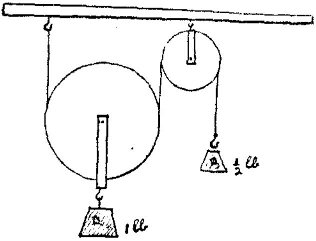


Tie a piece of rope to a hook T in the beam and let it pass under a pulley (p_1) with its hook downwards and having a cloth bag (B_1) of paddy attached to it. Pass the rope over the groove of a fixed pulley (P_2) in the same beam so that the ropes are all parallel as shown in the figure. To the other end of the rope attach another cloth bag B_2 and load it with paddy until the first bag B_1 begins to move up. Now weigh the bags B_1 and B_2. Repeat the above experiment, varying the contents of the bag B_1 and making corresponding changes in the contents of the bag B_2. Tabulate the weights of the bags B_1 and B_2 on the blackboard. Let the class compare the load and the effort and lead the pupils to note that the load is nearly twice the effort. Point out to them that the movable pulley itself has got weight which adds to the load. Weigh the pulley and add to the load and show that the load is nearly twice the effort. Inform pupils that when compared with the weights of the contents of the bags B_1 and B_2 the weight of the movable pulley may in general be considered negligible, inasmuch as only approximate results could be had. Let the class in any case realize that there is a marked advantage in using a pulley that can move and that it has been found possible that by using a greater number of pulleys of this type the effort to be applied to lift loads may be minimised.

(It is suggested that if there is a spring balance in the school better results can be obtained by using it in place of the fixed pulley mentioned above, the free end of the string being attached to the hook of the balance.)

The teacher will do well to remember the following line of argument to explain the mechanical advantage of the

movable pulley system and adopt it with a few bright pupils :—



Suppose a weight of 1 lb. is attached to the movable pulley. For the sake of convenience the weight of the pulley may be neglected. Since the weight of 1 lb. is supported by two strings, each will participate equally in carrying the weight and would be subject to a pull of ½ lb. The string fixed to the beam would pull down the beam with that force while the other string will be counterbalanced by a ½ lb. attached to the string passing over a fixed pulley. Thus the mechanical advantage may be found to be 2 by dividing 1 lb. by ½ lb. This mode of reasoning may be extended to work out the mechanical advantage when there is a system of movable pulleys.

Propose the following exercises :—

1. A punkah is found to offer a resistance equal to 20 lbs. Assuming that the pulleys move freely find with what force the punkah boy should pull the string. (*Ans.* 20 lbs.)

2. A man has to pull up a stone weighing 80 lbs. by means of a fixed pulley. With what force should he pull? (*Ans.* 80 lbs.)

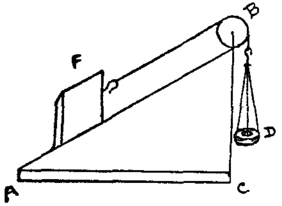
3. A bucket is attached to a movable pulley weighing 5 lbs. If the bucket when full of water weighs 55 lbs. find with what force a man must pull to haul up a bucketful of water. (*Ans.* 30 lbs.)

Section IV.—The inclined plane.

Refer pupils as to how huge blocks of stone are taken from the ground-level to the tops of temples and other high buildings under construction, how casks, bags and beams are lifted to a cart from the ground or from the boat to the canal-bund, etc. Point out how a sloping plank is made use of in all these cases. Lead pupils to infer that it is easier to pull the loads or roll them along the slope than vertically up. Recall to their experience that a cart which is unable to go straight uphill to the top of the hill finds it possible to reach the top along an incline, that it is easier for them to ascend a sloping flight of stairs than one more upright, that whereas they could not climb up a straight wall they could reach its top by an incline, that in baling out water by means of a mhote the bullocks are made to go along an incline, and so on.

Suggest that the above cases appear to point out a decided advantage in the effort to be gained by using an incline to carry a weight to a higher level and tell them that any such surface which can be said to be rigid and which is inclined to the horizontal is called an *inclined plane.*

Demonstrate and lead pupils to infer that the effort required to pull the same load from A to B along an inclined plane increases as the angle B A C increases by adopting a method like the following ;—

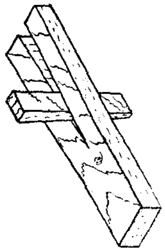


Let D be a scale-pan attached to a string passing over a pulley at B and let the other end of the string be attached to a heavy roller F. Add stones to D till the roller begins to move up. Then increase the angle C A B and show that the weights already in the pan are not enough to move the roller up. Show that by adding more weights to D the roller may be made to move up. Perform a series of similar experiments, gradually increasing the angle C A B.

Section V.—Other Machines.

(1) The Wedge.

Refer pupils to the use of the wedge-shaped piece of wood in sawing timber or splitting wood. Get from them that it is a piece of hard and tough material having two flat surfaces meeting at an angle forming a sharp edge. Point out that the action of a wedge does not consist in any continuous pressure being brought to bear upon it but by impacts with a hammer. Elicit from their experience that the longer and thinner the wedge is, the more efficient it works. Lead them to infer that this implies a very small slope and suggest from analogy that its working is based on the same principle as that of the inclined plane.

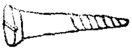


Propose to pupils the cutting of wood by a carpenter with his chisel and lead them to interpret it as a wedge. Ask them to give other examples of the application of the wedge such as the axe, the blacksmith's chisel, etc. Draw the pupils' attention to the shape of the prows of boats and get from them that their wedge-shape enables them to cleave through the water easily. Here allude to the cutting of fruit by means of a knife and let them interpret that the blade of the knife acts as a wedge. Tell them that the so-called sharpening of cutting tools is only the flattening of the wedge.

(2) The Screw.

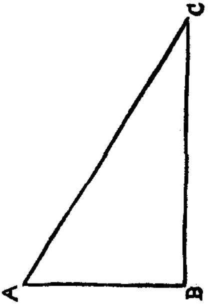
Refer to pupils' experience as to how with a cork-screw the corks of patent medicine bottles are easily removed, as to how by working with a nut on a screw the positions of certain bars in cycles are fixed, how by working with a screw the sluices in canals are opened or closed in spite of the force due to the pressure of the water, how by turning the screw the book-binder presses into a neat compact form what appears to be clumsy and cumbersome, how in the local "Peatsu" factory large bales of palmyra fibre are

pressed into short neat compact forms by working the screw, and so on. Point out that by mere pressing with one's hands directly on the articles such work cannot be done at all.

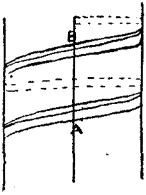


Practical Work.

Ask pupils to cut out on paper a right-angled triangle A B C and carefully wind it round a cylindrical pencil. Lead them to see that the slant side of the triangle forms a spiral on the pencil. Show them an ordinary screw and let them note the similarity in appearance to the thread of the screw. Let them next repeat the above experiment by taking a right-angled triangle with B C of the same length but with the angle B C A less in size and note that then the threads are closer together.



Invite pupils to consider two points A and B along a line parallel to the length of the cylindrical pencil on two consecutive threads. Let them imagine a small ant to follow the edge of the paper wound round the cylinder from A to B. Point out how it would have gone once round the cylinder and thus raised itself from A to B vertically in effect. Let them realize that it has only made use of an incline.



Refer to pupils' experience of its being easier to climb a slope obliquely than directly, that it is easier to

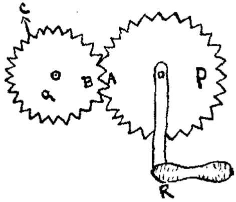
work a cork-screw into a cork than to pierce it and lead them to see that there is an advantage to be gained by the use of the screw.

Recall the fact that the screw resembles the inclined plane and point out that corresponding to the fact that the effort diminishes as the angle diminishes it may be stated that as the threads of the screw are closer the effort diminishes. Let pupils note that the screw in turning once completely round produces a linear motion equal to the distance between two consecutive threads of the screw, called also the *pitch of the screw.*

(3) The Cog-Wheels.

Show pupils the working of the gear in the sugar-cane crushing mill and point out that the bullocks turn a pole round and round and cause a toothed cylinder to rotate and as this cylinder rotates its teeth transmit the force to another toothed wheel and cause it to move. Quote also the cycle gear as an instance.

Show pupils two wheels P and Q fitted with teeth in their rims. Point out that the motion of the wheel P is transmitted to the wheel Q by the teeth of P successively fitting in with those of Q and that when the teeth strike they are in the line of centres of the wheels. (See the teeth A and B in the wheels.)

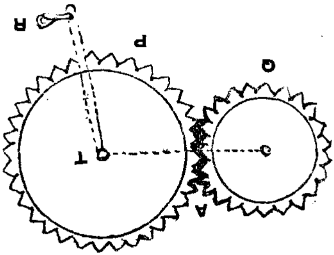


Point out that in the cycle the force from one wheel to another is transmitted by a chain. Let them count the number of teeth in the wheel Q and the corresponding number in the wheel P. Suppose they are 24 and 50. Point out by actual revolving that the tooth A in the wheel P strikes against the tooth B in the wheel Q and hence it moves with the same velocity as Q. Question how many teeth in the wheel P would have passed the line of centres when the wheel Q has once revolved round its centre. Point out that only 24 teeth would have passed the line of

centres and hence when the wheel P makes one revolution round its centre, 50 teeth in its circumference would have passed the line of centres and that as the wheel Q has only 24 teeth it would make 50/24 revolutions. Tell pupils that if one wheel is employed to drive the other, it can be called the '*Driver*' and the other wheel called the '*Follower*.' From the above lead pupils to see that in such a system *the number of revolutions per minute of the follower* × *the number of teeth on it* = *the number of revolutions per minute of the driver* × *the number of teeth on it.*

Propose :—

1. In the following figure a power of 10 lbs. is applied



to the crank T R which is 12 inches long. If the radius of the driver T A is 4″ what force will be transmitted at A ?

Lead pupils to work as follows :—

T A × the force transmitted at A = T R × the effort exerted at R.

The system may be taken as a lever of the second-class with the fulcrum at T, the load at A and the effort at R.

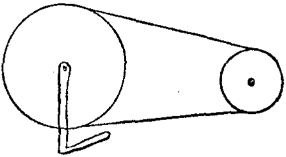
Hence the force transmitted at A equals 12′/4″ × 10 lbs., i.e., 30 lbs.).

2. A bicycle gear wheel has 40 teeth and drives the back wheel which has only 16 teeth. How many revolutions will the wheel of the bicycle make for one revolution of the pedal ?

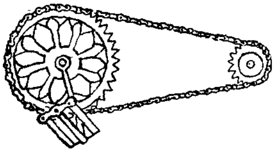
(In one revolution of the pedal, point out that 40 teeth rill pass by and so the follower would have made $\frac{40}{16}$ or $2\frac{1}{2}$ evolutions).

Repeat the above exercises by changing the numerical ata.

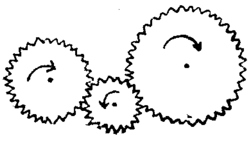
Point out to pupils that the driver and the driven move lways in opposite directions when the teeth gear into one nother, and when we want them to move in the same irection, tell them that it is usual to employ a driving hain as in the bicycle or a leather belting as is done in actories. Tell them also that yet another method is to mploy an idle wheel between the two.



Belting.



Chain Gearing.



Gearing with an idle wheel.

Propose :—

In the blacksmith's blower, two wheels of diameters 4 inches and 3 feet are connected by a belt. If by means of a handle, the larger wheel is turned so as to make 50 revolutions in a minute, how many revolutions will the smaller wheel make in a minute ?

(*Ans*. 450 revolutions.)

Section VI.—General Remarks.

In all the simple machines hitherto discussed, ask the pupils whether they would expect in full the calculated mechanical advantage. Suggest to them that whenever two objects come into contact, e.g., the lever and the load, the screw and the nut, the string and the pulley, the body and the inclined plane, in every one of these cases, friction, i.e., the rubbing between the surface of one part of the machine and another results in a great reduction of theoretical mechanical advantage. Impress on pupils that all our calculations are then very approximate and a liberal allowance must be made for the effects of friction on the machines. Also explain to them in the light of the above, the errors in the results obtained in the experiment described in the chapter and also why the mechanic always takes care to oil his machines or the pupil himself oils the cycle, the lock, the tools, etc., regularly.

Also tell the pupils that all the work put in the machine could not be got out of it and that friction again stands forward as the chief agent which robs away some of the work. Point out as an instance the heat in most machines as due to the friction through the agency of which the work applied is diminished and frittered away. Also tell them that the ratio of the work actually got out of the machine to the work done on it is called *the efficiency of the machine*.

Make the pupils clearly understand that the forces calculated in the questions above are merely the forces that would be required to keep the machine in perfect balance, that in the actual working of the machine, ignoring the effects of friction, a slightly greater force must be applied at the outset and when the body begins to move, it may sink to the theoretical value.

EXTRA EXAMPLES.

1. (*a*) A person has 2½ acres of dry lands. With an ordinary plough he can plough ⅝ of an acre a day. For a single ploughing he has to pay Rs. 1–4–0. How much will he have to spend for getting his lands ploughed once? At this rate how much will he have to spend for two ploughings? (*Ans.* Rs. 5; Rs. 10.)

(*b*) He wishes to apply farm-yard manure to his lands at 12 cart-loads for an acre. If a single cart-load of the manure costs Re. 1 and the cost of cartage to his fields is 8 annas extra, how much may he have to spend on the manure required for his lands? (*Ans.* Rs. 45.)

(*c*) For spreading the manure over an acre, 6 women should be employed for a day at 2 annas 6 pies each. If there be only 5 women available for work, for how many days should he engage them so as to have the manure spread over all his lands? How much may he have to spend for spreading the manure? (*Ans.* 3 days; Rs. 2–5–6.)

(*d*) If instead of farm-yard manure he goes in for castor-cake he may have to apply 15 bags of castor-cake to an acre of sugarcane. If he intends to raise sugarcane over 2½ acres and if castor-cake sells at Rs. 8–8–0 a bag, find how much he may have to spend for the cake necessary for the sugar cane cultivation. (*Ans.* Rs. 318–12–0.)

2. If 14 men and 8 women should be engaged for a day for digging, spreading castor-cake and earthing up an acre of sugarcane find how much should be spent for engaging the necessary number of coolies to do the same work over the lands, reckoning 4½ annas for a man and 2½ annas for a woman a day. (*Ans.* 12–15–6.)

3. If for cultivating jonna mixed with indigo the land has to be ploughed 6 times, find the cost of getting 2½ acres ploughed at 12 annas a ploughing pair a day. Assume a pair could plough only ½ an acre a day.

(*Ans.* Rs. 22–8–0.)

4. If, for growing ganti later on, the land should be ploughed 3 times, find the cost of ploughing 2½ acres, reckoning as before 12 annas a ploughing pair a day and that a pair could plough only ½ an acre a day.

(*Ans.* Rs. 11–4–0.)

5. A ryot has 3 acres of land, he wishes to raise Chodi. The land has to be ploughed 4 times. How many ploughing pairs should be engaged a day to have the ploughing done in 8 days. Assume that a pair could plough $\frac{3}{4}$ acre in a day. Also calculate the entire ploughing charges at Rs. 1-8-0 a pair a day. (*Ans.* 2 pairs a day; Rs. 24.)

6. A person has 15 acres of lands and on 10 of them he intends to raise sugar-cane and on the rest paddy. For an acre of sugarcane he has to apply 6 cart-loads of farm-yard manure and 10 bags of castor-cake and besides get 2,000 sheep penned for a day. For an acre of paddy cultivation he has to apply 10 cart-loads of farm-yard manure. Find the entire cost of the manure required at Rs. 2-8-0 for 500 sheep for a day, Rs. 5 for a bag of castor-cake and Re. 1-8-0 for a cart-load of farm-yard manure. (*Ans.* Rs. 675.)

7. For the cultivation of gingelly '*Tholakari*' a person has to give three ploughings and pen 1,000 sheep for a day for an acre before sowing. Calculate the cost of the ploughing and manuring required before sowing on $2\frac{1}{2}$ acres at Rs. 2-8-0 for 500 sheep a day and Re. 1 for a single ploughing. (*Ans.* Rs. 20.)

8. A ryot has a nursery bed of ten cents. He could sow in the nursery at 40 kunchams of paddy seed per acre. At 6 annas for a kuncham of seed find the cost of the seed which he sows in the bed. (*Ans.* Rs. 1-8-0.)

9. *Only $\frac{3}{5}$ of the above seed would sprout.* The selected seed in our farm sells at 8 annas a kuncham and $\frac{9}{10}$ of it could sprout. Which of the two seeds would be more economical to go in for? (*Ans.* The latter variety.)

10. Some time ago, people were sowing at 20 kunchams of seed for an acre and now they find after the demonstration done by the Agricultural Department that 5 kunchams will do. Find in money value the savings in seed which a cultivator owning $2\frac{1}{2}$ acres of lands would effect by adopting the advice offered by the Agricultural Department. Assume that the seed sells at 8 annas a kuncham.

(*Ans.* Rs. 18-12-0.)

11. In 5 acres of *mettu bhumi* a ryot sows at 4 kunchams of dry-land paddy mixed with one seer of red gram and half a viss of cotton seed per acre. Find the cost of the

seed required at 8 annas a viss of cotton seed, and 5 annas a kuncham of dry paddy. (*Ans.* Rs. 7–15–6.)

12. A landlord finds that as his lands are not quite good 10 seers of jonna have to be sown. Find the cost of the jonna at 6 seers a rupee. By soaking them in copper-sulphate solution as recommended by the Agricultural Department he finds he has to spend only 6 pies for a seer of seed, and thereby finds that he avoids a loss of Rs. 7–12–0 worth of crops from disease. Find the extra net income he gets by adopting the precautionary method. (*Ans.* Rs. 1–10–8; Rs. 7–7–0.)

13. While sowing groundnuts as a pure crop 3 pairs of cattle with 3 men to drive and 6 women extra are able to do an acre in an half a day. A ryot wishes to have groundnut sown over 25 acres. How much will he have to spend for sowing at 12 annas for a pair of cattle and a man for half a day and 2 annas 6 pies for a woman a day.

(*Ans.* Rs. 79–10–6.)

14. A person has to raise ganti in 5 acres of lands. Out of them 3 acres are first rate lands which require 1½ seers of seed for an acre and the rest are second rate lands requiring 3 seers of seed for an acre. Find how many seers of seed will be required on the whole.

(*Ans.* 10½ seers.)

15. A person contemplates to have *Gogu* raised in 5 acres of his lands. The seed required is at the rate of 4 kunchams for an acre and a kuncham of the seed sells at 7 annas. Find the cost of the seed required.

(*Ans.* Rs. 8–12–0.)

16. To treat cholam seed sufficient for sowing an acre against short smut, six tolas of copper sulphate are required. A ryot owning 100 acres treats his seed with copper sulphate before sowing; but his neighbour owning an equal area fails to treat his seed and incurs a loss of 10 per cent of his crop. What is the difference in the net income of these two ryots? (Cost of copper sulphate is Rs. 45 per cwt., and the average income from an acre of land is Rs. 40.)

(*Ans.* Rs. 394–10–3.)

17. In the year 1914–15 the price of copper sulphate was Rs. 25 per cwt. and 1,300 acres were sown in the Ceded Districts with cholam seed treated with copper sulphate solution (7 tolas of copper sulphate were used

to treat seed sufficient to sow an acre). (1) What was the cost of the copper sulphate used? (2) What would the ryots have lost by not treating their seed with copper sulphate solution, granting a loss of 12 per cent in the crop thereby and the average income from an acre to be Rs. 85? (*Ans.* (1) Rs. 50-12-6; (2) Rs. 5,450.)

18. During 1915-16, 17,000 acres were sown with cholam seed treated with copper sulphate, 6 tolas of copper sulphate were used to treat seed required to sow one acre and the price of copper sulphate was Rs. 35 per cwt. (1) What was the quantity as well as the cost of copper sulphate used? (2) What would have been the loss in that area if the seed were not treated with copper sulphate solution, granting the loss caused by the smut thereby to be 15 per cent of the area of the crop and the average income from an acre to be Rs. 55?
(*Ans.* (1) 22 cwt. 86 lb. costing Rs. 796-14-0;
(2) Rs. 140,250.)

19. A landlord finds that 12 women coolies should be engaged for a day in order to transplant an acre with ragi. He has 5 acres to be so transplanted. If he wants to have the entire transplantation done in 8 days, how many women should be engaged for a day? Calculate how much of wages he should give them daily at 2 annas 6 pies a woman a day. (*Ans.* 20 women; Rs. 3-2-0.)

20. A landlord has to plant onion seed-bulbs in 2 acres of lands. 10 women could transplant an acre in a day. If coolies are not available in plenty so that 5 women only are available for the daily work, how long should he employ them to have the transplantation done, and how much will he have to spend as wages at 3 annas a cooly a day? (*Ans.* 4 days, Rs. 3-12-0.)

21. How much should a ryot spend on paddy seedlings required for transplanting 6 acres at Rs. 2-8-0 for ½ an acre? (*Ans.* Rs. 30.)

22. A ryot in a village requires 65 bundles of onion seedlings for transplantation in his lands. At Gollaprole good seedlings are sold at 5 bundles a beda. If the trainage to and fro is Re. 0-3-6 from his village, how much may he have to spend to secure the seedlings in his village? (*Ans.* Rs. 1-0-6.)

23. A person finds that he requires 1,000 plantain suckers for planting in his fields. At Rs. 6-4-0 for every 100 suckers, how much may he have to spend for the purchase of the requisite number of suckers ?

(*Ans.* Rs. 62-8-0.)

24. A ryot plants in an acre of lands 900 plantain suckers. For digging 900 holes 1 foot deep he should employ 4 men coolies a day; for lifting suckers, 4 men coolies a day; for planting, 8 men coolies a day and for pressing, 18 men coolies a day. If each man cooly demands Re. 0-4-6 as wages, find how much he may have to spend for having these operations carried out in 2 acres 50 cents of his lands. (*Ans.* Rs. 23-14-6.)

25. Chilly seedlings sell at 5 bundles for a beda. A person finds that he requires 100 bundles for his lands. How much may he have to spend for the seedlings and conveyance if the cost of fetching the seedlings from the nursery (where they are sold) to the field (where they are to be transplanted) involves an extra charge of Re. 1.

(*Ans.* Rs. 3-8-0.)

26. Sugarcane is bought in puttis. A putti of cane gives 1,500 sets. 12,000 sets are required for planting an acre. How many puttis of cane should be bought for raising sugarcane plantation in 5 acres of lands ? Find the cost of the sets required at Rs. 4 a putti.

(*Ans.* 40 puttis ; Rs. 160.)

27. One has to engage 6 men and 10 women for harvesting sugarcane, stripping, carrying, cutting into sets and planting an acre in a day. How many men and women will have to be engaged for having these operations done in 5 acres of lands in 2 days ? If each man demands 3 annas 6 pies a day and each woman 2 annas, find the cost of the labour involved.

(*Ans.* 15 men and 25 women ; Rs. 12-13-0.)

28. A ryot engages 3 women to lift ragi seedlings and 15 women to plant an acre in a day. If 8 acres of lands have to be planted in 2 days, how many women coolies will have to be engaged ? Calculate the total wages that should be given to them at 2 annas 6 pies a day. (*Ans.* 72 women coolies ; Rs. 22-8-0.)

29. 15 women could weed an acre of groundnuts in a day. If 20 women work for 2 days, find how many acres of land will be weeded. (*Ans.* $2\frac{2}{3}$ acres.)

30. For planting turmeric over 2 acres, 3 ploughing pairs with 3 boys to drive will have to be engaged with 6 women extra to dibble. A ryot has to plant 12 acres under turmeric. What will be the cost of planting if each pair of cattle is paid at Rs. 1–2–0, a boy at 3 annas and a woman at 2 annas 6 pies per day?

(*Ans.* Rs. 29–4–0.)

31. 10 women could weed out 1 acre of land in a day. A person has $7\frac{1}{2}$ acres of lands for such weeding being done by hand. If he employs only 15 women a day, in how many days will the weeding be finished and how much will he be spending as wages at 2 annas 6 pies a woman a day? What will be the cost of 3 such weedings?

(*Ans.* 5 days; Rs. 11–11–6; Rs. 35–2–6.)

32. By using the Sampson harrow in a young ganti crop, $2\frac{1}{2}$ acres could be covered in a day by a pair of oxen. Calculate the cost of getting the harrow worked over 7 acres 50 cents at Rs. 1–2–0 a pair a day, the harrow being supplied free. (*Ans.* Rs. 3–6–0.)

33. With a picottah worked by 3 men from a pit-well, an acre of sugarcane crop can be irrigated in 3 days. Each cooly is paid at 5 annas a day. If 12 such waterings are to be given in the course of three months what cost will be incurred for watering 4 acres of sugarcane crop? (*Ans.* Rs. 135.)

34. If the capacity of the picottah bucket is 10 cubic feet and if a man lifts 40 buckets in an hour, find how many inches of water would be supplied near the discharge point to a field, half an acre in extent, during a day of 8 working hours. If a ryot is to irrigate half an acre of sugarcane crop requiring watering equivalent to 50″, how many days of 8 working hours each will the field have to be watered?

(*Ans.* 1·8″ nearly; about 28 days.)

35. If turmeric requires 10″ of water an acre and if the rainfall has helped to the extent of 6″ with water how many cubic feet of water has to be supplied by watering from other sources? (*Ans.* 14,520 cubic feet.)

36. A well can irrigate only 4 acres of lands under turmeric. A person has 3 such wells in his garden lands. Find how many acres of turmeric he may cultivate. If turmeric requires 10″ of water and onions 6″ find how many acres of onions a well can irrigate.

(*Ans.* 12 acres ; $6\frac{2}{3}$ acres.)

37. Cambodia cotton grown during dry weather requires 10″ of water per acre. A cultivator finds that he has wells enough to raise 2 acres of turmeric and $2\frac{1}{2}$ acres of onions, the former requiring 10″ of watering and the latter 6″ of water. Find how many acres of Cambodia cotton he may cultivate depending solely on the above supply. (Answer the above question substituting sweet potatoes requiring 12″ of water and sugarcane requiring 50″ of water in place of Cambodia cotton.)

(*Ans.* 3 acres 50 cents ; $2\frac{11}{12}$ acres, 70 cents.)

38. In the cultivation of sugarcane crop over an acre, a ryot has to engage 15 men to wrap and 5 boys to remove rubbish and weeds for the first time, 20 men to wrap and 10 boys to remove rubbish for the second time, 25 men to wrap and 2 boys to remove rubbish for the third time and the same number for the fourth time also. If a ryot owns 4 acres of sugarcane, what will it cost him to give 4 such wrappings for the whole area? Assume the daily wages of a man at 4 annas and of a boy at 2 annas a day. (*Ans.* Rs. 68-8-0.)

39. Bamboos sell at Rs. 5 per thousand. A cultivator finds that he requires 500 bamboos for propping the sugarcane in an acre. A man will fix 300 bamboos or tie 500 bamboos a day. If 3,600 bamboos have to be so fixed on the ground and the rest tied cross-wise, calculate the entire wages that may have to be given for propping the canes in 4 acres. Assume the wages to be at 4 annas a man a day. Also calculate the cost of the bamboos required.

(*Ans.* Rs. 15, Rs. 102.)

40. For attracting moths in a groundnut crop a ryot has to keep a light-trap in a field for two months. The trap consumes 6 pies worth of kerosine oil a day. Besides a watchman has to be engaged at Rs. 5 a month to drive away the jackals during the period. How much will have to be spent on the above items during the period.

(*Ans.* Rs. 6-14-0.)

41. By putting a light-trap 7,400 moths which were causing injury to the crops were destroyed. Of them half were females. Each female can lay 1,200 eggs. If 15 per cent of these produced caterpillars after hatching find how many caterpillars were destroyed thereby? At that rate how many moths would 15 light-traps catch?

(*Ans.* 666,000; 11,100.)

42. The cost for a light-trap is as follows:—The lamp costs 8 annas, the tin-tray 8 annas and the bamboo frame-work for the stand 2 annas. What is the cost 15 lighttraps required for a village? (*Ans.* Rs. 16-14-0.)

43. The oil consumed each night costs 8 pies per light. The light has to be thus kept for a fortnight. What do the lighting charges come to for 15 such lights for the fortnight? (*Ans.* Rs. 8-12-0.)

44. In some years it has been found that 50 per cent of the crop may be destroyed by insect pest, especially in jonna, ganti, groundnut and pulses. If the value of the usual crop may be estimated to be Rs. 50 to Rs. 80 per acre, estimate the value of the crop that may be destroyed in 500 acres.) (*Ans.* Rs. 12,500, to Rs. 20,000.)

45. A landlord got 750 kunchams of paddy from 2½ acres of land. How much is this per acre?

(*Ans.* 300 kunchams.)

46. A ryot finds that 3 men and 15 women can reap 1 acre 25 cents in a day. If he has 6 acres 25 cents of paddy lands and he wants that the reaping should be finished in 3 days, how many men and how many women will he have to employ a day? Calculate the total wages for reaping at 3 annas a man a day and 2 annas a woman a day. (*Ans.* 5 men and 25 women. Rs. 12-8-0.)

47. For stacking paddy sheaves on the floor close by 2 men and 4 women a day are necessary for an acre. A ryot has 5 acres of paddy lands. How many men and how many women should he employ so as to have the stacking done in a day? (*Ans.* 10 men, 20 women.)

48. For pulling out Bengal-gram 5 women should be employed to do an acre in a day. A landlord employs 25 women for 5 days consecutively. How many acres of Bengal-gram could be so pulled out? (*Ans.* 25 acres.)

59. 15 women are required for cutting off the ear-heads of gantlu in an acre a day and 8 women for cutting off the straw from the lands later on. If 6 acres are under ganti and if 12 women work at it daily, find in how many days the ear-heads could be cut off and in what time the straw. Calculate the total wages at 2 annas a woman a day. (*Ans.* $7\frac{1}{2}$ days ; 4 days ; Rs. 14–12–0.)

50. For removing sweet potatoes from an acre of lands, in a day, a ryot has to employ 40 men to dig and 30 women to turn over and collect the vines and tubers. If there are 40 cents of land under sweet potatoes in a garden and if they should be removed in 2 days, how many men and how many women should be employed daily ?

(*Ans.* 8 men and 6 women a day.)

51. For digging, lifting, carrying and cleaning onion from an acre 10 men and 60 women should be employed for a day. If I set 5 men and 30 women to work for each of 6 days, how many acres will be done by them ?

(*Ans.* 3 acres.)

52. A batch of 5 men, 6 women and 2 boys will cut sugarcane enough for 4 charges or boilings. If 12 boilings are arranged for, how many men, women and boys should be employed to cut enough canes and what will be their total wages at 6 annas a man a day, 3 annas for a woman or a boy a day ? (*Ans.* Rs. 10–12–0.)

53. For extracting juice and making jaggery out of the canes in his gardens, a landlord finds that the mill hire comes to Rs. 25 and 26 men have to be engaged at 4 annas a man a day. Find how much he has to spend on the whole. (*Ans.* Rs. 49.)

54. For harvesting and curing tobacco grown in an acre of lands, a landlord finds that he has to employ 120 men on the whole, each man working for a day and demanding $3\frac{1}{2}$ annas as the daily wages. At this rate how much should another landlord having $2\frac{1}{2}$ acres of tobacco spend for harvesting and curing ? (*Ans.* Rs. 65–10–0.)

55. To harvest an acre of tobacco in a day 4 men have to be engaged to cut out stalks and 2 men and 10 women to carry the produce to the curing shed. If 7 acres of tobacco have to be so harvested, find how many men and women have to be engaged for a day ?

(*Ans.* 42 men and 70 women.)

66. For stripping and bundling the tobacco got from an acre, 6 men are required for a day, for drying 12 women, for heaping 6 men and finally for opening, sorting and rebundling the leaves and sprinkling salt (or sugar) water on them, 8 men and 12 women. If a man has to be given $3\frac{1}{4}$ annas a day and a woman 2 annas a day, find how much a ryot will have to spend who has $2\frac{1}{2}$ acres of tobacco to be stripped and bundled.

(*Ans.* Rs. 18–7–0.)

57. A cultivator finds that he must employ 12 men and 48 women daily to harvest the onions in 5 acres in 5 days. Calculate how much he will have to spend at 3 annas a man a day and 1 anna 9 pies a woman a day.

(*Ans.* Rs. 37–8–0.)

58. For pulling, cutting, bundling and carrying gogu from an acre, 10 men and 6 women have to be employed a day. If the above operations have to be done on 7 acres, how many men and women will have to be paid for on the whole? Calculate their wages at 3 annas a man a day and 2 annas a woman a day.

(*Ans.* 70 men and 42 women, Rs. 18–6–0.)

59. 10 men or 10 women will pull out gogu from an acre in a day. A ryot employs 5 men and 20 women for 2 days. What extent of gogu will be pulled out? (*Ans.* 5 acres.)

60. For bundling alone, a ryot finds that 51 women are required for $8\frac{1}{2}$ acres of gogu. How many women are required to bundle an acre of gogu? (*Ans.* 6 women.)

61. For retting gogu from $2\frac{1}{2}$ acres, 5 men have to be employed for a day. How many will be required for retting gogu from 50 cents? (*Ans.* 1 man.)

62. One man to supply stalks and 10 women to strip will finish up the stripping of gogu from 1 acre in 2 days. At 3 annas a day for a man and 1 anna 6 pies for a woman a day, find the wages to be given for stripping the produce from an acre. (*Ans.* Rs. 1–2–0.)

63. For threshing and winnowing paddy a landlord finds that 8 men, 1 woman and 10 pairs of cattle should be engaged a day for finishing up the produce from an acre. If he has three acres, how many men, women and pairs will he have to engage?

(*Ans.* 24 men, 8 women and 30 pairs of cattle.)

64. For threshing jonna from an acre in a day 3 men and 2 pairs of cattle are required. If thereby 200 kunchams are threshed out, find how long the pair will take to thresh out 1,600 kunchams. If the threshing should be done in 2 days, how many men and how many pairs of cattle will have to be engaged?

(*Ans.* 8 days, 12 men and 8 pairs of cattle.)

65. In picking cotton it is found that 1 woman can pick half a maund a day. If in one picking an acre can yield ¼ maund and if a cultivator has 32 acres under cotton, how many women should be employed to have the picking done in 4 days? (*Ans.* 4 women.)

66. A team of 16 cattle tied in four hanthis and 7 men will thresh out in a day one garisa of paddy out of a sufficient quantity of stalks heaped up. If an acre yields ¼ a garisa of paddy, find how many cattle and how many men will have to be engaged to thresh out the yield expected from 16 acres in 24 days? (*Ans.* 64 cattle and 28 men.)

67. A person gets 200 kunchams of paddy from an acre of land. He has an acre and a half under paddy. He disposes of the produce in the threshing ground at Rs. 6 a putti. How much of money will he get? (Reckon 20 kunchams for a putti.) (*Ans.* Rs. 90.)

68. By cultivating Cambodia cotton in an acre of his lands a ryot got 2½ puttis or candies of kappas. Valuing a maund of kappas at Rs. 2, find the money value of the yield from 3 acres at the above rate. (*Ans.* Rs. 300.)

69. A landlord gets an yield of 800 kunchams of groundnut from 2½ acres of his lands. Estimate how many kunchams of groundnut he has got for an acre. Also, find the value of the nuts obtained from an acre at 5 kunchams a rupee. (*Ans.* 320 kunchams; Rs. 64.)

70. A cultivator gets 200 kunchams of ragi and 2½ cart-loads of straw from an acre of his lands. If he requires 150 kunchams of ragi alone for his use but wants 7½ cart-loads of straw find whether he could meet his needs by selling the ragi at 4 annas a kuncham and getting the straw at Rs. 2-8-0 a cart-load. (*Ans.* Yes.)

71. A ryot finds that 5 acres of his lands fetched him 250 puttis of turmeric. At this rate find how much he may expect from a plot 57 cents in extent. Find the money

value of turmeric that would be obtained from the plot at Rs. 10 a putti. (*Ans.* 28½ puttis, Rs. 285.)

72. B. Deviah buys 4 acres of lands. From the day in Chaitram when he gets rains he engages 2 ploughing pairs a day and gets the land ploughed in 20 days. If he pays for them at 12 annas each a day, find how much he spends for ploughing alone. (*Ans.* Rs. 30.)

He applies 6 cart-loads of farm-yard manure and 10 bags of castor-cake for an acre. Find how much he spends on the above manure for his lands. Assume 12 annas for a cart-load of farm-yard manure and Rs. 7–8–0 for a bag of castor-cake. (*Ans.* Rs. 318.)

If he is advised to penn 2,000 sheep for a day in addition to the above, for how many days should he penn a flock of 400 sheep and find the charges he will have to incur at Rs. 2 a flock a day. (*Ans.* 5 days; Rs. 10.)

73. A ryot owning 4 acres of lands is advised to plant 12,000 sets of sugarcane for an acre. A putti of canes gives him 1,500 sets. How many puttis should he go in for? At Rs. 4 a putti what will be the cost of the sets required for his lands? (*Ans.* 32 puttis; Rs. 128.)

74. A ryot owning 4 acres of lands engages 6 men and 10 women to cut, strip, carry and plant sugarcane in 1 acre. A man is paid at 4 annas a day and a woman at 2½ annas a day. How much will he have to give as wages at the end of each day? (*Ans.* Rs. 12–4–0.)

75. If each set planted contains 5 *kanupus* and 12,000 sets are planted in an acre, how many canes may a ryot expect to get from a sugarcane plantation of 4 acres, provided each *kanupu* produces a cane? (*Ans.* 240,000.)

76. For watering sugarcane once in 10 days during the non-rainy season, a ryot spends at Rs. 1–4–0 a day. Calculate how much he has to spend for giving 80 such waterings. (*Ans.* Rs. 100.)

77. If for propping sugarcane a ryot requires 2,000 bamboos (thin size) and 13,000 bamboos (thick size) for an acre, find the cost of the bamboos required for 4 acres at Rs. 7–8–0 a hundred for the thicker variety and Rs. 6 a hundred for the thinner variety. (*Ans.* Rs. 870.)

If for fixing every 300 bamboos he has to employ 1 man a day, find in what time two of his *Palers* would

finish the propping. Calculate how much of wages they have to be given at 4 annas each day.

(*Ans.* 22 days ; Rs. 11.)

If for wrapping the canes he employs for a day 30 men to wrap and 10 boys to remove rubbish during the first time, 40 men and 20 boys during the second time, 25 men and 2 boys during the third time, as well as during the fourth time, calculate the cost of the wrapping at 3 annas 6 pies a man a day and 2 annas a boy a day.

(*Ans.* Rs. 30-8-0.)

78. If for making jaggery from the cane a ryot has to employ in addition to 2 pairs of cattle, a batch of 6 men, 6 women and 2 boys for cutting, cleaning, stripping, topping and milling the cane from 5 cents in a day so as to be ready for boilings, calculate how many pairs of cattle, men, women and boys he will have to employ for 4 acres of cane. Find the wages which should be given to them at 6 annas a man a day, 3 annas for a woman or a boy and Re. 1-8-0, for a pair of cattle a day.

(*Ans.* 160 pairs of cattle, 480 men, 480 women and 160 boys ; Rs. 456.)

He finds that a three-roller mill with two good pairs of animals would crush 2 tons of clean cane by working in shifts in 12 hours. If 20 tons of clean cane be obtained from 1 acre, find how many such mills should work in order to crush the entire canes from his lands in 5 days of 12 working hours each. (*Ans.* 8 mills.)

79. If 3,000 lb. of juice may be expected from 2 tons of canes and if a pan could hold only 600 lb. of juice, find how many boilings will be necessary to mill 30 tons of canes. If 1 acre of cane yields 20 tons of clean cane, from which was obtained 5,600 lb. of jaggery, find what fraction of the weight of the cane has been obtained as jaggery. (*Ans.* 75 boilings ; $\frac{1}{8}$ of the weight.)

80. If Messrs. P. & Co. offered to pay at Rs. 500 per acre of sugarcane crop which will yield 4 tons of jaggery and if, by cutting and crushing, the owner of the crop could get Rs. 3-8-3 for a maund (25 lb.) of jaggery, find to what extent he would be a gainer or loser by doing the work himself. Assume the cost of extracting the juice and making jaggery to be Rs. 120 per acre.

(*Ans.* Gainer by Rs. 640 per acre.)

81. A ryot finds that to raise tobacco he has to get 5,000 sheep penned for a day over an acre. If a flock of 300 sheep is available for Rs. 2-4-0 a day, find how much he may have to spend on behalf of the manure for 1 acre and 40 cents of lands. Also, find the cost of the seed required at 5 tolas per acre worth 4 annas a tola weight.

(*Ans.* Rs. 52-8-0; Rs. 1-12-0.)

If, for harvesting and curing the *Pogaku* from an acre, 25 men and 30 women have to be engaged, find the cost of curing the produce from 1 acre 40 cents at 4 annas 6 pies a man and 2 annas a woman a day. If the other items of expenditure in the cultivation amount to Rs. 50 per acre, find the total cost of cultivation per acre.

(*Ans.* Rs. 15-1-6; Rs. 60-12-6.)

If a ryot gets 2⅝ puttis of tobacco from 1 acre 50 cents of lands, estimate the yield per acre.

(*Ans.* 1¾ puttis per acre.)

Find the value of the tobacco obtained per acre above at Rs. 80 a putti. (*Ans.* Rs. 140.)

82. A cultivator intends to raise *pasupu* in an acre of well-irigated land. He gives three ploughings at Rs. 3-0-0 per ploughing. He applies 20 cart-loads of farm-yard manure and spends at Rs. 1-8-0 per cart-load including spreading charges. He buys *pasupu* rhizomes to the extent of 2 puttis at Rs. 12 a putti. How much will he have to spend for ploughing, manuring and for the rhizomes on the whole? (*Ans.* Rs. 63.)

If he plants the rhizomes one foot apart each way, find how many plants he expects to sprout in an acre. (Assume that a square field is taken into account.)

(*Ans.* 42,025.)

For planting the rhizomes in an acre he employs 12 men for two days paying at 4 annas a man a day. Find how much he has to give as coolies' wages for planting.

(*Ans.* Rs. 6.)

For watering the crop once he spends at Rs. 3 per acre. If he gives 20 waterings for the crop calculate how much he spends on the whole for watering.

(*Ans.* Rs. 60.)

For gathering the rhizomes from an acre he employs a plough for three days paying Rs. 2 a plough a day and

8 men and 12 women, besides, for each of these three days at 4 annas a man and 2 annas a woman a day. If he has also to give a cart-fare of Rs. 2 for carrying the roots home, calculate how much the picking charges come to.

(*Ans.* Rs. 18-8-0.)

If he finds there is an yield of 20 puttis of green pasupu and a putti sells at Rs. 55, find how much he may get from the sale of the rhizomes. (*Ans.* Rs. 33.)

If instead he takes to boiling and drying, spending Rs. 4 worth of fire-wood for a putti and Re. 1 extra for drying, turning, removing, rubbing, etc., find how much he may have to spend under these items for the entire quantity of turmeric he gets. (*Ans.* Rs. 33.)

Estimate the total cost of cultivation assuming that items of cultivation not mentioned above come to Rs. 20 extra. (*Ans.* Rs. 195.)

If out of the produce obtained he has to give his landlord ½ the produce, estimate whether he is a gainer or loser by taking to the cultivation.

(*Ans.* Gainer by Rs. 45.)

83. The following are the particulars relating to the cultivation of wet paddy :—

Month.	Particulars of operation.	Men at 4 annas.	Women at 4 annas.	Cattle at 12 annas.	Seed manure, etc.
	Seed bed 10 cents.				
December ...	Ploughings, 2 ...	½	...	½	...
February ...	Do. 1 ...	¼	...	¼	...
April ...	Do. 1 ...	¼	...	¼	...
May ...	Penning sheep ...	...	...	...	250
Do. ...	Ploughing, 1 ...	¼	...	¼	...
June ...	Seed	...	...	...	16 seers.
Do. ...	Sowing, ploughing, brush harrowing, etc.	½	...	½	...

Month.	Particulars of operations.	Men at 4 annas.	Women at 4 annas.	Cattle at 12 annas.	Seed manure, etc.
	Cultivation of Land.				
July ...	Puddlings, 4 ...	8	...	8	...
Do. ...	Trimming bunds ...	3	...	...	...
Do. ...	Manure	...	...	...	12 cart-loads.
Do. ...	Carting and spreading.	3	1	1	...
Do. ...	Pulling seedlings and carrying to fields.	5	...	...	...
Do. ...	Transplanting ...	...	12	...	...
Do. ...	Watering and watching.	1	...	...	...
August ...	Do.	1	...	...	...
September.	Do.	1	...	...	...
Do.	Weeding	...	12	...	...
October ...	Watering and watching.	1	...	...	...
November.	Do.	2	...	...	...
December ...	Harvesting, bundling, stacking, etc.	4	12	...	...
January ...	Threshing and winnowing.	1½	1½	2½	...
February and March.	Carting grain and straw, etc.	2	...	1	...

Assuming the charges to be Rs. 2-8-0 for penning 500 sheep, Re. 1 for a cart-load of manure and 2 annas for a seer of seed, calculate the total cost of the cultivation.

(*Ans.* Rs. 38-9-0.)

If an yield of 300 kunchams of paddy were obtained as well as 2 cart-loads of straw, find the money value of the yield at Rs 6-10-0 for a putti of paddy and Rs. 5 for a cart-load of straw. (*Ans.* Rs. 110.)

If the above details relate to the cultivation of paddy in one acre, calculate the net profit in the cultivation of 5 acres at the above rate. (*Ans.* Rs. 357-8-0.)

84. The following details relate to the cultivation of (irrigated) ragi in an acre of land :—

Month.	Particulars of operation.	Men at 4 annas.	Women or boys at 2 annas.	Cattle at 12 annas.	Seed manure, etc.
	Seed bed.				
October ...	Ploughing	½	..	½	...
November.	Manure	...	...	...	2 cart-loads.
Do.	Carting and spreading.	½	¼	¼	...
Do.	Ploughing	½	...	½	...
Do.	Seed	...	...	..	3 seers.
Do.	Forming bed, sowing and covering.	1	...	...	...
Do.	Irrigation	8	...	...	...
	Cultivation of Land.				
December.	Manure	...	...	...	12 cart-loads.
Do.	Carting and spreading.	8	1	1	...
Do.	Ploughing 2 times ...	8	...	8	...
Do.	Making beds and channels.	4	...	...	...
Do.	Pulling seedlings and carrying	3	...	...	...
Do.	Transplanting ...	...	12	...	...
Do.	Irrigation (preliminary to planting).	...	...	...	3"
January	Hoeing	...	12	...	...
Do. ...	Irrigation 3 times ...	...	...	4½"	...
February and March.	Do. twice ...	...	...	3"	...
Do.	Harvesting, etc. ...	2	10	1½	...
Do.	Threshing and winnowing.	6	1	8	...

Calculate the total cost of cultivation, assuming the charges to be Re. 1 for a cart-load of farm-yard manure, 2 annas for a seer of seed and Re. 1-8-0 for 1½" of irrigation. (*Ans.* Rs. 51-10-0.)

If there was an yield of 200 kunchams of ragi valued at 2 kunchams a rupee and 2 cart-loads of straw valued at Rs. 3 each, find the net profit in the cultivation.

(*Ans.* Rs. 54-6-0.)

85. The following details relate to the cultivation of ragi (dry) in an acre of lands :—

Month.	Particulars of operation.	Men at 4 annas.	Women or boys at 2 annas.	Cattle pairs at 12 annas a pair.	Seed manure, etc.
	Seed bed.				
December ...	Ploughing seed bed.	$\frac{1}{8}$	...	$\frac{1}{8}$	...
February ...	Do.	$\frac{1}{4}$	...	$\frac{1}{4}$	...
April ...	Do.	$\frac{1}{4}$	...	$\frac{1}{4}$	...
May ..	Do.	$\frac{1}{4}$	...	$\frac{1}{4}$	...
June ...	Sheep or cattle penning.	...	...	...	100
Do. ...	Ploughing	$\frac{1}{4}$	...	$\frac{1}{4}$	...
Do. ...	Seed sowing ...	...	...	...	4 seer.
Do. ...	Ploughing and harrowing.	$\frac{1}{8}$	...	$\frac{1}{8}$	...
Do. ...	Ploughing land ...	3	...	3	...
July ...	Sheep penning ...	...	...	...	400
Do. ...	Ploughing land ...	3	...	3	...
Do. ...	Pulling seedlings and carrying.	5	...	...	...
Do. ...	Planting in plough-furrows.	2	8	2	...
August ...	Weeding	...	12	...	...
September.	Harvesting ...	...	16	..	...
Do.	Straw	2	4	...	...
Do.	Threshing	$1\frac{1}{4}$	$1\frac{1}{4}$	$2\frac{1}{4}$	...

Find the net profit in the cultivation assuming the yield to be 120 kunchams of ragi and a cart-load of straw. (Take Re. 1 to be the cost of penning 100 sheep for a day, $2\frac{1}{2}$ annas as the cost of a seer of seed, Rs. 6 the price of a cart-load of straw and 8 annas to be the price of a kuncham of ragi.) (*Ans.* Rs. 41–3–0.)

86. A capital of Rs. 50,000 is required for purchasing a rice-mill and erecting a factory and an extra capital of Rs. 1,50,000 for the working expenses. How many shares of Rs. 25 each may be secured for getting the amount required? (*Ans.* 8,000.)

87. A cow gives 2 seers of milk a day. From the milk could be got half a seer of ghee every four days besides 2 seers of curd a day with the cream removed. Which is more profitable and by how much—to sell the

milk at 4 annas a seer or to convert the milk into curd and ghee and sell the curd at 3 annas a seer and the ghee at Rs. 1–4–0 a seer ?

(*Ans.* The latter by 12 annas every four days.)

88. Which is the more profitable of the two arrangements:—to sell a busta of 26 kunchams of Akullu for Rs. 18 without bag or to sell at 5½ seers of Akullu a rupee (1 kuncham = 4 seers) ? (*Ans.* The latter.)

89. Which is the more profitable of the two arrangements :—to sell a busta of 22½ kunchams of Doosi Sannelu at Rs. 20 a busta without bag or at 4½ seers a rupee (1 kuncham = 4 seers) ? (*Ans.* The latter.)

90. It costs Rs. 3 to get 100 tender bamboos for rattan work. From each stick a basket (butta) could be made. A basket sells at 2 annas (Beda). In one day a cooly will make 10 baskets. What will he be able to earn as wages in basket-making during a month of 30 days if he buys *vethiru* at the above rate, weaves baskets and sells them as aforesaid ? (*Ans.* Rs. 28–8–0.)

91. A person has some straw in his native village, where a cart-load is selling at Rs. 7. He engages carts at Rs. 1–4–0 each, carries the straw to the neighbouring towns, converts each cart-load of straw into 35 smaller bundles and sells the bundles at 8 annas each. If the extra charges for making into smaller bundles and selling a cart-load amount to (Re. 1) find how much he would gain by so selling 10 cart-loads of straw. (*Ans.* Rs. 82–8–0.)

92. The cost of 1,000 bricks (*Goda Ittika*) is Rs. 4–8–0. In a brick-kiln there are 17 holes and for each hole it is counted that there would be 8,000 bricks. Find the value, at the above rate, of the total number of bricks in the brick kiln. (*Ans.* Rs. 612.)

93. For every 1,360 *pachi Ittika* (unburnt bricks) supplied to a brick-kiln, the contractor realizes 1,100 as good *Goda Ittika* (burnt bricks). To get 49,500 of good *Goda Ittika*, find how many *Pachi Ittikas* will have to be prepared. (*Ans.* 61,200.)

94. Two coolies, one at 8 annas a day and the other at 8 annas a day are necessary to get 500 mud bricks (*pachi ittikas*) prepared in a day. How many coolies should be

they pull out? At 3 annas a day for a man and 1 anna 6 pies a day for a woman, calculate what whould have to be given as wages for that day.

(*Ans.* 7 6/8 acres. Rs. 5-2-6.)

101. The tank where palm fibre is put in for boiling in a Peatsu factory is 1½ yards broad, 1 yard high and 3 yards long. Find its volume. (*Ans.* 4½ c. yds.)

102. Estimate the volume in c.ft. of rectangular bales of Peatsu (fibre) of the following dimensions:—

Bale No.	Length.	Breadth.	Height.
A	23″	19″	15″
B	24″	18″	14″
C	25″	13″	18″
D	25″	14″	17″
E	25″	17″	16″

If bale A weighs 1 cwt., estimate also the weights of each of the other bales to the nearest lb.

(*Ans.* A 6,555 c. ins., C 5,850 c. ins.,
D 5,950 c. ins., E 6,800 c. ins.,
B 6,048 c. ins.)
(*b*) B 103 lb., C 100 lb., D 102 lb., E 116 lb.)

103. A beam (Dhulam) of *Nallamadhi* wood, 9 muras long, 1 ft. high, and 1 ft. wide sells at Rs. 8. Find at this rate the value of a beam of the same kind of wood but of the following dimensions:—

18 muras long, 1½ ft. high, ½ ft. wide.

(*Ans.* Rs. 12.)

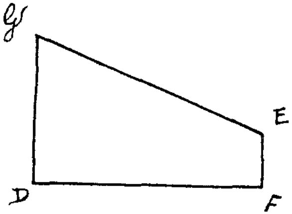
If 1 mura = 17″, how much does the above price come to for a c.ft.? (*Ans.* Re. 1-8-5 a c. ft.)

104. One *dukki* = ⅛″ of rain. One day there was 1¼ *dukki* of rain. How much of water fell on an acre?

(*Ans.* 2,722½ c.ft.)

105. For levelling fields and removing earth, wages are calculated at 16 pies per cubic yard of earth removed. An acre of land is lowered in depth by 1½ ft. What would the charges come to? (*Ans.* Rs. 201-10-8.)

106. In terracing a hill-slope to render it flat, GD, EF, the depths dug out at either end are found to be 3 ft. and 2 ft., respectively. If the area covered by the terrace be 12 acres, find the volume of the earth removed and the wages at 18 pies per c.yd. Take the average of the depths above specified for purposes of reckoning.



(*Ans*. Rs. 4,537-8-0.)

107. A brick measures 8″ in length, 4″ in width and 2″ in height. A brick-stack is 24 ft. wide, 1 ft. high and 30 ft. long. Estimate how many bricks there are likely to be in it. (*Ans*. 19,440 bricks.)

108. There are two ways of estimating the cost of rubble for basements: (*a*) to pay the contractors a flat rate of Re. 1-8-0 per cubic yard or (*b*) to get the rubble in cart-loads at 2 cubic yards for 3 cart-loads giving 8 annas as hire per cart-load and 9 annas as the price of a cart-load of stone. If a basement is to be 30 ft. long, 10 ft. high and 2 ft. thick, estimate the cost at either rate of supplying the rubble required. Find which is the cheaper and by how much it is cheaper.

(*Ans*. Rs. 33-5 4 : Rs. 35-6-8 : Rs. 2-1-4.)

109. In a count of eggs in different clusters laid by a moth it was found that the number was as follows:—

Eggmass.	Number of eggs.
1st	109
2nd	187
3rd	154
4th	129
5th	137
6th	138
7th	141
8th	126

What is the average number per cluster?

(*Ans*. 140 per cluster.)

110. The girth of a log of wood as measured in 6 different places at equal intervals along the stem has been found to be 76″, 70″, 69″, 68″, 63″. Find the average girth of the log. (*Ans.* 68 2/3.)

111. The relation of husk to grain is 11 per cent by weight in *minumulu*, 24 per cent by weight in *pesalu*, and 10 per cent by weight in *bobbara*. How many lb. of each grain will give 20 lb. of unhusked grain?

(*Ans.* Minumulu 22·5 lb., Pesalu 26·8 per cent, Bobbara 22 lb.)

112. In a mango-tope there are 420 trees, all badly infested with mango-hoppers. The owner sprays half the number of trees with fish-oil soap solution at an expense of 7 annas per tree. The sprayed trees yield on an average 225 fruits per tree and the unsprayed ones 105 fruits per tree. It the price of mangoes is Rs. 4–12–0 per 100, what profit does he make by spraying?

(*Ans.* Rs. 1,105–2–0.)

113. It takes 30 days for a hairy-caterpillar moth to pass through its entire life cycle. A female moth before it dies lays a mass of 650 eggs of which half will be males and half females. If on the first January a moth laid a mass of eggs, what will be the number of its descendants on the 1st of March, granting that all the descendants survive? (*Ans.* 68,656,250.)

114. How many caterpillars less would there be if 80 per cent of the female moth in Ex. 113 above are collected and destroyed either by hand-picking by boys or by light traps before they lay eggs early in February?

(*Ans.* 65,910,000.)

115. The graft-mango gardens belonging to a Razu at Alamonda are badly infested with hopper pests. The Razu is advised to spray the mango trees. Each tree requires on the average four gallons of fish-oil soap solution for each of seven applications at one per week, and the normal of soap cultivation is made up of 1 lb. of soap to ten gallons of water. What will be the cost of the soap required to spray 250 trees when a cwt. sells at Rs. 18 8–0?

(*Ans.* Rs. 109–6–0.)

116. A man ploughs a field, 50 yards long. He goes up the field 176 times in a day's ploughing and back the

same number of times. How many miles has he walked in the day ? (*Ans.* 10 miles.)

117. Two women set out daily from a village to sell ghee in a neighbouring town which is $3\frac{1}{2}$ miles away. They go on an average five days a week throughout the year. If the ghee was sold in the village to a single dealer how many miles would be saved to the women in a year ?
(*Ans.* 1,820 miles for each.)

118. Nine hundred bundles of paddy seedling have to be carried from the seed-bed to the field, a distance of 250 yards. By putting a plank over a watercourse 75 yards can be saved. If each cooly carried 15 bundles what will be the total distance saved ? (*Ans.* 4,500 yards.)

119. For building a wall 1,260 bricks are required. If a girl can carry seven bricks at a time how many journeys will she have to make in building a wall $2\frac{1}{2}$ times as long, other dimensions remaining unaltered ? (*Ans.* 450.)

120. A cattle-dealer bought six animals for Rs. 500. One died. For how much should he sell each of the rest on the average in order to get Rs. 75 profit in the transaction ?
(*Ans.* Rs. 115.)

121. A chain is one link too short. A rectangular field measured with it is two chains wide and five chains long. What are its real dimensions and actual area ?

(*Ans.* Length = 4·95 chains,
Breadth = 1·98 ,,
Area = 9·93 sq. chains.)

122. A cart-tyre is 112″ in circumference. If it expands on heating by 1/56 of its length, what is its circumference when it is ready to be put on the wheel ?
(*Ans.* 114″.)

123. Out of 30,850 pounds excreted by a bullock during the year, 4/5 is water and the rest is dry substance; of the dry substance 3/1,000 is nitrogen. If a cultivator burns ¾ of the dry substance by way of dung cakes, how many lb. of nitrogen which may have been used as manure does he lose thereby ? Valuing nitrogen at Rs. 50 for every 176 lb., calculate in money value the nitrogen lost thereby by a ryot rearing 88 pairs of bullocks in a village.
(*Ans.* 13·9 lb. nearly ; Rs. 694–2.)

124. It has been found that a good wool-bearing ram is capable of giving 3¾ lb. of wool in a year. How many such rams may be reared to get 375 lb. of wool and what quantity of wool would be got from two flocks of 500 rams?

(*Ans.* 100 ; 1,875 lb.)

125.

Village number.	Outturn in lb. per acre of paddy.	
	Transplanted.	Broad-casted.
1	3,349	1,910
2	3,520	2,180
3	3,220	1,990
4	2,020	1,448
5	1,760	780
6	2,430	1,440
7	2,580	1,660
8	1,237	1,148

Calculate the average outturn in lb. per acre obtained by transplanting paddy seedlings and by sowing broadcast, respectively. (*Ans.* 2,514½ ; 1,569½.)

126. A rectangular field is 500 links by 450 links. Calculate the produce that would be got from it—

(1) at 240 kunchams of *Bobbili-ganti* per acre
(2) at 60 kunchams of *korra* per acre.
(3) at 150 kunchams of mooca jonna per acre
(4) at 50 kunchams of gingelly per acre.
(5) at 200 kunchams of ragi per acre.

(*Ans.* 540, 135, 337½, 112½, 450.)

127. In years having heavy rainfall the sugar-cane plots which received wrapping and propping operations were found to yield 3,460 lb. more of jaggery per acre than unwrapped cane plots. Find the increase in the quantity of jaggery obtained by adopting wrapping and propping in 1,000 acres of sugar-cane cultivation?

(*Ans.* 1,652 tons.)

128. A ryot wishes to bag 2,240 lb. of paddy. He can put 112 lb. into each bag and wants 18″ of string to tie up each bag. How many yards of string will he require altogether? (*Ans.* 10 yards.)

129. An improved plough makes a furrow 6″ wide. If a plough-man drives his cattle 10 miles in a day, what is the area he has ploughed? (*Ans.* 67 cent nearly.)

130. Estimate the cost of white-washing the inner surface of the four walls of the hall you are in, at a pie per square foot, given the dimensions of the hall to be 40 ft. by 20 ft. by 12 ft. and the area not to be white-washed owing to the existence of doors, windows, etc., covering 145 square feet. (*Ans.* Rs. 6-11-11.)

131. What is the cost of roofing the building you occupy (or other suitable building) the length being 40 ft., the sloping width on each side of the roof being 14 ft., and the cost of roofing being at the rate of Rs. 25 for 100 square feet? (*Ans.* Rs. 280.)

132. For a pair of bullocks the cattle-shed must be 7 ft. broad, 10 ft. long and a length of 10 ft. more should be allotted for each additional pair of cattle. How much of space should be allowed for cattle shed providing accommodation for 20 cattle? (*Ans.* 700 sq. ft.)

133. A man measures a field by pacing it and thus finds its area to be 2·52 acres. He thinks that his pace is 30″ but it is really 32″. What is the true area of the field? (*Ans.* 2·87 acres to the nearest cent.)

134. Experiments show that spacing single plants (paddy) 4″, 6″ or 9″ apart showed little difference in the total yield, the plants throwing out more tillers in the second and third cases and so making up for wider spacing. Find the gain of seedlings in bundles of 400 each in planting an acre of ground, i.e., 10 chains by 1 chain by spacing 9 inches instead of 6 inches. (*Ans.* 262½ bundles.)

135. An approach road has to be opened across a farm to lead to the farm-granary. If the road should be 3 furlongs in length and 12 ft. in width, find the area that would be occupied by it. (*Ans.* 55 cents nearly.)

136. The cost of making jaggery from sugar-cane is at Rs. 2-4-0 a putti. What is the cost of making a kantlam? (*Ans.* Re. 0-14-5.)

137. The cost of making jaggery is at Rs. 1-8-0 for 1,000 canes, while the cane cost 9 pies each extra. Taking that 1,000 canes yield ¾ putti of jaggery what is the cost of a putti of jaggery? (*Ans.* Rs. 92-8-10.)

138. It costs Rs. 3-8-0 worth of firewood to manufacture 5 cwt., of jaggery, during the first two days until dried

megass could be had. How much is this (1) per ton and (2) per lb. of jaggery made? (*Ans*. Rs. 14. 1 pie nearly.)

139. A pair of bullocks could plough 50 cents in a day. How many pairs of bullocks should be engaged to plough 8 acres in two days? (*Ans*. 8 pairs.)

140. A pair of animals walk 21 yds. in going once around a sugar-cane mill. If it takes 15 revolutions for obtaining each pot of juice, and if 25 pots are enough for one boiling, how far do the animals walk round in getting juice enough for 3 boilings? (*Ans*. 13 miles and 745 yds.)

141. Two carts set out for a market 15 miles off. One goes at 3 miles an hour and rests for 10 minutes, after every 3 miles. The other goes at 4 miles an hour and rests for 10 minutes after every $2\frac{1}{2}$ miles. Which gets to the market first? (*Ans*. the latter.)

142. Ten women can plant an acre of paddy in a day if supplied with seedlings. Seven women undertake to plant a block of 11 acres. After half the planting has been done 2 women fall ill; how many days would it take to plant the whole area? (*Ans*. 19 days nearly.)

143. A brush harrow covers 5 acres a day. It is set to harrow a field of 25 acres; on the third day another harrow which can do the whole field in ten days is set to work in addition. In what time will the work be completed?

(*Ans*. 4 1/3 days.)

144. By spending Rs. 15 in buying fish manure for his paddy lands a ryot obtained an increased yield of 80 kunchams of paddy. What is the increased net profit, reckoning 3 kunchams for a rupee? (*Ans*. Rs. 11–10–8.)

145. The mango-hopper used to reduce the yield of mangoes in a mango tree from 2,000 to 10. If spraying had been done with fish-oil-soap and crude oil emulsion, the yield would not have deteriorated. A contractor had taken 100 mango trees on lease from each of which he expected 2,500 mangoes. What would be the loss in the number of mangoes if spraying had not been attempted against the mango-hopper? (*Ans*. 248, 750 mangoes.)

146. A ryot requires 8 kunchams of paddy seed for one acre of land if showing broadcast is adopted. But if he adopts transplantation by single seedling 3 kunchams will do.

How much of seed could he save if he has 25 acres of p
lands to be cultivated? Also, find the value of the
saved thereby at 2 kunchams a rupee.

(*Ans.* 125 kunchams, Rs. 62-8

147. It costs a ryot 4 annas to make a maund of jag
by following the old ways. A raw man using the n
process incurred an expense of 3 annas in making the s
kind of jaggery. A man experienced in the new w
spends only two annas for a maund. If 30 puttis of jag
have to be made, what will be the expenditure incurre
the three men?

(*Ans.* I, Rs. 50; II, Rs. 112-8-0, III, Rs.

148. How many miles a day of six working hours d
boy walk when driving a pair of bullocks in a cane mi
he walks 8 ft. from the centre of the mill and goes rou
three times a minute and rests 10 minutes after each h
(Assume that the circumference of a circle is 3 1/7 tim
diameter.) (*Ans.* 8 6/7 miles nea

149. A sugar-boiler does not skim the juice when
ing and thereby obtain 3 per cent more of jaggery.
dirty jaggery fetches Rs. 3-4-0 a maund, while if he
skimmed he would have got a clean jaggery which he w
have sold at Rs. 3-10-0 a maund. How much does he
in 100 maunds? (*Ans.* Rs. 36-1

150. A spent Rs. 150 on his lands and got back Rs
by way of produce. B spent on the same area Rs. 60
got back Rs. 230. Which made the bigger profit? W
got a greater return for the capital invested?

(*Ans.* The first by Rs. 20 the second 157 per c

151. The following table shows the average amou
nitrogen in 100 parts of the excrements of certain ani

	Nitrogen in dung.	Nitrogen in urine.
Sheep	0·7	1·4
Horse	0·5	1·2
Cow	0·3	0·8

By what per cent is nitrogen richer in the dung of
sheep than in that of the horse, in that of the horse th
that of the cow?

Answer the above question with reference to the urine. How many times the quantity of nitrogen in the dung is that in the urine of each animal?

(*Ans.* 40 per cent, 60·7 per cent, 16·7 per cent,)
50 per cent, twice 2·4 times and 2·7 times.)

152. Stored green-gram if infected with a grain weevil used to lose 10 per cent in weight. If fumigated with carbon-bisulphide, it lost only 2 per cent in weight. How many lb. of green-gram does a ryot gain by fumigating his stored green-gram of 500 kunchams which would otherwise have been infected with the grain weevil? Assume that one kuncham of green gram weighs 9¾ lb. and the cost of fumigating is ½ a kuncham. (*Ans.* 380 lb.)

153. 16⅔ per cent of the harvested crop of paddy was destroyed by stem-borers. If the harvested crop was 600 kunchams, find the quantity destroyed by the stem-borers. (*Ans.* 100 kunchams.)

154. Sweet potato-weevil destroyed 15 per cent of the crop of potatoes in a field. If the field would have produced 1,000 lb. of potatoes without the destruction, calculate the loss incurred in actual production.

(*Ans.* 150 lb.)

155. It has been calculated that a sugar-cane plantation will yield 60 puttis of cane if *B. 208* be planted or 75 puttis of cane if Red Mauritius be planted. By what per cent would the yield increase by cultivating Red Mauritius instead of *B. 208*? What would be the increase in the value of jaggery if B. 208 jaggery sells at Rs. 5 more a putti than Red Mauritius? (*Ans.* 25 per cent.)

156. Buffaloes' milk contains water 82 per cent, fat 8 per cent, proteid 4 per cent, sugar 5 per cent and ash 1 per cent. What is the weight of each ingredient in 67½ seers of the milk? [The seer here is a local unit of weight].

(*Ans.*—55·3 seers, 5·4 seers, 2·7 seers., 3·4 seers, ·7 seer.)

157. In castor-cake 6·4 per cent by weight of the manure is nitrogen, 2·6 per cent is phosphoric acid and 1·2 per cent is potash. Find the weight of each ingredient in 1,500 lb., of the cake. (*Ans.* 96, 39, 18 lb.)

158. Eight lb. of buffaloes' milk gives 1 lb. of cream and 12 lb. of cows' milk gives the same amount of cream. Find what per cent of each milk is cream?

(*Ans.* $12\frac{1}{2}$ per cent, $8\frac{1}{3}$ per cent.)

159. Seventy lb. of buffaloes' milk gives 3 lb. of ghee. Find the percentage by weight of ghee that could be obtained.

(*Ans.* 4·3 per cent nearly.)

160. Four lb. of buffaloes' milk gives 12 oz. of cream and 4 oz. of butter. In cream 30 per cent is butter and the rest butter-milk. Find the quantity of butter that could be obtained from 124 lb. of the milk.

(*Ans.* 14 lb. 12 oz. nearly.)

161. A man buys a sugar-mill for Rs. 180 and in undertaking to prepare jaggery from canes supplied by cultivators gains every year 10 per cent of his investment. What is his monthly gain? (*Ans.* Rs. 1–8–0.)

162. In a mango-garden containing 240 trees 75 were badly infested with hoppers and though each tree required only the normal quantity of solution—4 gallons and the usual number of operations 7—once every week, it had to be sprayed with 25 extra doses of the insecticide each time; 100 trees received the normal doses as usual; of the remaining trees 35 were found less infected with insects and it was found sufficient to give the trees 4 periodical applications instead of 7, but being bigger than the average size trees each received 5 gallons of the mixture each time. The remaining trees received but one application, each at 4 gallons. What did it cost the gardener on the insecticide for the whole garden and what was the average cost of the tree on the whole, the cost of insecticide being Rs. 17–8–0? (*Ans.* Rs. 97–9–3, Re. 0–6–6.)

163. There are 30 families of cowherds in a village and each family produces 2 viss of ghee every month. Each family employs a man and pays him 4 annas a day to take the ghee to the neighbouring town and sell it. It takes 2 days for each man to sell the ghee and return to his village. All the ghee can be carried by one man in a single cart (which costs 12 annas a day) over and above a man's daily wages and it will take him 4 days to sell the ghee and return. Which of the systems of carriage will be the more expensive and by how much? (*Ans.* Rs. 11.)

164. A family purchases its monthly requirements in a co-operative stores, the father holding 3 shares of Rs. 5 each, the mother 1 share and each of the three adult children one share. Their monthly purchases amount to Rs. 30 per mensem, on which the society allows a bonus at three pies per rupee. If a dividend of 5 per cent is declared at the end of the year, what will the family as a whole earn by way of dividend and bonus for a year?

(*Ans.* Rs. 7-6-0.)

165. The price of a pair of dhoties in a village is Rs. 1-8-0. The price of 120 pairs ot dhoties at the neighbouring town is Rs. 140. The price of a bottle of kerosine oil in the village shop is Re. 0-1-6 and the price of two tins of kerosine oil in the town is Rs. 3-8-0. There are 24 bottles of kerosine oil in each tin. One hundred members of the co-operative society require one pair of dhoties and four bottles of kerosine oil each and they request members of panchayat to purchase these at the neighbouring town. The panchayatdars accordingly purchase these dhoties and the oil in the town and bring them in a cart to the village and distribute them to the members. The cost of carrying 100 pairs of dhoties and 20 tons of kerosine oil in a cart from the town to the village is Rs. 3-2-0. What profit has been made by buying in this manner instead of in the local shop?

(*Ans.* Re. 0-6-2 ; Rs. 38-8-0.)

166. The price of a viss of 3 lbs. of Cambodia cotton seed in the retail shop in a village is Re. 0-2-3. The price of a bag of 100 lb. at a big shop in a neighbouring town is Rs. 3-10-4. The price of a measure of indigo seed is Re. 0-5-0 in the village and a bag of 40 measures is selling at Rs. 10 in the town. Fifty ryots of the village want to buy 10 lb. of cotton seed and a bag of indigo seed each and they send an agent to make the purchase in the town and he buys and distributes the things among the ryots. The cost of carriage comes to Rs. 9-6-0. What profit has been made by each and by all altogether by buying in this manner instead of at the local shop?

(*Ans.* Rs. 2-6-8, Rs. 120-13-4.)

167. The selling price of sweet potatoes in a village is Rs. 6 per bag of 200 lb. An officer of the Army Supply Department offers to buy at Rs. 8 per bag if the villagers

can sell not less than 5,000 bags. Fifty members of the co-operative society in the village agree to supply the quantity and each supplies 100 bags. The 5,000 bags are delivered to the Military office and the officer pays Rs. 40,000. What is the profit made by each member and by all together in selling the produce jointly instead of individually in the village market?

(*Ans.* Rs. 200, Rs. 10,000.)

168. There are 254 cultivators in a village requiring selected seed for sowing. If each of them goes to a distant farm where selected seed is sold, buys the seed and spends 3 annas extra and 4 hours in transit, how much of money and time could be saved by the whole village if one of them goes and spends 4 hours and Rs. 2 in bringing the seed in a cart? (*Ans.* Rs. 45-10-0, 1,012 hours.)

169. By providing for proper drainage the yield of a piece of wet land increased by 80 per cent. The drainage arrangement costs Rs. 5 per acre. If before providing for drainage the land yielded 200 kunchams, what was its yield after drainage? Also calculate the net gain at 3 kunchams a rupee. (*Ans.* 360 kunchams, Rs. 48-5-4.)

170. Five-eighths of the manure given by cattle is liquid and is generally wasted by cultivators. What is the percentage of liquid in the manure given by cattle? If in spite of such wastage a cultivator gets Rs. 70 worth of manure from his cattle, find how much more he would get by preserving liquid manure. Assume that the cost of manure is proportional to the quantity of liquid or solid contained in it. (*Ans.* 62½ per cent, Rs. 18-10-8.)

171. The following particulars are about the local and improved methods of butter-making:—

	Milk taken.	Butter.	Ghee.
Local method	... 12 lb.	... 6 tolas	... 4 tolas.
Improved method ...	12 lb.	... 8 tolas	... 6 tolas.

Find the percentage of butter to milk and that if ghee to milk in each case. Also find the percentage of increase of ghee in each case in the improved method over that obtained by the local method.

(*Ans.* 1¼ per cent, ⅚ per cent, 1⅔ per cent, 1¼ per cent, 50 per cent, increase in ghee.)

172. A ryot borrowed Rs. 100 from a co-operative bank at the rate of 1 pie per rupee per month. Another ryot borrowed the same amount from a sowcar at Rs. 1-8-0 per cent, per month. At the end of each year, each pays off the interest due and one-fourth of the principal. What sum is paid as interest by each?

(*Ans.* Rs. 15-10-0; Rs. 45.)

173. A ryot buys a plot of land for Rs. 8,500. From *Ashada Sudda Panchami* to *Karthikai Purnami* the land is under Maharajabogam and yields 600 kunchams of paddy worth 6 annas a kuncham. The ryot follows it by gingelly at the beginning of Pushyam up to the end of Phalgunam and gets 180 kunchams worth 12 annas a kuncham. Assuming the cost of cultivation to be Rs. 110 and that there are no other crops raised during the year, calculate the rate of interest the ryot realizes from his lands.

(*Ans.* 5 per cent per annum.)

174. A plantain tree (*Karpurachakara Keli*) yields a bunch of 155 on the average in the course of a year. A ryot has 150 plantain trees in his field. At the end of the 8th month after the cultivation he feels the necessity for money and hence sells in advance at 8 annas per hundred the plantains he could supply. If at the end of the 12th month the sale price per 100 is Rs. 1-4-0 what sum of money would he have gained if there was a co-operative bank which would have lent him money at *Muppavala Vaddi*, without forcing him to sell the plantain crop in advance at the low rate? (*Ans.* Rs. 170-14-2.)

175. Selected seed sufficient to sow 200 acres was sold in one season. If this would provide seed for 20,000 acres next year, calculate at this rate how many acres could be sown with selected seed which would be obtained thereby at the end of two years. (*Ans.* 2,00,000 acres.)

176. A ryot had cattle worth Rs. 350. Owing to the shortage of food and fodder he mortgaged them and purchased provisions on credit at 65 per cent over and above the price he may have to pay for ready money. If their ready money price was 150 rupees and he redeemed the cattle after two months on repayment of the loan at 36 per cent interest per year, how much did he then pay? If there was a co-operative bank lending money at 1 dabbu (4 pies) per rupee per month, how much would he have gained? (*Ans.* Rs, 262-4-7, Rs. 106-1-7.)

177. When land is served with bone-meal, the value of the manure depreciates by 30 per cent during the first year, by 40 per cent during the second year and 25 per cent during the third year. A garden is served with 2 tons of bone-meal at the beginning of each of two years. Find the quantity of bone-meal contained in the garden soil at the end of the third year. (*Ans.* 1,955¼ lb.)

178. In a cane cutting factory it has been found that in 45 hours 135 puttis of cane were crushed and 12½ per cent of it was obtained as jaggery. Find in what time the factory would crush one putti of jaggery and how many puttis of canes should be crushed for the purpose?

(*Ans.* $2\frac{2}{3}$ hours, 8 puttis of jaggery.)

179. Wire-netting with meshes of particular dimensions and net 3 feet wide costs 14 annas a yard length. A field of sugarcane is 200 yards long and 100 yards wide. If wire-netting should be provided to a height of 6 feet from the ground-level, find the cost of the net required.

(*Ans.* Rs. 1,050.)

180. Company cotton gives 30 per cent lint to Kappas. Village cotton gives only 25 per cent. How many candies of each variety of cotton should be secured to get 3 candies of lint?

(*Ans.* 10 candies of company cotton and 12 candies of village cotton.)

181. If the yield of company cotton is 20 per cent higher, how many acres of village cotton would you have to sow to get the same yield of lint as from 10 acres?

(*Ans.* $8\frac{1}{3}$ acres.)

182. A person borrowed Rs. 1,000 at one rupee per Rs. 100 per month, interest payable at the end of each period of 6 months. At the end of a half year he paid away Rs. 450. What amount should he pay at the end of the year to clear his debt? (*Ans.* Rs. 646–9–7.)

183. Mr. Satyanarayana borrowed Rs. 800 from Mr. B. Narayanamurti at 1½ per cent per month with a view to raising sugarcane cultivation in his lands. Twelve months hence he thought of repaying the debt with the interest due, but as he had to meet marriage expenses he was not able to repay the loan and hence a new pronote was executed to the effect that Satyanarayana had borrowed afresh the amount due by the old loan, and the old pronote

was then destroyed. If 11 months after the second pronote was executed, Mr. Satyanarayana clears his debt find how much he may have to repay ? (*Ans.* Rs. 1,046-8-0.)

184. The value of a sugarcane mill at the end of a year is 8 per cent less than that at the beginning of the year owing to its wear and tear. If at the beginning of a year the mill was valued at Rs. 150 find how much it may be valued at the beginning of the second year and how much at the beginning of the third year.
(*Ans.* Rs. 138, Rs. 126-15-4.)

185. Gourisam bought lands for Rs. 10,000. He paid only Rs. 7,000 cash. From that time he began to pay Rs. 1,500 at the end of each half-year in addition to the interest then due at 1½ rupees per Rs. 100 per month. Calculate how much he would be paying at the end of the first half-year, and at the end of the first-year. (*Ans.* Rs. 1,770, Rs. 1,635.)

186. For a payment of Rs. 75 the Post Office issued a war loan certificate which entitled the holder to get Rs. 100 at the end of five years. What rate of simple interest per cent per annum does this come to on money invested ? (*Ans.* 6·7 per cent nearly.)

187. If you remit Rs. 95 to-day you would be credited with having lent Rs. 100 to the Government of India at 5 per cent interest per annum. What rate of interest per cent per annum does this work out on the money you invest ? (*Ans.* 5·3 per cent nearly.)

188. The water lifted from a well flows along a long channel to the field. 10 per cent is lost from the leakiness of the bucket and 15 per cent from seepage in the channel. On an average 160 cubic feet an hour is lifted from the well. How much of water will be lost in the course of seven days of 8 working hours each ? (*Ans.* 2,240 c.ft.)

189. To prepare mortar 3 baskets of sand are mixed with one basket of lime. What is the quantity of lime and sand required for 100 cubic feet of mortar ?
(*Ans.* 75 c.ft. of sand, and 25 c.ft. of lime.)

190. An acre of paddy cultivation in wet lands will give ¼ garisa of paddy and 4 cart-loads of straw. The tenant (*Raiyatu*) will give the landlord (*Bhukamanthu*)

half the produce of paddy alone. If a garisa of paddy sells at Rs. 150 and a cart-load of straw sells at Rs. 7–8–0 find the money value of the share obtained by the landlord as well as by the tenant from an acre of cultivation. (*Ans.* Landlord Rs. 75, Tenant Rs. 105.)

191. The quantity of water baled out by a $2\frac{1}{2}$ centrifugal pump is 20 cubic feet per minute. Find out what depth of water will be supplied to $1\frac{1}{4}$ acres of paddy in a day of 8 working hours. (*Ans.* 2″.)

192. A garisa of lime contains 10 cart-loads and sells at Rs. 12–8–0. For putting on Bengal terrace 10′ by 10′ a person goes in for 1,500 flat tiles at Rs. 3–8–0 a thousand, half a cart-load of lime, spends 8 annas for grinding the lime and engages two people for a day at 8 annas each for other work. Find the total cost incurred.

(*Ans.* Rs. 3–14–0.)

193. A putti of sugarcane gives $\frac{3}{4}$ putti of juice and from the juice is obtained 1/8 putti of jaggery. How much of jaggery could be obtained from a putti of juice ?

(*Ans.* 1/6 putti of jaggery)

194. The following table gives the prices which *mangoes of different varieties fetch in a market :—*

Name of variety.		Cost of a single fruit.			Cost per 100.		
		RS.	A.	P.	RS.	A.	P.
Jehangir ...	...	1	0	0	...		
Collector ...	...		...		8	0	0
Swarnareka	...		...		6	0	0
Rasumas ...	...		...		3	0	0
Anders ...	...		...		15	0	0
Bangenpalle	...		...		20	0	0
Uvas ...	...		...		12	0	0
Gannern ...	...		...		4	0	0
Baramas for pickles			...		3	8	0

A landlord gets *from his gardens 18 mangoes of* Jehangir variety, 150 of Collector, 250 of Swarnareka, 500 Rasumas, 800 of Anders, 1,500 Bangenpalle, and 1,600 of each of the remaining varieties. If he disposes them off at the above rates, how much would be realise ?

(*Ans. Rs. 792.*)

195. The villagers' estimate for the straw obtained from a crop is as follows :—

If a putti of grain is obtained there would be $1\frac{1}{2}$ puttis of straw. A ryot gets 12 garisalu of paddy. Find how many cart-loads of straw he may be expected to get. (Assume 30 puttis for a cart-load.)

(*Ans.* 18 cart-loads of straw.)

196. A groundnut factory pays Rs. 3-8-0 for a bag of 90 lb. of groundnut kernel. The weight of oil extracted is 9/20 of the weight of the powdered kernel supplied and the rest is cake. The factory sells the cake at Rs. 6-4-0 for a bag of 164 lb. and the oil at Rs. 120 for a candy of 500 lb. How much of money does it realize for its labours in buying 4,100 bags of 90 lb. each of kernel, extracting oil and cake and disposing them off ?

(*Ans.* Rs. 33,236-6-0.)

197. Given the span of a gable QR = 12 feet, the carpenter wants to find what length to choose for a rafter PQ. He constructs a right angled triangle ABE, BE = 6" AE = 4", angle AEB = 90°. Explain how by measuring AB he could calculate the length of PQ. Actually draw such a triangle and calculate the length of the rafter required.

198. If an oil-mill be erected at a cost of Rs. 3,000 and a working capital Rs. 70,000 be had for running the mill, the annual income from the oil and cake obtained by crushing gingelly, groundnut and mustard alone would amount at the current market rate to Rs. 4,35,600. If the cost of extraction amounts to Rs. 3,98,428, what is the balance left as yearly savings ?

What per cent dividend would be declared thereby on money invested ? (*Ans.* Rs. 37,172 ; 50·9 per cent.)

199. A *'Jaha'* in the Anakapalle town means a building plot 10 yards long and 8 yards wide. It is normally valued at Rs. 200. B. Hanumantha Rao thinks of disposing of a plot 24 yards long and 20 yards wide at the above rate. How much may he expect to get by selling the plot ? (*Ans.* Rs. 1,200.)

200. An *Agraharam* consists of 16 *Vrithis* and a *Vrithi* is subdivided into 16 visams. A ryot owns 2 visamulu in an agraharam of 1,800 acres. How many acres

of land does he actually own? (Express fractions of an acre to the nearest cent.) (*Ans.* 10 acres 16 cents.)

201. To raise the foundation for a wall 24 feet long, 3 feet wide and 8 feet high, rough stone is supplied at Rs. 3 a cubic yard and Rs. 6 extra is charged for a cubic yard as working charges. What will the total charges come to for raising the foundation for the wall? (*Ans.* Rs. 128.)

202. A merchant buys chillies at Rs. 3-8-0 a maund and sells them at 4 kunchams a rupee. If a maund contains 20 kunchams, how much would he gain in buying and selling 20 maunds? (*Ans.* Rs. 30.)

203. A merchant buys 100 maunds of Anakapalle sugar at Rs. 2-12-0 a maund, sends the sugar to Samalkota at a cost of 1 anna a maund, brings it back to the Anakappalle market paying the same charge, passes it off as Samalkota sugar and sells it at Rs. 3-6-0 a maund. How much does he gain on the whole? (*Ans.* Rs. 50.)

204. A landlord buys 50 bags of castor-cake of $1\frac{1}{2}$ cwt. each at Rs. 5 a bag on the 1st of March when the cake is cheap. He pays an interest of Re. 1 per cent for the money he requires. On July 1st when he has to apply the manure to his fields he finds that there is a dryage of 5 per cent in the cake he has stored and at the same time notes that a bag then sells at Rs. 8 in the market. Has he gained or lost by purchasing in advance and by how much? (*Ans.* Gained Rs. 120.)

205. The cost of insecticide has to be reckoned for spraying 100 trees in a mango garden. The Entomologist says that the quantity required for each tree comes to 3 lb. of fish oil soap and that 1 cwt. costs Rs. 16. At this rate calculate the total cost of the insecticide required for the garden. (*Ans.* Rs. 42-13-9.)

206. By using a wooden mill for extracting juice it is found that only 66 per cent of the juice is extracted while by using an iron mill 70 per cent of the juice is extracted. Find the wastage in juice in crushing 10,000 canes each weighing 3 lb., on the average by the wooden mill instead of the iron mill. (*Ans.* 1,200 lb. of juice.)

207. The following details refer to some fertilisers.

Name of fertilizer.		Percentage of nitrogen	Price for a bag of 1½ cwt.
			Rs. A. P.
1. Pungam cake ...	...	3	2 8 0
2. Margossa cake ...	...	3½	3 12 0
3. Black-castor cake	...	5	6 0 0
4. Groundnut cake	...	7	6 0 0
5. White castor cake	...	7	10 0 0
6. Fish guano ...	...	8	10 0 0

I want to prepare a fertilizer to supply 25 lb. of nitrogen at a cost not exceeding Rs. 20. Which of the fertilizers may I choose ?

(*Ans.* Any of the fertilizers except castor-cake.)

If I want to apply 25 lb. of nitrogen to an acre of land, find how much of each fertilizer I should take and what the cost would come to.

(*Ans.* 833 lb. of Pungam cake, Rs. 12-6-5 ; 714 lb. of margossa cake, Rs. 16–12–10 ; 500 lb. of black castor-cake, Rs. 17–13–9 ; 357 lb. of groundnut cake, Rs. 12–12–1 ; 312 lb. of fish guano, Rs. 18–9–7.)

208. There is a ridge in the middle of a field 300 feet long and 50 feet wide with an average height of 2½ feet. It has to be removed. For manual labour the coolies charge at 2 annas a cubic yard. How much would the labouring charges then come to ? (*Ans.* Rs. 173–9–9.)

A buckscraper carrying 4½ cubic feet each trip with a pair of bullocks and one man makes 30 trips in an hour. After working for how many hours could it remove the entire ridge ? If the owner of the buckscraper charges Rs. 1–8–0 per day of 8 hours for allowing the buckscraper being used and also for sending in a man and a pair of bullocks, in how many days could the whole ridge be removed and what would the total charges come to ?

(*Ans.* 278 hours, 34¾ days, Rs. 5,242.)

209. A pair of calves, a year old, just weaned from the mother is purchased for Rs. 18 in the market, is fed for 3 successive years at Rs. 2 a month on the average in good pasture land and sold at the end of the period for Rs. 200. How much does the rearer get for his labours ?

(*Ans.* Rs. 110.)

.210. A milk-vendor bought a buffalo for Rs. 80. The buffalo gave milk for 9 months after the date of purchase and then dried off. The average daily yield was 4 seers worth 4 annas a seer and the cost of feeding came to 8 annas a day. When she became dry, she was sold for Rs. 30. How much does the vendor gain or lose, in having gone in for the buffalo? (*Ans.* gain Rs. 85.)

211. Another milk-vendor bought the above buffalo for Rs. 30. She calved 5 months hence, when she was valued at Rs. 100. The cost of maintaining during the period came to Rs. 10 per month. How much does he gain on the transaction? Which do you think to be more economical:—to dispose the buffalo off as the previous milk-vendor or retain her without selling?

(*Ans.* Rs. 20. The latter.)

212. With a well of 20 feet lift worked with a single mhote a ryot finds that an acre of land is irrigated in 3 days with a working day of 8 hours. He knows that his garden crops require an irrigation once in 12 days. How many acres of garden crops could he raise with the above well? (*Ans.* 4 acres.)

If he has a garden containing 8 acres, how many such wells should he have? (*Ans.* 2 wells.)

213. A bag contains 500 rupee-coins. How many visams would a man be lifting when he carries the above bag? (*Ans.* $4\frac{1}{6}$ visams of 120 tolas each.)

214. A ryot finds that he requires 3 cart-loads of maize or straw to feed a pair of bullocks for a year and 4 cart-loads to maintain a pair of buffaloes for the same period. If he has 3 pairs of buffaloes and 4 pair of bullocks find how many cart-loads of maize stalk or straw he should procure to feed his animals for the year. Also find the value of the fodder-stuff at Rs. 7–8–0 a cart-load.

(*Ans.* 25 cart-loads, Rs. 187-8-0.)

215. In the farm it is found that a pair of bullocks require 30 lb. of straw a day. It is known that a ton of straw comes to $2\frac{1}{2}$ cart-loads and an acre of average crop will give 5 cart-loads of straw. At this rate how many acres of paddy land should a ryot cultivate so that he may get enough straw for his 4 pairs of bullocks during a year?

(*Ans.* $9\frac{3}{4}$ acres nearly.)

216. A ryot finds that green chillies sell at 3 annas a viss. Instead of selling at this rate he gets them dried and stored. The loss in dryage is 75 per cent and the cost of labour in drying and storing is 1 anna per viss. If afterwards he sells the chillies at 14 annas a viss, find his gain or loss from a crop of 2 puttis of chillies. (*Ans.* Rs. 30.)

217. Paddy seed with a germination capacity of 96 per cent sells at 7 seers a rupee and another with a germination capacity of 50 per cent sells at $8\frac{1}{4}$ seers a rupee. Which of the two varieties is more economical to buy?

(*Ans.* The former.)

218. A ryot finds that 50 seers of seed are required for his lands when the seed has a germination capacity of 96 per cent. If he is able to get seed which has a germination capacity of 60 per cent, how much of the latter variety should he sow to get sufficient seedlings for his lands?

(*Ans.* 80 seers.)

219. In a garden there were 12 mango trees which were sprayed at a cost of 3 annas per tree. The out-turn of mangoes from them came to 5,590. In the same garden 12 more mango trees were left unsprayed and though they were of the same type the out-turn of mangoes from them came to 1,010 because of the mango-hopper pest. Valuing mangoes at Rs. 4 per 100, find what net profit is got from the first 12 trees. How much may the gardener be expected to have lost if he had not sprayed these 12 trees, and how much may he have expected to get if he had sprayed all the 24 trees? (*Ans.* Rs. 180–15–2, Rs 442–11–2.)

220. A merchant buys 500 bags of paddy at Rs. 6 a bag when paddy is cheap, stores it in a granary paying a monthly rent of Rs. 5 and disposes of the bags 10 months hence. If there has been a shortage of 10 per cent in the paddy owing to insect damages, etc., at what price should he sell so as to gain Rs. 200 on the whole?

(*Ans.* Rs. 7–3–7 a bag.)

221. A ryot sends 4 servants and 5 carts to the Madgole Forest, obtains permit and gets 2,000 bamboos after incurring permit charges at Rs. 2 per thousand bamboos and cutting charges and delivery at cart-stand at Rs. 5 per thousand. This takes him 12 days, during which he pays his servants at 4 annas a day and the cart-owners 12 annas a day. Another ryot purchases bamboos at Rs. 44 per

thousand in the Thummapala market. How much d the former ryot gain by getting the bamboos direct fr the forest instead of buying in the market?

(*Ans*. Rs. 1]

222. In a ganti-crop when it is young, 10 men necessary for thinning an acre in a day. But by us a Sampson harrow and a pair of cattle to work in the fie 5 acres could be done in a day. If 20 acres of the c have to be so thinned, which of the two would cost less a by how much, the employment of men alone or the of the harrow? (Assume the wages of a man to be 4 an a day and the charges for hiring a cattle pair with a n to drive to be Rs. 1-4-0 a day).

(*Ans*. The former by Rs.

223. In a horse-gram crop, some creeping weeds found to grow which bear seeds just like horse-gram. B horse-gram and the weeds are threshed out and the s of the weeds forms 25 per cent of the produce obtain If the produce obtained sells at 7 annas a kuncham a pure horse-gram at 9 annas a kuncham, find how muc ryot would gain if he took care to remove the weeds time at a cost of Rs. 2-2-0 on the whole and got only kunchams of pure horse-gram instead of 200 kunchams the mixed crop. (*Ans*. Rs. 2

224. It costs Rs. 18 to remove *hariali* by crow-barri from an acre of lands during the summer. By using improved plough the grass could be removed at a cost Rs. 8 from an acre of lands A ryot has 4½ acres of la from which he has to remove the grass. How much m will he have to spend in crow-barring than in using an i proved plough? (*Ans*. Rs. 5

225. In puddling it has been found that a coun plough could do ½ an acre in a day. But if t lands be swampy and dangerous for cattle and hen men be set to dig with spade, it is found that 20 m can do an acre in a day. Which of these two methods cheaper to adopt, and by how much in 4½ acres of n swampy lands? Assume the daily wages of a man to 4 annas and the daily hire for a ploughing pair 12 annas.

(*Ans*. The former by Rs. 10-6-

226. An ordinary three-roller sugarcane mill has be found to extract juice from sugarcane to the extent of

per cent whereas Mohen's mill extracts juice to the extent of 69·5 per cent. By crushing a ton of cane, how many lb. more of juice could be obtained by using Mohen's mill instead of an ordinary three roller mill? If 1 ton of juice gives 1/6 ton of jaggery and a maund of 25 lb. of jaggery sells at Rs. 3-2-0, estimate the increase in value of the jaggery that may be obtained from the above juice.

(*Ans.* 168 *lb.* Rs. 3-8-0.)

227. Fish-manure (beach-dried) mixed with sand sells at Rs. 50 a ton whereas the quality unmixed with sand sells at Rs. 60 a ton. If the sand so mixed with the manure is 1/5 of the total weight of the manure, find which is cheaper to go in for, the former or the latter.

(*Ans.* The latter.)

228. A ryot sells his groundnuts by wetting and gets at the rate of Rs. 20 per candy of 500 lb. Another ryot sells thoroughly dried stuff at Rs. 24 for a candy. The precentage of loss in drying is 10. If he gets 10 candies of groundnuts from his fields. find how much he would gain by selling them after drying. (*Ans.* Rs. 16.)

229. A ryot cleans and dries his ragi and then sells it at Rs. 10 for a bag of 166 lb. whereas another ryot who has not cleaned and dried his stuff properly sells at Rs. 9 a bag. There is a loss of 5 per cent in weight in cleaning and drying. Find how much a ryot would gain by cleaning and drying before selling 20 bags. (*Ans* Rs. 9.)